2. Examples of simple models and constraints

- interplay theory – expt.
- use simple observables

2.1 4th generation

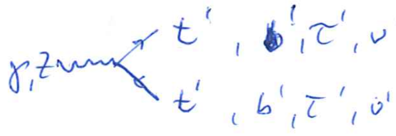
(Djouadi, Lenz '12)
(and Lenz, Adv. in H.E.Phys. '12)

Motivation: · Why 3 generations? ⇝ Is there a 4th generation?
$\leadsto \binom{u}{d}\binom{c}{s}\binom{t}{b}\binom{t'}{b'}$ + leptons
- model has "non-decoupling" properties
(effects don't become small if $m_{4th\,gen.} \to \infty$)
- can be definitely falsified using combination of observ.

Model: $SM4 := SM + \underbrace{4th\ gen.\ (t', b', \tau', \nu')}$
identical gauge quantum numbers as 3rd gen,
Yukawa coupl. to Higgs: same structure as other gen. but diff. values

simplification here: neglect generation mixing

Feynman rules:



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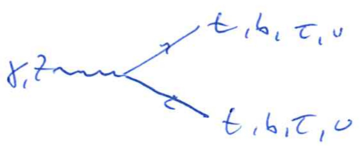

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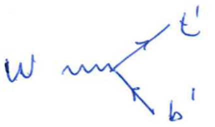

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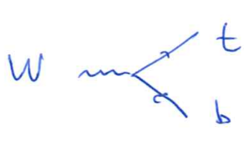

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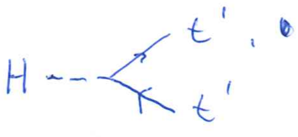
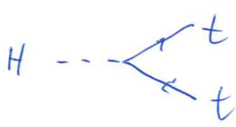


$= -i\, Y_{t'}/\sqrt{2} = -i\, \frac{m_{t'}}{v}$



(SM) $= -i\, Y_t/\sqrt{2} = -i\, \frac{m_t}{v}$

etc.

1st constraint: Invisible Z-width, measured at LEP

LEP ⇒ total Z-decay rate $\Gamma_{Z,tot}$ $= 2{,}4952\,(23)\,\mathrm{GeV}$

partial Z-decay rates $\Gamma_{Z\to(SM,\,visible)} = \ldots$

⟹ "invisible Z-decay rate" $\Gamma_{Z\to invisible} = 0{,}04974\,(25)\,\mathrm{GeV}$

SM theory ⇒ perfect agreement of visible decays; $\Gamma^{SM}_{Z\to\nu\bar\nu_{gen\,1,2,3}} = 0{,}05017\,(2)\,\mathrm{GeV}$

⇒ ~~add~~ if Z  would be possible, $\Gamma_{Z\to invisible}$ should be $\frac{4}{3}$ of measured value

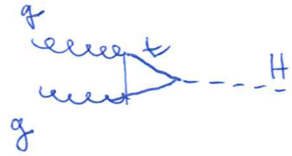
↯

⇒ $\boxed{\nu' \text{ must be heavy}, \quad m_{\nu'} > \frac{M_Z}{2}}$

~ ~~4th~~ gen very diff. from other neutrinos!

$\boxed{\text{obviously, also } t', b' \text{ must be heavier than top, otherwise discovered}}$

2nd constraint: Higgs production and decay, at LHC

dominant mode in SM: 

strong gauge coupl. — from loop integration

$$\sim \frac{g_s^2}{16\pi^2}\cdot Y_t \cdot \underbrace{f(m_t)}_{\sim \frac{1}{m_t}\ \text{(dimension)}}$$

Loop factor — $Y_t = \frac{\sqrt{2}\,m_t}{v}$

⇒ SM-amplitude $T_{fi}^{SM} \sim \frac{g_s^2}{16\pi^2}\cdot\frac{1}{v}$, indep. of m_t!

non-decoupling because coupling ~ mass

⇒ in SM4:  $T_{fi}^{SM4} = 3\cdot T_{fi}^{SM}$

⇒ $\boxed{\sigma^{SM4}_{pp\to H} \approx 9\cdot\sigma^{SM}_{pp\to H}}$ (note: higher-order corr.??)

Higgs decays: $H \to b\bar{b},\ \tau\bar{\tau},\ \ldots$ $\quad H \to W W^* \to W\, u\, d$ $\quad H \to \gamma\gamma,\ gg$

SM: dominant $H \to b\bar{b}$, similar in SM4: $\Gamma^{SM4}_{Hbb} \approx \Gamma^{SM}_{Hbb}$

$\Gamma_{H\to\gamma\gamma}$: W-loop $\to \gamma\gamma$ + $t, b, \tau, \ldots$-loop $\Rightarrow$ complicated sum

detailed calc. $\Rightarrow$ $\Gamma^{SM4}_{H\to\gamma\gamma} = \sim(0{,}01 \ldots 0.1)\ \Gamma^{SM}_{H\to\gamma\gamma}$

$\Gamma^{SM4}_{H\to WW^*} = \sim(0.2) \cdot \Gamma^{SM}_{H\to WW^*}$

(depends on $m_{\nu'}, m_{\tau'}, m_{t'}, m_{b'}$)

additional possibility: $H \to \nu'\nu'$ invisible

(if $m_{\nu'} < \frac{M_H}{2}$

($\Rightarrow$ $= 45 \ldots 62$ GeV))

$$\Rightarrow \Gamma^{SM4}_{H\to total} = \Gamma_{H\to b\bar{b}} + \ldots + \begin{cases} 0 \\ \Gamma_{H\to\nu'\nu'} & \text{if possible} \end{cases}$$

$$\approx \Gamma^{SM}_{H\to tot} + \begin{cases} 0 \\ \Gamma_{H\to\nu'\nu'} \end{cases} \equiv \Gamma^{SM}_{H\to tot}\left(1 + \Delta_{H\nu'\nu'}\right)$$

