Physics Beyond the Standard Model — Exercise Sheet 6

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1. **MSSM Superpotential:** The superpotential of the minimal supersymmetric standard model (MSSM) with one generation of quarks and leptons is given by

$$W_{\text{MSSM}} = y_d H_d \cdot QD + y_u H_u \cdot QU + y_e H_d \cdot LE - \mu H_d \cdot H_u. \tag{1}$$

The appearing chiral superfields are the two Higgs doublets H_u, H_d, the left-handed quark doublet Q, the left-handed lepton doublet L, the right-handed quark and lepton singlets U, D, E. The notation of the component fields and further details are given in Tab. 1.

The Yukawa coupling parameters are $y_{d,u,l}$. The SU(2)-invariant dot product of SU(2) doublets is defined by $H_d \cdot Q = H_d^1 Q^2 - H_d^2 Q^1 = \epsilon_{ij} H_d^i Q^j$ etc.

a) Write down the corresponding Lagrangian for the terms involving fermions. Show that you get appropriate mass terms for quarks and leptons if you assume the vacuum expectation values (VEVs)

$$H_d = \begin{pmatrix} v_d \\ 0 \end{pmatrix}, \qquad H_u = \begin{pmatrix} 0 \\ v_u \end{pmatrix} \tag{2}$$

(Note these VEVs are annihilated by electric charge $Q = T^3 + Y$)

Question: how do the numerical values of the Yukawa couplings differ from the corresponding SM values? Consider in particular the case $v_u/v_d \approx 50$.

superfield	components	$SU(3)_C \times SU(2)_L \times U(1)_Y$
Φ	$A, \quad \psi$	
V	$\lambda, \quad A_\mu$	
Q	$\tilde{q}_L = \binom{\tilde{u}_L}{\tilde{d}_L}, \quad q_L = \binom{u_L}{d_L}$	$(3, 2, 1/6)$
U	$\tilde{u}_R^\dagger, \quad u_R$	$(3^*, 1, -2/3)$
D	$\tilde{d}_R^\dagger, \quad d_R$	$(3^*, 1, 1/3)$
L	$\tilde{l}_L = \binom{\tilde{\nu}_L}{\tilde{e}_L}, \quad l_L = \binom{\nu_L}{e_L}$	$(1, 2, -1/2)$
E	$\tilde{e}_R^\dagger, \quad e_R$	$(1, 1, 1)$
H_d	$H_d, \quad \psi_{H_d}$	$(1, 2, -1/2)$
H_u	$H_u, \quad \psi_{H_u}$	$(1, 2, 1/2)$
v'	$\lambda', \quad B_\mu$	$(1, 1, 0)$
V^a	$\lambda^a, \quad W_\mu^a$	$(1, 3, 0)$
V_s^a	$\lambda_s^a, \quad G_\mu^a$	$(8, 1, 0)$

Table 1: MSSM superfields, their components and charges. The first two lines show the generic notation given in the lecture. Note that we use the same notation for the Higgs superfields and Higgs scalar components. $SU(3)_C$-generators: 3: $T_s^a = \frac{\lambda^a}{2}$, 3^*: $T_s^a = -\frac{\lambda^*}{2} = -\frac{\lambda^T}{2}$ (with Gell-Mann matrices λ^a, $a = 1 \ldots 8$). $SU(2)_L$-generators: 2: $T^a = \frac{\sigma^a}{2}$. $SU(3)_C$ octet and $SU(2)_L$ triplet are adjoint representations given by structure constants.

b) Sketch the Feynman rules for the interactions between fermions and scalars (there are interactions involving Higgs scalars and interactions involving Higgsinos). What are possible decay modes of a Higgsino $\tilde{H}_d$ and of a Higgsino $\tilde{H}_u$?

c) Write down all the solutions of the equations of motion for all F-auxiliary fields. Then write down the Lagrangian for 3- and 4-scalar interactions (which results after eliminating the F-auxiliary fields). Sketch the Feynman rules.

d) Now replace the Higgs scalars by their VEVs given above, and determine all mass terms for stops (i.e. here: all terms bilinear in the $\tilde{u}_{L,R}$). Show that you get a non-diagonal mass matrix in the basis $\binom{\tilde{u}_L}{\tilde{u}_R}$.

