Physics Beyond the Standard Model — Exercise Sheet 8

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1. **FCNC-free Yukawa Lagrangian**

   Show that no Higgs-mediated tree-level FCNCs appear if the Yukawa Lagrangian has the form

$$\mathcal{L}_{\text{Yuk}} = Y^d_{ij}\bar{Q}^0{}_i\mathcal{F}_d(\{\Phi_k\})d^0_{j\text{R}} + Y^u_{ij}\bar{Q}^0{}_i\mathcal{F}_u(\{\Phi_k\})u^0_{j\text{R}} + Y^l_{ij}\bar{L}^0{}_i\mathcal{F}_l(\{\Phi_k\})e^0_{j\text{R}} + \text{H.c.}\,, \qquad (1)$$

   where $\mathcal{F}_{u,d,l}(\{\Phi_k\})$ are functions of Higgs fields $\{\Phi_k\}$, which are $SU(2)_\text{L}$ doublets and possess the following properties

   i) independent of the fermionic flavour indices i, j

   ii) $\mathcal{F}_u$ transforming like an $SU(2)_\text{L}$ doublet with hypercharge $Y = -1/2$ and $\mathcal{F}_d$ and $\mathcal{F}_l$ with hypercharge $Y = 1/2$.

   Here i, j are generation (flavour) indices. Q^0 and L^0 are $SU(2)_\text{L}$ quark and lepton doublet weak eigenstates. d^0_R, u^0_R and e^0_R are $SU(2)_\text{L}$ singlet weak eigenstates. The matrices Y^f_{ij} are in general non-diagonal.

   (a) Refer to Standard Model books and the introduction of the CKM matrix and quark and lepton mass eigenstates and the diagonalization of the Yukawa matrices. Applying the same procedure here, answer whether Higgs couplings to quarks and leptons contain flavour changing neutral currents.

   (b) Give some examples of $\mathcal{F}_{u,d,l}(\{\Phi_k\})$. Each example consitutes a separate two-Higgs doublet model.

   Literature: James Wells, `https://arxiv.org/abs/0909.4541v1`

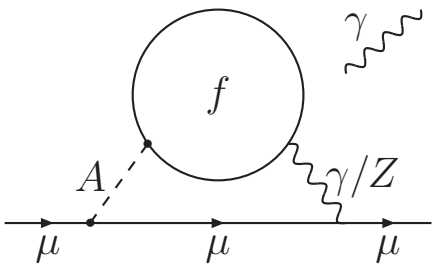
2. **Type X Yukawa interaction in the 2HDM**

   In the type X Yukawa interaction we impose the Z_2 transformation: $\Phi_1 \to -\Phi_1$, $e^0_{j\text{R}} \to -e^0_{j\text{R}}$, and $\mathcal{S} \to \mathcal{S}$ for all the other fields. The Z_2-invariant Yukawa interaction term can be written as

$$-\mathcal{L}^\text{X}_\text{Y} = \bar{Q}^0_i\Delta^u_{ij}\Phi^c_2 u^0_{j\text{R}} + \bar{Q}^0_i\Delta^d_{ij}\Phi_2 d^0_{j\text{R}} + \bar{L}^0_i\Gamma^d_{ij}\Phi_1 e^0_{j\text{R}}\,. \qquad (2)$$

   (a) Referring to the lecture notes, diagonalize the mass matrices and calculate the Yukawa interaction couplings of the fermions to the neutral and charged Higgs fields.

   (b) As a phenomenological example we show here a Barr-Zee type Feynman diagram contributing to the muon anomalous magnetic moment (the external photon can couple to all charged particles). For which fermion f in the inner loop is the Feynman diagram enhanced by $\tan^2\beta$? (Note that $\tan\beta$ can be realistically as large as ~ 60.)



3. **Leptoquark models:**

   Consider an extension of the SM by leptoquarks ϕ. Out of the 10 possible types of leptoquarks (see particle data group review on leptoquarks), we take the two options for interactions

$$\mathcal{L}_{LQ1} = \lambda_{IJ}\phi_1 \bar{d}^I P_L L^J + h.c. \tag{3}$$
$$\mathcal{L}_{LQ2} = \lambda'_{IJ}\bar{Q}^I P_R l^J \phi_2 + \lambda''_{IJ}\bar{u}^I P_L L^J \cdot \phi_2 + h.c. \tag{4}$$

   where L^I, Q^I are the SM doublets of left-handed leptons and quarks of generation I, and similar for the right-handed singlets l^I, u^I, d^I.

   a) What must the hypercharges of ϕ_1 and ϕ_2 be in order for these interactions to be gauge invariant? In case 2, check that both terms are gauge invariant for the same hypercharge assignment.

   b) Write down Feynman diagrams for the three interactions. In case 2, the leptoquark is an SU(2)$_L$ doublet $\phi_2 = \binom{\phi_2^1}{\phi_2^2}$, and the dot product is defined as $L \cdot \phi_2 = \nu\phi_2^2 - l\phi_2^1$.

   c) In which case(s) can you obtain a leptoquark tree-level contribution to the process $b \to ce\bar{\nu}_e$? Which couplings are involved in the Feynman diagram?

   d) In which case(s) can you obtain a leptoquark one-loop contribution to $g-2$ of the muon, i.e. to the muon-muon-photon interaction? Which couplings are involved?

   Literature: Particle Data Group review
   http://pdg.lbl.gov/2020/reviews/rpp2020-rev-leptoquark-quantum-numbers.pdf