$$BR(H\to ii)^{SM4} = \frac{\Gamma^{SM4}_{H\to ii}}{\Gamma^{SM4}_{H\to tot}}$$

measured at LHC		predicted in SM4	
$\sigma_{pp\to H} \cdot BR(H\to\gamma\gamma)$	$= SM * 1.16(18)$	$= SM \cdot 9 \cdot$ $= SM \cdot (<0.27)$	$\dfrac{\overbrace{\Gamma^{SM4}_{H\to\gamma\gamma}}^{(0.01\ldots)}}{\Gamma_{H\to tot}}\ \dfrac{1}{1+\Delta_{H\nu'\nu'}}$
$\sigma_{pp\to H} \cdot BR(H\to b\bar{b})$	$= SM * 0.95(22)$	$= SM \cdot 9 \cdot$ $= SM \cdot (6.3 \ldots 27)$	$\dfrac{\Gamma^{SM \simeq SM4}_{H\to b\bar{b}}}{\Gamma_{H\to tot}}\ \dfrac{1}{1+\Delta_{H\nu'\nu'}}$
$\sigma_{pp\to H} \cdot BR(H\to WW^*)$	$= SM * 1.08(18)$	$= SM \cdot 9 \cdot$ $= SM(1\ldots9)$ [Lenz]	$\dfrac{\Gamma_{H\to WW^*}}{\Gamma_{H\to tot}}\ \dfrac{0.2}{1+\Delta_{H\nu'\nu'}}$
...			

$\Longrightarrow$ $\boxed{\text{impossible to agree with all measurements simultaneously!}}$ ~~most direct: 2H~~

"final result" [Lenz, Adv. H.E.Ph. '12]

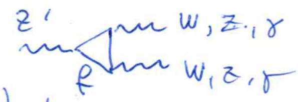
2.2 Z' models

(~ review PDG Z')

Z' := extra neutral vector boson

Motivation:
- simplest case of extra vector boson
- often contained in GUTs, other SM-extensions (Susy), gauged symmetry for flavour

Theory considerations:

- Huge class of models: different choices of Z' couplings to quarks, leptons/generations, to Higgs, to additional unknown, new particles
- vector boson $\xrightarrow{\text{renormalizability}}$ gauge invariance ~ simplest choice $U(1)'$
  but also more complicated (~ Z'', Z''', ...)
- if $U(1)'$ should originate from non-abelian (GUT?) group
  ⟹ quantum numbers quantized (like Hypercharge)
- if Z' is massive, $U(1)'$ should be spontan. broken
  → some scalar field with VEV couples to Z' (SM-Higgs or new scalar)
- anomaly cancellation: gauge inv. must not be broken ~ diagrams 

  must cancel
  ~ $\sum_f$ (combination of Z', Z, W, γ charges) $= 0$ must hold

Goals:
- some generic expt. properties at LHC etc, EWPO (mixing)
- $L_\mu - L_\tau$ as specific unusual example
- kin. mix, dark photon " "

Concrete $U(1)'$ models: ~~gauge~~

covar. derivative $D^\mu = \partial^\mu + i g_s G^\mu + i g_w W^\mu + i g_y Y B^\mu + i g' Y' Z'^\mu$

g' : new gauge coupling
Y' : $U(1)'$ generator → must be chosen for every quark, lept., Higgs

some examples choice:

$U(1)_{B-L}$: $Y'_{\text{quarks}} = \frac{1}{3}$, $Y'_{\text{lept.}} = -1$, $Y'_{\text{Higgs}} = 0$

$U(1)_{L_\mu - L_\tau}$: $Y'_{\text{Muon}, \nu_\mu} = 1$, $Y'_{\tau, \nu_\tau} = -1$, $Y'_{\text{rest}} = 0$

$U(1)_{N\,(E_6SSM)}$: $Y'_{u_{L,R}} = \pm 1$, $Y'_{d_{L,R}} = 1, -2$, $Y_{e_R} = -1, \ldots$, $Y'_{H_1} = -3$

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later

~~mass mixing:~~

~~clearly need extra scalars with VEV: $\Phi_{SM}, \Phi_{\text{new}}, \ldots = \Phi_i$~~

~~⤳ mass matrix for gauge bosons:~~

$$\cancel{(M^2)_{ab} = \sum_i \langle \Phi_i \rangle^\dagger \{ g_a T_a, g_b T_b \} \langle \Phi_i \rangle}$$

~~⤳ can lead to mixing Z' with W^3, B~~

~~e.g. off-diag. term~~

$$\cancel{(M^2)_{BZ'} = \sum_i \langle \Phi_i \rangle^\dagger g_y g' 2 Y_i Y'_i \langle \Phi_i \rangle}$$

~~this is $\neq 0$ if $\exists$ scalar Φ_i with $Y_i, Y'_i \neq 0$~~

~~e.g. $U(1)_N$ unavoidable~~

~ result of diagonalization with angle ϑ:

mass eigenstate gauge bosons γ, Z_1, Z_2

couplings of $Z_1 = -\cos\vartheta \cdot (Z_{SM}) + \sin\vartheta\,(Z')$

$Z_2 = -\sin\vartheta\,(Z_{SM}\text{-coupl.}) + \cos\vartheta\,(Z'\text{-coupl.})$

same effect see H-singlet extension, exercise

later

exp. constraints: EWPO / LEP:

Z-couplings very precisely measured, agree with SM

$\Rightarrow$ very strong constraints on mixing

(if either ~~offe-di~~

$$M^2 = \begin{pmatrix} M_Z^2 & \text{very small} \\ \text{very small} & M_{Z'}^2 \end{pmatrix} \quad \text{or} \quad \begin{pmatrix} M_Z^2 & \text{big} \\ \text{big} & \text{very big} \end{pmatrix}$$

very heavy!

first

kinetic mixing (rarely interesting, but here!)