2. **MSSM Higgs/Higgsino sector, Peccei-Quinn symmetry (3 points):**

   The superpotential of the minimal supersymmetric standard model (MSSM) with one generation of quarks and leptons is given by

$$W_{\text{MSSM}} = y_d H_d \cdot QD + y_u H_u \cdot QU + y_e H_d \cdot LE - \mu H_d \cdot H_u. \tag{3}$$

   For details see Sheet 6.

   a) Consider the superpotential in the limit $\mu = 0$. Show that in this limit the entire supersymmetric part of the MSSM Lagrangian has a global $\text{U}(1)_{\text{PQ}}$ symmetry[1] under which

$$H_u \to e^{i\alpha} H_u \qquad H_d \to e^{i\alpha} H_d \qquad D \to e^{-i\alpha} D \qquad \ldots \tag{4}$$

   (And provide transformations of all other superfields such that this transformation is indeed a symmetry!)

   b) Consider the soft SUSY breaking Lagrangian of the MSSM. Show that it is invariant under the same $\text{U}(1)_{\text{PQ}}$ symmetry in the limit $B\mu = 0$.

   c) In the vacuum the two Higgs doublets have vacuum expectation values

$$H_d = \begin{pmatrix} v_d \\ 0 \end{pmatrix}, \qquad H_u = \begin{pmatrix} 0 \\ v_u \end{pmatrix}. \tag{5}$$

   Is the $\text{U}(1)_{\text{PQ}}$ symmetry spontaneously broken? Note that if Yes, you would expect a massless Goldstone boson. For $\mu, B\mu \neq 0$ this would imply that one Higgs mass eigenvalue is proportional to $\mu, B\mu$.

3. **Proton decay and R-parity violation (2 points):**

   Write down a Feynman diagram which could give rise to proton decay in the MSSM with the following R-parity violating terms:

   a) Additional term UDD in the superpotential — what could be the final state of the proton decay?

   b) Additional term LQD in the superpotential — what could be the final state of the proton decay?

[1] Such a symmetry is commonly called Peccei-Quinn symmetry.

4. **Flavour violating soft SUSY breaking parameters (2 points):**

   Assume that the MSSM Lagrangian contains the soft SUSY breaking term

$$\mathcal{L}_{\text{soft}} = \ldots + (M_Q^2)_{32}\,(\tilde{t}_L^\dagger,\, \tilde{b}_L^\dagger)\begin{pmatrix}\tilde{c}_L\\ \tilde{s}_L\end{pmatrix} + h.c. \tag{6}$$

   and treat this $(M_Q^2)_{32}$ term as a small interaction (i.e. giving rise to bilinear Feynman rules). Show that there exists a Feynman diagram which gives contributes to the process

$$b \to s\gamma \tag{7}$$

   at the one-loop level proportional to $eg_s^2(M_Q^2)_{32}$, where g_s is the strong gauge coupling.

5. **MSSM Higgs potential (3 points):**

   The quartic terms of the MSSM Higgs potential are exclusively given by the D-terms for the $\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$ gauge groups (why?):

$$\mathcal{L}^D_{\text{Higgspot.}} \;=\; -\frac{g_W^2}{2}\left(H_d^\dagger T^a H_d + H_u^\dagger T^a H_u\right)^2 - \frac{g_Y^2}{2}\left(Y_{H_d} H_d^\dagger H_d + Y_{H_u} H_u^\dagger H_u\right)^2 \tag{8}$$

   where $T^a = \sigma^2/2$, the square implies a sum over a, and $Y_{H_u} = 1/2$, $Y_{H_d} = -1/2$. Note the following relations for doublets ϕ, ϕ_i:

$$\phi^\dagger\sigma^a\phi = -(\phi^C)^\dagger\sigma^a\phi^C, \qquad (\phi_1^\dagger\sigma^a\phi_1)\,(\phi_2^\dagger\sigma^a\phi_2) = 2\phi_1^\dagger\phi_2\,\phi_2^\dagger\phi_1 - \phi_1^\dagger\phi_1\,\phi_2^\dagger\phi_2. \tag{9}$$

   a) Identify $H_d^C = \Phi_1$ and $H_u = \Phi_2$ to map the MSSM Higgs sector to the one of the 2HDM, see e.g. Sheet 4. Argue first that the coefficients $\lambda_{1,2,3,4}$ of the general 2HDM Higgs potential from Sheet 4, eq. (2), are all proportional to $g^2_{W,Y}$. Argue also without calculation that $\lambda_5 = 0$.

   b) Determine the values of $\lambda_{1,2,3,4,5}$. Hint: some sample results are $\lambda_5 = 0$, $\lambda_1 = \lambda_2 \propto (g_W^2 + g_Y^2)$, $\lambda_4 \propto g_W^2$.

   Note: this calculation is the basis for the prediction that the lightest CP-even Higgs boson in the MSSM has a tree-level mass $\leq M_Z$, a result of utmost phenomenological interest.