Physics Beyond the Standard Model — Exercise Sheet 4

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1. **Analysis of the general vacuum of the 2HDM**

   Literature: J.L. Diaz-Cruz, A. Méndez, Nucl. Phys. B380 39 (1992)

   After a proper $SU(2) \times U(1)$ rotation we obtain the most general form of the VEV of the two Higgs-doublets, Φ_1 and Φ_2

$$\langle \Phi_1 \rangle_{\mathrm{V}}^{\mathrm{T}} = (0, v_1), \qquad \langle \Phi_2 \rangle_{\mathrm{V}}^{\mathrm{T}} = (u, v_2 e^{i\xi}) \tag{1}$$

   where v_1, v_2, u and ξ are real ($v_1 > 0$). We consider the CP-conserving and soft Z_2 breaking potential

$$\begin{aligned} V^{\mathrm{2HDM}} &= m_{11}^2 \Phi_1^\dagger \Phi_1 + m_{22}^2 \Phi_2^\dagger \Phi_2 + m_{12}^2 (\Phi_1^\dagger \Phi_2 + \Phi_2^\dagger \Phi_1) \\ &+ \lambda_1 (\Phi_1^\dagger \Phi_1)^2 + \lambda_2 (\Phi_2^\dagger \Phi_2)^2 + \lambda_3 (\Phi_1^\dagger \Phi_1)(\Phi_2^\dagger \Phi_2) \\ &+ \lambda_4 (\Phi_1^\dagger \Phi_2)(\Phi_2^\dagger \Phi_1) + \frac{1}{2}\lambda_5 ((\Phi_1^\dagger \Phi_2)^2 + (\Phi_2^\dagger \Phi_1)^2) , \end{aligned} \tag{2}$$

   and write the Higgs doublets with 8 real scalar fields

$$\Phi_1 = \begin{pmatrix} \phi_1 + i\phi_2 \\ v_1 + \phi_3 + i\phi_4 \end{pmatrix}, \qquad \Phi_2 = \begin{pmatrix} u + \phi_5 + i\phi_6 \\ v_2 e^{i\xi} + \phi_7 + i\phi_8 \end{pmatrix}, \tag{3}$$

   where $\langle \phi_i \rangle_{\mathrm{V}} = 0$.

   (a) Verify the conditions for the potential extrema obtained from

$$\left. \frac{\partial V}{\partial \phi_i} \right|_{\phi_k = 0} = 0 \, . \tag{4}$$

   i. $u(m_{12}^2 + (\lambda_4 + \lambda_5) v_1 v_2 \cos\xi) = 0$
   ii. $u v_1 v_2 \sin\xi (\lambda_4 - \lambda_5) = 0$
   iii. $m_{11}^2 v_1 + m_{12}^2 v_2 \cos\xi + 2\lambda_1 v_1^3 + \lambda_3 v_1 (u^2 + v_2^2) + v_1 v_2^2 (\lambda_4 + \lambda_5 (\cos^2\xi - \sin^2\xi)) = 0$
   iv. $v_2 \sin\xi (m_{12}^2 + 2\lambda_5 v_1 v_2 \cos\xi) = 0$
   v. $u(m_{22}^2 + 2\lambda_2 (u^2 + v_2^2) + \lambda_3 v_1^2 = 0$
   vi. $m_{22}^2 v_2 \cos\xi + m_{12}^2 v_1 + 2\lambda_2 v_2 (u^2 + v_2^2) \cos\xi + \lambda_{345} v_1^2 v_2 \cos\xi = 0$
   vii. $v_2 \sin\xi (m_{22}^2 + 2\lambda_2 (u^2 + v_2^2) + (\lambda_3 + \lambda_4 - \lambda_5) v_1^2) = 0$

   (b) Find the conditions for v_1, v_2, u, and ξ satisfying the equations above and discuss the cases for $m_{12}^2 = 0$ and $m_{12}^2 \neq 0$.