special gauge inv. term: $B^{\mu\nu} Z'_{\mu\nu}$ (field strengths of $U(1)_Y, U(1)'$)

$$\sim \mathcal{L}_{\text{gauge kin}} = \dots - \frac{1}{4} B^{\mu\nu} B_{\mu\nu} - \frac{1}{4} Z'^{\mu\nu} Z'_{\mu\nu} - \frac{\sin\chi}{2} Z'^{\mu\nu} B_{\mu\nu}$$

$$\mathcal{L}_{\text{int}} = \dots B^\mu \cdot j_{B\mu} + Z'^\mu j'_\mu$$

useful rewrite

$$-\frac{1}{4} BB - \frac{1}{4} Z'Z' - \frac{\sin\chi}{2} Z'B = -\frac{1}{4}(B + \sin\chi Z')^2 + \frac{1}{4}\sin^2\chi Z'Z' - \frac{1}{4} Z'Z'$$

define: $\tilde{B} =$

$\ni g_W W^3 T^3 + g_Y Y \tilde{B} + \tilde{Z}(-t_\chi g_Y Y + \frac{1}{2c_\chi} g' Y')$

$$(M^2)_{ab} = \quad \begin{pmatrix} W \\ B \\ Z' \end{pmatrix} = \begin{pmatrix} 1 & & \\ & 1 & -t \\ & 0 & 1/c \end{pmatrix} \begin{pmatrix} W \\ \\ \end{pmatrix}$$

kinetic mixing (here interesting; in other models often usually absent)

special gauge invariant term:

$$B^{\mu\nu} Z'_{\mu\nu} \quad \text{(fieldstrengths of } U(1)_Y, U(1)'\text{)}$$

$$\rightsquigarrow \mathcal{L}_{\text{gauge kin}} = \dots -\frac{1}{4} B^{\mu\nu} B_{\mu\nu} - \frac{1}{4} Z'^{\mu\nu} Z'_{\mu\nu} - \frac{\sin\chi}{2} Z'^{\mu\nu} B_{\mu\nu}$$

$$\mathcal{L}_{\text{int}} = \dots B^{\mu} j_{B\,\mu} + Z'^{\mu} j'_{\mu}$$

useful to rewrite:

$$-\frac{1}{4} B^2 - \frac{1}{4} Z'^2 - \frac{s_\chi}{2} B Z' = -\frac{1}{4} (B + s_\chi Z')^2 - \frac{c_\chi^2}{4} Z^2$$

define new fields (non-unitary since kin. terms not normalized)

$$\begin{pmatrix} \tilde{B} \\ \tilde{Z}' \end{pmatrix} = \begin{pmatrix} 1 & s_\chi \\ 0 & c_\chi \end{pmatrix} \begin{pmatrix} B \\ Z' \end{pmatrix}$$

$$\begin{pmatrix} B \\ Z' \end{pmatrix} = \begin{pmatrix} 1 & -t_\chi \\ 0 & \frac{1}{c_\chi} \end{pmatrix} \begin{pmatrix} \tilde{B} \\ \tilde{Z}' \end{pmatrix}$$

$\rightsquigarrow$ now normal kinetic terms $-\frac{1}{4} \tilde{B}^{\mu\nu} \tilde{B}_{\mu\nu} - \frac{1}{4} \tilde{Z}'^{\mu\nu} \tilde{Z}'_{\mu\nu}$

and:

$$\mathcal{L}_{\text{int}} = \dots + \tilde{B}^{\mu} j_{B\mu} + \tilde{Z}'^{\mu} \left(-\tan\chi\, j_{B\mu} + \frac{1}{c_\chi} j'_{\mu} \right)$$

has SM-couplings

has original Z'-couplings + perturbations

note: field transf. is not unique: one could do another unitary transf. without changing kin. terms mass mixing: extension of SM Z, W masses

define as in SM:

$$\boxed{\begin{aligned} \frac{\sin\theta}{\cos\theta} &:= \frac{g_Y}{g_W}, \quad \begin{pmatrix} A \\ Z \end{pmatrix} = \begin{pmatrix} c & s \\ -s & c \end{pmatrix} \begin{pmatrix} B \\ W^3 \end{pmatrix} \\ e &:= s_\theta g_W = c_\theta g_Y, \quad g_Z := \frac{g_W}{c_\theta} = \frac{e}{s_\theta c_\theta} \\ Q &:= T^3 + Y \quad (= \text{generators acting on fields}) \\ T^Z &:= T^3 - s_\theta^2 Q \end{aligned}}$$

$$\leadsto \boxed{g_W W^{a\mu} T^a + g_Y Y B^\mu = e Q A^\mu + g_Z T^Z Z^\mu + g_W (T^1 W^{1\mu} + T^2 W^{2\mu})}$$

(equally valid after $B \to \tilde{B}$)

abbrev. $\tilde{Z}'^\mu (-t_\alpha Y g_Y + \frac{1}{c_\alpha} Y' g') =: \tilde{Z}^\mu \tilde{g}' \tilde{Y}'$

$\leadsto$ neutral gauge boson masses:

$$\mathcal{L} = \dots \sum_{\text{scalars } \phi} |D^\mu \phi|^2 = \dots \frac{1}{2} (A \ Z \ Z') \mathcal{M}^2 \begin{pmatrix} A \\ Z \\ Z' \end{pmatrix}$$

$$(\mathcal{M}^2)_{ab} = \sum_{\text{scalars}} \langle\phi\rangle_0^\dagger \{ g_a T_a, g_b T_b \} \langle\phi\rangle_0$$

since $Q \langle\phi\rangle_0 = 0$ ($\Leftrightarrow$ photon massless, $U(1)_{e.m.}$ unbroken)

$(\mathcal{M}^2)_{Ai} = 0, \quad i = A, Z, Z'$

and

$$(\mathcal{M}^2)_{ZZ} = 2 g_Z^2 \sum_\phi \langle\phi\rangle_0^\dagger T^{Z^2} \langle\phi\rangle_0$$

$$(\mathcal{M}^2)_{Z'Z'} = 2 \tilde{g}'^2 \sum_\phi \langle\phi\rangle_0^\dagger \tilde{Y}'^2 \langle\phi\rangle_0$$

