Standard Model Theory – Tutorial 3

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1. **Determinate the most genereal form of a Lagrangian symmetric under** $SU(2)$

   The gauge group of the Standard Model is given by

   $$SU(3)_C \otimes SU(2)_L \otimes U1_Y \tag{1}$$

   Let us condsider only the $SU(2)$ part for the moment. Under $SU(2)$ the fields transform as

   $$\psi_R \to \psi'_P = \psi_R, \qquad \psi_L \to \psi'_L = e^{i\tau\theta_a(x)\tau^a}\psi_L \tag{2}$$

   Write down the most generic Lagrangian build out of the matter fields of the Standard Model and $SU(2)$ gauge theory sector.

2. **Linear sigma model** Consider a theory with a multiplett of N scalar fields

   $$\Phi = \{\phi_1, \ldots, \phi_N\} \tag{3}$$

   transforming under $SO(N)$ together with the Lagrangian

   $$\mathcal{L} = \frac{1}{2}(\partial_\mu \Phi^t)(\partial^\mu \Phi) - V(\Phi) \tag{4}$$

   $$V(\Phi) = \frac{\lambda}{4}\left(|\Phi|^2 - \frac{\mu^2}{\lambda}\right)^2 \tag{5}$$

   (a) Derive the properties of the generators of the group from the properties of the group elements.

   (b) Count the numer of generators to obtain the dimension of the group.

   (c) Find the minimum of the energy.

   (d) Find an explicit state for Φ_0.

   (e) Determine the number of massless and massive scalar fields after spontaneous symmetry breaking.