   (c) What is the symmetry of the vacuum if $u = 0$ and if $u \neq 0$? (*i.e.* which linear combination(s) of the four generators T^a and Y (with $a = 1, 2, 3$) annihilates the vacuum $\langle \Phi_{1,2} \rangle_{\mathrm{V}}$). What is the consequence for the gauge boson masses?

2. **Flavour Aligned Two-Higgs-Doublet Model (ATHDM):**
Another way to prevent the Higgs-mediated FCNC is to relate Σ^f to Δ^f by introducing free parameters η^f $(f = u, d, l)$. In the Higgs basis ($\beta = 0$ fixed) the Yukawa Lagrangian is

$$-\mathcal{L}_{\rm Y} = \bar{Q}^0(\Sigma^u\Phi_v^c + \Delta^u\Phi_\perp^c)u_{\rm R}^0 + \bar{Q}^0(\Sigma^d\Phi_v + \Delta^d\Phi_\perp)d_{\rm R}^0 + \bar{L}^0(\Sigma^l\Phi_v + \Delta^l\Phi_\perp)e_{\rm R}^0\,, \quad (5)$$

where

$$\Phi_v = \begin{pmatrix} G^+ \\ \frac{1}{\sqrt{2}}(v + S_1 + iG^0) \end{pmatrix}, \quad \Phi_\perp = \begin{pmatrix} H^+ \\ \frac{1}{\sqrt{2}}(S_2 + iS_3) \end{pmatrix}.$$

$Q^0 = \binom{u_{\rm L}^0}{d_{\rm L}^0}$, $L^0 = \binom{\nu_{\rm L}}{l_{\rm L}^0}$ and $f_{\rm R}^0$ are fermion weak eigenstates. Focusing only on the neutral scalar fields the Yukawa interaction Lagrangian in terms of the fermion mass eigenstates $f_{\rm L}$ and $f_{\rm R}$ can be written as

$$-\mathcal{L}_{\rm Y} \ni \sum_{f=u,d,l} \bar{f}_{\rm L} M^f_{\rm diag.} f_{\rm R} + \bar{u}_{\rm L}[Y_H^u H + Y_h^u h + iY_A^u A]u_{\rm R} + \sum_{f=d,l} \bar{f}_{\rm L}[Y_H^f H + Y_h^f h + iY_A^f A]f_{\rm R}\,. \quad (6)$$

Follow the steps given below and find out mass matrices $M^f_{\rm diag.}$ and the Yukawa coupling matrices $Y^f_{H,h,A}$.
a)Write only the vev and neutral scalar field terms in the the Yukawa Lagrangian Eq. 5 using

$$\Delta^u = \eta_u^*\Sigma^u\,, \quad \Delta^d = \eta_d\Sigma^d\,, \quad \Delta^l = \eta_d\Sigma^l\,.$$

b) The Yukawa interaction matrices Σ^f can be diagonalized by appropriate unitary matrices $V^f_{\rm L/R}$.

$$y^f = V_{\rm L}^f\Sigma^f V_{\rm R}^{u\dagger}\,,$$

where y^fs are diagonal matrices. The fermion mass eigenstates $f_{\rm L/R}$ are defined as $f_{\rm L} = V_{\rm L}^f f_{\rm L}^0$ and $f_{\rm R} = V_{\rm R}^f f_{\rm R}^0$. Focus only on the fermion mass terms, i.e. the terms obtained from $\mathcal{L}_{\rm Y}$ by replacing the Higgs fields by their vevs. Show that you can write them in the form

$$\mathcal{L}_{mass} = \sum_{f=u,d,l} \bar{f}_{\rm L} M^f_{\rm diag.} f_{\rm R}\,,$$

and find the relation between $M^f_{\rm diag.}$ and y^f.
c)The neutral Higgs boson fields h, H, and A are defined as

$$\begin{pmatrix} H \\ h \end{pmatrix} = R^\alpha \begin{pmatrix} S_1 \\ S_2 \end{pmatrix}, \quad A = S_3\,.$$

Show that the full Yukawa Lagrangian expresse in terms of the mass eigenstates $f_{\rm L/R}$ can be written as Eq. 6 and find the values of the Yukawa coupling matrices $Y^f_{H,h,A}$.