$\rightarrow$ there must be scalars with $Y' \neq 0$ (either SM Higgs or new)

$$(\mathcal{M}^2)_{ZZ'} = 2 g_Z \tilde{g}' \sum_\phi \langle\phi\rangle_0^\dagger T^Z \tilde{Y}' \langle\phi\rangle_0$$

$\rightarrow$ $\boxed{\text{mixing} \neq 0 \text{ if } \exists\, \phi \text{ with } T^Z \langle\phi\rangle_0 \neq 0 \text{ and } \tilde{Y}' \langle\phi\rangle_0 \neq 0}$

diagonalization: $\leadsto$ with rotation $\begin{pmatrix} Z \\ Z' \end{pmatrix} = \begin{pmatrix} c_\alpha & -s_\alpha \\ +s_\alpha & c_\alpha \end{pmatrix} \begin{pmatrix} Z_1 \\ Z_2 \end{pmatrix}$ $\leadsto$ couplings of $Z_1 \sim \cos\alpha (Z_{SM}) + \sin\alpha (Z')$

exp. constraints on mixing: EWPO/LEP:

Z - couplings very precisely measured (1‰)

$\Rightarrow$ very strong constraints on mixing

- mass eigenstate with mass $\simeq M_Z^{exp}$ needed
- and with couplings $\simeq$ SM-Z-couplings

$\Rightarrow$ mixing angle for diagonalization must be very small

$$\boxed{\mathcal{M}^2_{Z-Z'} = \begin{pmatrix} M^2_{Z\,SM} & \text{very small} \\ \text{very small} & M^2_{Z'} \end{pmatrix} \quad \text{or} \quad = \begin{pmatrix} M^2_{Z\,SM} & \text{big} \\ \text{big} & \underline{\text{very}}\ \text{big} \end{pmatrix}}$$

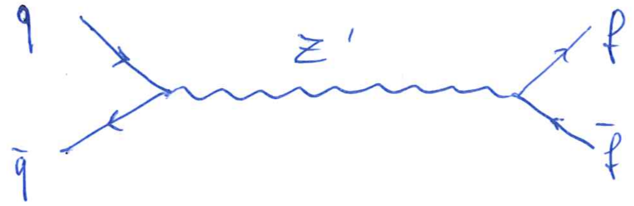
Z' very heavy!

-

2.3 Predictions in Z' models – typical observables

if Z' couples to q and leptons with gauge coupling $g' \sim g_{W,Y}$

$\Rightarrow$



$f = e, \mu, \ldots$

very easy to see at LHC

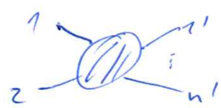
$\Downarrow$ $\boxed{\text{develop required theory !}}$

2.3.1 QFT Appendix

$\rightarrow\ iT_{fi}$ probability ampl. for $i \to f$

cross section σ: $\quad d\sigma \cdot \dfrac{(\#\text{ target part.})\cdot(\#\text{ beam part})}{\text{time}\cdot\text{area}} = d\left(\dfrac{\#\text{ scattered part}}{\text{time}}\right)$

$$d\sigma = \frac{|T_{fi}|^2\, d\Phi_n}{4\sqrt{(p_1p_2)^2 - m_1^2 m_2^2}}$$



decay rate Γ: $\quad d\Gamma \cdot (\#\text{part.}) = d(\#\text{ decays/time})$

$$d\Gamma = \frac{|T_{fi}|^2\, d\Phi_n}{2m_1}$$

phase space:

$$d\Phi_n := (2\pi)^4 \delta^{(4)}(p_1 + p_2 - p_1' - \ldots - p_n')\, \frac{d^3 p_1'}{(2\pi)^3 2E_1'} \cdots \frac{d^3 p_n'}{(2\pi)^3 2E_n'} \qquad E_i' = \sqrt{\vec{p}_i'^2 + m_i^2}$$

note: $4\sqrt{(p_1p_2)^2 - m_1^2 m_2^2} \xrightarrow[s=(p_1+p_2)^2 \approx 2p_1p_2]{E_{1,2} \gg m_{1,2}} 2s$

CMS: $\int d\Phi_2\, f(E_1', E_2') = \frac{1}{8\pi} f\left(\frac{E}{2}, \frac{E}{2}\right) \qquad (E = p_1^0 + p_2^0)$

spinors in T_{fi}:

 $= u(p,s)$

 $= \bar{u}(p,s)$

 $= \bar{v}(p,s)$

 $= v(p,s)$

vectors in T_{fi}:

 $= \varepsilon^{\mu}(p,s)$

 $= \varepsilon^{\mu *}(p,s)$

$$\sum_{s=\pm\frac{1}{2}} u(p,s)\bar{u}(p,s) = \not{p} + m$$

$$\sum_{s=\pm\frac{1}{2}} v(p,s)\bar{v}(p,s) = \not{p} - m$$

$$\sum_s \varepsilon^{\mu}\varepsilon^{\nu *} = P^{\mu\nu}(p)$$

depends on gauge choice
often: $P^{\mu\nu} = -g^{\mu\nu}$

useful currents:

p_1  p_2 $\sim \bar{v}(p_2)\gamma^{\mu}u(p_1)$ or similar

$$\left| \text{} \right|^2 \sim \underbrace{\bar{v}(p_2)\gamma^{\mu}u(p_1)\cdot\bar{u}(p_1)\gamma^{\mu'}v(p_2)}\ldots$$

$= T^{\mu\mu'}$ tensor, depends on $p_{1/2}, s_{1/2}$

spin sums:

$$\sum_{s_1 s_2} T^{\mu\mu'} = \mathrm{Tr}\left(\gamma^{\mu}(\not{p}_1 + m_1)\gamma^{\mu'}(\not{p}_2 + m_2)\right)$$

traces:

$$\mathrm{Tr}(\gamma^{\mu}\gamma^{\nu}) = 4g^{\mu\nu}$$

$$\mathrm{Tr}(\not{p}_1\gamma^{\mu}\not{p}_2\gamma^{\nu}) = 4\left(p_1^{\mu}p_2^{\nu} + p_1^{\nu}p_2^{\mu} - g^{\mu\nu}\,p_1\cdot p_2\right)$$

propagators:

 $\sim \frac{1}{p^2 - M^2}$

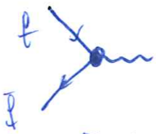
$\sim$ higher orders  $\sim \frac{1}{p^2 - M^2 + \Sigma(p)}$

close to $p^2 = M^2$ $\approx \frac{1}{p^2 - M^2 + iM\Gamma} =: D(p^2)$

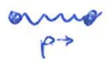
Im - part!

2.3.2 

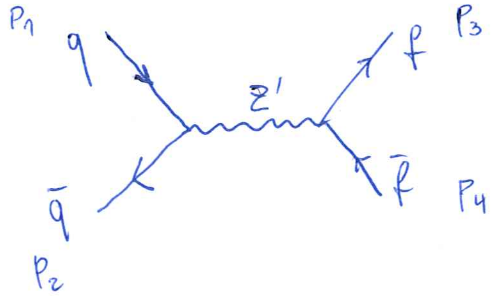
<u>assume Feynman rules</u> $\qquad \mathcal{L}_{int} = -g_q \bar{q} \not{Z}' q - g_f \bar{f} \not{Z}' f$



$= -i g_f \gamma^\mu \qquad$ (no γ_5)



$= -i g_{\mu\nu} D(p^2)$

iT_{fi} 

$$= \bar{u}_3 (-i g_f \gamma^\mu) v_4 \cdot \frac{-i g_{\mu\nu}}{p^2 - M^2 + iM\Gamma} \bar{v}_2 (-i g_q \gamma^\nu) u_1$$

$$= -i g_f g_q \left[\bar{u}_3 \gamma^\mu v_4\right] \frac{1}{p^2 - M^2 + iM\Gamma} \left[\bar{v}_2 \gamma_\mu u_1\right]$$

assume $m_{1,2,3,4} \cong 0$: $(p_1+p_2)^2 = 2p_1p_2 = s = 2p_3p_4$, $(p_{1,2}^\mu) = (E,0,0,\pm E)$

$p_1p_3 = E^2(1-c_\theta)$ etc $\qquad (p_{3,4}^\mu) = (E, \pm s_\theta E, 0, \pm c_\theta E)$

~~problem~~

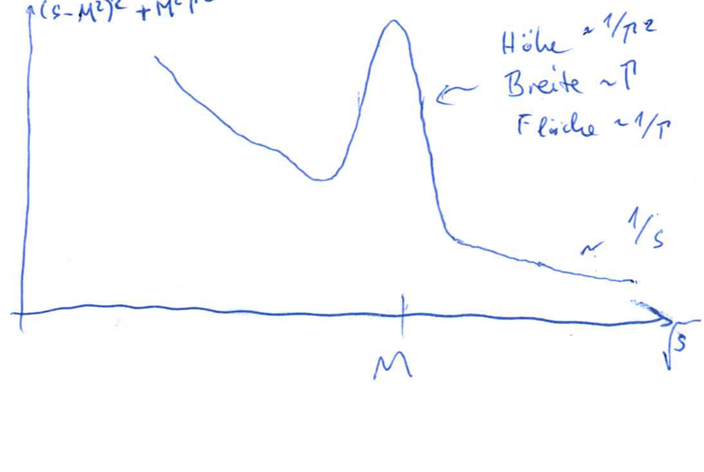
$$\overline{|T_{fi}|^2} := \frac{1}{4} \sum_{s_{1234}} |T_{fi}|^2 = \frac{1}{4} g_f^2 g_q^2 \sum T_{34}^{\mu\mu'} T_{12\,\mu\mu'} |D(p^2)|^2$$

^ # i-state spin configs

$$= \frac{1}{4} g_f^2 g_q^2 |D(s)|^2 \cdot 4s^2 \underbrace{(1+c_\theta^2)}_{\frac{4}{3} \text{ im Mittel in } \int d\Phi_2 \to \int d\cos\theta}$$

$$\Rightarrow \boxed{\sigma_{q\bar{q}\to f\bar{f}} = \frac{1}{2s} \int d\Phi_2 \, \overline{|T_{fi}|^2} = \frac{1}{12\pi} s |D(s)|^2 g_f^2 g_q^2}$$

$$s|D(s)|^2 = \frac{s}{(s-M^2)^2 + M^2\Gamma^2}$$



→ very easy to see !

→ study theory relations for σ :

intuition :

$$\text{probab.}(q\bar{q} \to f\bar{f}) \approx \text{prob.}(q\bar{q} \to Z') * \text{prob.}(Z' \to f\bar{f})$$

"almost" stable particle / resonance

(approx . since Z'-spin is not observed, exact relation for amplitudes and definite Z'-spin)

$$\rightsquigarrow \quad \sigma_{q\bar{q} \to Z'} = \frac{1}{2s} \int d\Phi_{\underset{\underset{1}{\uparrow}}{1}} \overline{|T_{fi}^{q\bar{q}Z'}|^2}$$

also summed / averaged over Z'-spin

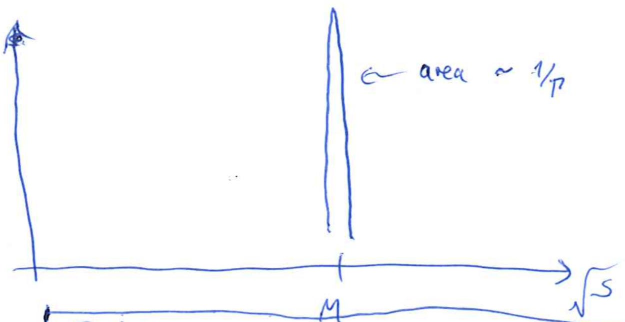
$$= g_q^2 \pi \, \delta(s - M^2)$$

$$\Gamma_{Z' \to f\bar{f}} = \frac{1}{2M} \int d\Phi_2 \overline{|T_{fi}^{Z'f\bar{f}}|^2}$$

$$= \frac{g_f^2 M}{12\pi}$$

indeed :

$$\sigma_{q\bar{q} \to Z'} \cdot \underbrace{\frac{\Gamma_{Z' \to f\bar{f}}}{\Gamma}}_{\text{prob.}(Z' \to f\bar{f})} = \frac{1}{12\pi} g_f^2 g_q^2 \frac{M\pi}{\Gamma} \underbrace{\delta(s - M^2)}_{= \frac{\delta(\sqrt{s} - M)}{2M}}$$



$$\boxed{D(s) = \frac{1}{(s - M^2)^2 + M^2\Gamma^2} \xrightarrow[\Gamma \to 0]{} \frac{\pi \, \delta(s - M^2)}{M\Gamma}}$$

⇒ this shows that the above is a well-defined limit of $\sigma_{q\bar{q} \to f\bar{f}}$.

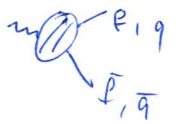
another way to rewrite:

$$\boxed{\sigma_{q\bar{q}\to f\bar{f}} = \frac{4(2S_{Z'}+1)}{(2S_q+1)(2S_{\bar{q}}+1)} \cdot \frac{4\pi s}{M^2} \cdot |D(s)|^2 \cdot \Gamma_{f\bar{f}} \cdot \Gamma_{q\bar{q}}}$$

spins of initial state / resonance

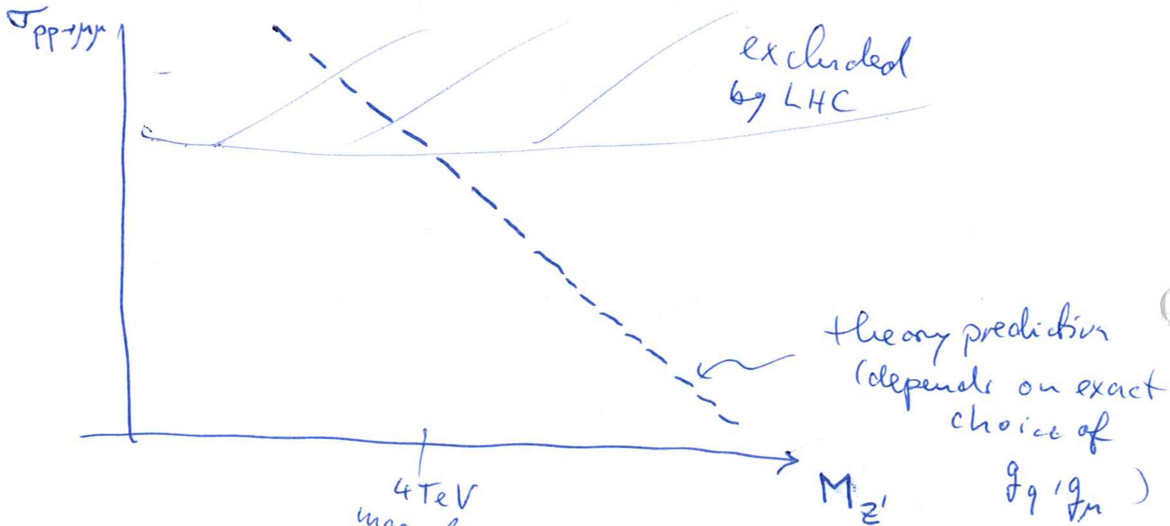
resonance

decays ↔



e.g. Langacker, App. F

compare with exp.



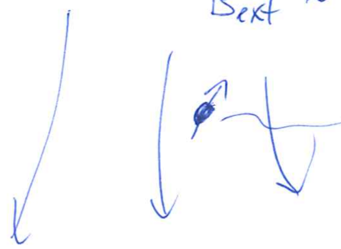
mass limits around 4-5 TeV!

2.4 g − 2 and new physics

2.4.1 General theory

Magnetic dipole moment (MDM) and Lorentz force

$\vec{B}_{ext} \approx$ homogen.



small object

classically / non-relativistically: $H = \frac{(\vec{p} - eQ\vec{A}_{ex})^2}{2m} - \vec{B}_{ex} \cdot \vec{\mu}$

defines MDM

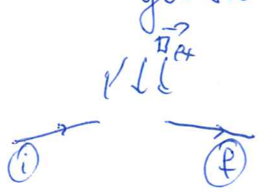
MDM for quantum particle − assume

$$\vec{\mu} = g \cdot \frac{Qe}{2m} \cdot \vec{S}$$

eff. Lagrangian density → $\vec{S} = \int d^3x \; \vec{s}(x)$ spin density operator

$$\mathcal{L}_{int} = + g \frac{Qe}{2m} \vec{s} \cdot \vec{B}_{ex}$$

general QM: scattering for small interaction:



$$S_{fi} = -i \int dt \, \langle f | \hat{H}_{int}(t) | i \rangle = +i \int d^4x \, \langle f | \hat{\mathcal{L}}_{int}(x) | i \rangle$$

Definition: scattering in full QFT: if we can write

$$S_{fi} = i (2\pi)^4 \delta^{(4)}(p_i + q - p_f) \; g \frac{Qe}{2m} \langle f | \vec{s}(0) | i \rangle \cdot \vec{\tilde{B}}_{ex}(q)$$

(in non-relativ. limit and at $\mathcal{O}(B_{ex})$), then this defines the full MDM and g

$q = p_f - p_i$

Spin $\frac{1}{2}$:

angular mom. operator $= \int d^3x\, \bar\psi \gamma^0 \left(i (x^\mu \partial^\nu - x^\nu \partial^\mu) + \frac{\sigma^{\mu\nu}}{2} \right) \psi = \hat{J}^{\mu\nu}$

~~Spin dens~~

$$\rightsquigarrow \langle \phi\, e^-, p_f, s_f \,|\, \hat{S}^{\mu\nu}(0) \,|\, e^-, p_i, s_i \rangle = \bar u(f) \gamma^0 \frac{\sigma^{\mu\nu}}{2} u(i)$$

$\sigma^{\mu\nu} = \frac{i}{2}[\gamma^\mu, \gamma^\nu]$

$\not p_i\, u(i) = m\, u(i)$, $\bar u(f) \not p_f = m\, \bar u(f)$

non-relativistically: $\not p \approx m\gamma^0 \rightarrow \bar u \gamma^0 = \bar u$ in

Gordon identity:

between $\bar u(f) \dots u(i)$:

$$2m\gamma^\mu = m\gamma^\mu + \gamma^\mu m = \not p_f \gamma^\mu + \gamma^\mu \not p_i$$

$$= \frac{1}{2}\{\not p_f, \gamma^\mu\} + \frac{1}{2}[\not p_f, \gamma^\mu] + \frac{1}{2}\{\gamma^\mu, \not p_i\} + \frac{1}{2}[\gamma^\mu, \not p_i]$$

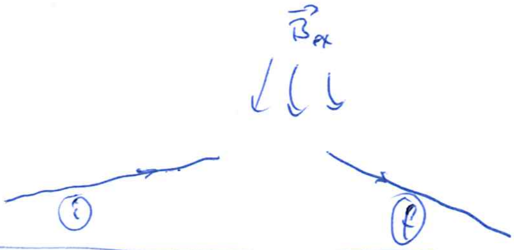
$$= p_f^\mu + p_i^\mu + \frac{1}{2}[\not p_f - \not p_i, \gamma^\mu]$$

$$= (p_f + p_i)^\mu + i\sigma^{\mu\nu} q_\nu$$

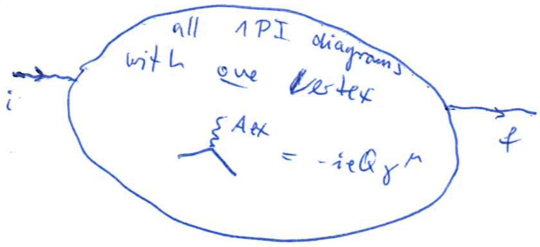
scattering $e^- - \vec B_{ex}$:

QFT with $A^\mu(x) \longrightarrow A^\mu(x) + A^\mu_{ex}(x)$

quantized field ; ext., classical field



$S_{fi} =$



$$S_{fi} =: i(2\pi)^{(4)} \delta^{(4)}(p_f - p_i - q)\, \bar u(f)\, i\Gamma^\mu(p_i, p_f)\, u(i)\, \tilde A_{ex,\mu}(q)$$

most general result:

between $\bar u \dots u$

$\Gamma^\mu \sim \gamma^\mu, (p_f + p_i)^\mu, q^\mu$ $\left(\xleftrightarrow{\text{Gordon}} \sigma^{\mu\nu} q_\nu \right)$

but $A^\mu_{ex} \to A^\mu_{ex} + \partial^\mu \Lambda$ / $\tilde A^\mu_{ex} \to \tilde A^\mu_{ex} + q^\mu \tilde\Lambda$

must not have an effect $\Rightarrow \Gamma^\mu$ cannot contain q^μ.

hence:

$$\boxed{\begin{aligned}\Gamma^\mu(p_i, p_f) &= -eQ\left\{\gamma^\mu F_1(q^2) + \frac{i\sigma^{\mu\nu}q_\nu}{2m}F_2(q^2)\right\}\\ &= -eQ\left\{\gamma^\mu(F_1+F_2) - \frac{(p_i+p_f)^\mu}{2m}F_2\right\}\\ &= -eQ\left\{\frac{(p_i+p_f)^\mu}{2m}F_1 + \frac{i\sigma^{\mu\nu}q_\nu}{2m}(F_1+F_2)\right\}\end{aligned}}$$

and

$$S_{fi} = \sim \text{Lorentzforce} + \frac{ieQ}{2m}(2\pi)^4\delta^{(4)}(p_f-p_i-q)\cdot\left(F_1(q^2)+F_2(q^2)\right)$$

$$* \, 2\langle f|s^{\mu\nu}(0)|i\rangle \cdot \underbrace{\left(-iq_\nu \tilde{A}^{ex}_\mu(q)\right)}_{\frac{1}{2}\tilde{F}^{ex}_{\nu\mu}(q)}$$

$$\underbrace{s_z \mathrel{\hat{=}} s^{12}, \quad B_z \mathrel{\hat{=}} \partial_1 A^2 - \partial_2 A^1 = -F_{12}, \quad \vec{s}\cdot\vec{B} \mathrel{\hat{=}} -\tfrac{1}{2}s_{\mu\nu}F^{\mu\nu}}_{=\langle\vec{s}\rangle\cdot\tilde{\vec{B}}_{ex}(q)}$$

hom. field, $q\to 0$ $\Longrightarrow$ $\boxed{g = 2\left(F_1(0) + F_2(0)\right)}$

On-shell definition of charge

$e := e^{OS} :=$ eff. charge for $q\to 0$ processes / class. limit

$\rightsquigarrow$ require δe such that  (on shell, $q\to 0$)

$\Leftrightarrow F_1(0) = 1$ after renorm.

$$\rightsquigarrow \boxed{F_1(0) = 1, \qquad g = 2\left(1 + F_2(0)\right)}$$

anomalous magn. moment "g-2"

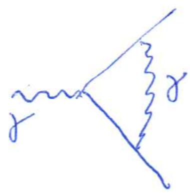
tree-level: $F_1 \equiv 1, \; F_2 \equiv 0 \quad \rightsquigarrow \; g = 2$

loops, $F_2(0) \neq 0 \quad \rightsquigarrow$ $\boxed{g = 2(1+a), \quad a = \frac{g-2}{2}}$ "anomaly"

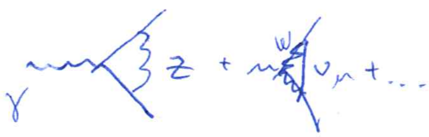
$\boxed{a = F_2(0)}$

recipe: $a \Leftrightarrow F_2(0) \Leftrightarrow (p_i + p_f)^\mu$ - term in Γ^μ.

loop results:



$\longrightarrow \quad a_\mu = \frac{\alpha}{2\pi}$ (Schwinger) $\sim 10^{-3}$



$\longrightarrow \quad a_\mu^{weak} \sim \frac{\alpha}{s_W^2} \cdot \frac{m_\mu^2}{M_W^2} = \sim 15 \cdot 10^{-10}$

status:

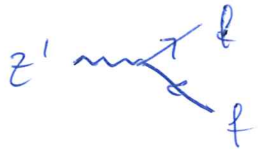
$$\boxed{a_\mu^{Exp} - a_\mu^{SM} \approx 30\,(8) \cdot 10^{-10}}$$

→ constraint on BSM

→ some BSM models can explain Δa_μ!

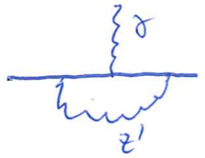
2.4.2 Simple BSM contributions

Z' model with $U(1)_{L_\mu - L_\tau}$



$$= -ig'\gamma^\mu \begin{cases} 1 & (f = \nu_\mu, \mu) \\ -1 & (f = \nu_\tau, \tau) \\ 0 & (\text{else}) \end{cases}$$

$\rightsquigarrow$

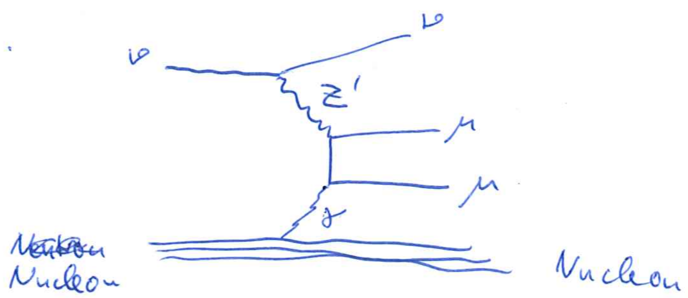


$$\Rightarrow \boxed{a_\mu^{(Z')} = \frac{g'^2}{12\pi^2} \frac{m_\mu^2}{M_{Z'}^2}}$$

- decoupling as $1/M_{Z'}^2$
- if $g' \approx g_{SM}$ and $M_{Z'} \approx M_Z \rightarrow a_\mu^{(Z')} \sim 10^{-9}$

  $\rightsquigarrow$ interesting!
- not forbidden by LHC/LEP since no coupl. to e^-, quarks
- however: would also contribute to

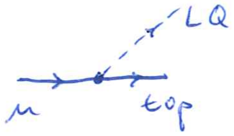
"neutrino trident production":



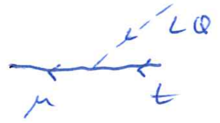
$\Longrightarrow$ excluded unless $M_{Z'}$ very small $\lesssim$ few GeV

(while $\frac{g'}{M_{Z'}} \approx \frac{g_{SM}}{M_Z}$)

Leptoquark model



$$= i\,(c_L P_L + c_R P_R)$$



$$= i\,(c_L^* P_R + c_R^* P_L)$$

$$\rightsquigarrow \quad a_\mu^{(LQ)} = \frac{m_\mu}{16\pi^2}\left\{ \frac{m_\mu}{12 M_{LQ}^2}\left(|c_L|^2 + |c_R|^2\right) f_1(x) + \frac{2 m_t}{3 M_{LQ}^2}\, \mathrm{Re}(c_L c_R^*)\, f_2(x) \right\}$$

$$x = m_t^2 / M_{LQ}^2 \quad , \quad f_i(1) = 1$$

- diag. term: $\sim \frac{m_\mu^2}{M_{LQ}^2}$ decoupling, magnitude similar to Z'-case
- off-diag $c_L c_R$-term: m_t instead of m_μ !?

  $\Rightarrow$ could explain Δa_μ with heavier M_{LQ} if $c_L c_R \neq 0$ and not too small !

Interpretation:

$$a_\mu \Leftrightarrow \bar{u}\,\sigma^{\mu\nu} u = \overline{u_L}\,\sigma^{\mu\nu} u_R + \overline{u_R}\,\sigma^{\mu\nu} u_L$$

$\boxed{\text{"chirality flip"}}$

$$\Leftrightarrow \mathcal{L}_{eff} = \sim \overline{\mu_L}\,\sigma^{\mu\nu}\mu_R$$

- SM gauge invariance req. $\underset{\text{doublet}}{\underset{\uparrow}{\bar{L}}}\,\sigma^{\mu\nu}\,\underset{\text{singlet}}{\underset{\uparrow}{\mu_R}}\,\underset{\text{VEV of some doublet}}{\underset{\uparrow}{\langle \Phi \rangle}}$
- breaks "chiral sym.": $\mu_R \rightarrow e^{i\alpha}\mu_R$

In all models: $a_\mu \sim$ VEV $*$ (chiral sym.-breaking parameter)

very often: ch. sym. broken by $\mathcal{L}_{Yuk} = y_\mu \cdot \bar{L}\,\mu_R\,\Phi$

e.g. here: combination $c_L\, y_{top}\, c_R$ also breaks ch. sym.

$\Rightarrow$ very often $a_\mu \sim m_\mu^2 / M_{BSM}^2$

here $a_\mu \sim m_\mu c_L c_R y_{top} v / M_{BSM}^2$