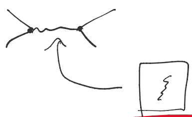
Monday, 20 April 2020 11:39

Aufgabe 2

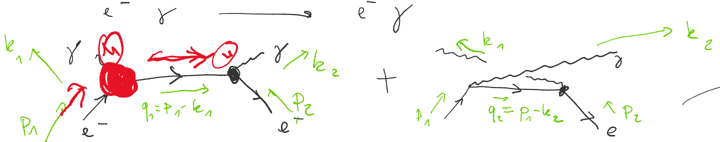
a) $e^+ e^- \longrightarrow \mu^+ \mu^-$



$\longrightarrow$ BRS-Inv. $\longrightarrow$ "Slavnov-Taylor-Identitäten" $\Longrightarrow$ phys. S-Matrix ist unabh. von ξ.

b) QED Compton-Streuung

$e^- \gamma \longrightarrow e^- \gamma$



$q_2 = -p_2 + k_1$

$$iT_{fi} = i\left(\mathcal{M}^{(1)}_{\mu\nu} + \mathcal{M}^{(2)}_{\mu\nu}\right) \varepsilon^{\mu *}(k_1, \lambda_1)\, \varepsilon^{\nu *}(k_2, \lambda_2)$$

$$\mathcal{M}^{(1)}_{\mu\nu} = \bar{v}(p_2, \sigma_2)\,[-ieQ\gamma_\nu]\, \frac{i}{\not{q}_1 - m_e}\, [-ieQ\gamma_\mu]\, u(p_1, \sigma_1)$$

$$\mathcal{M}^{(2)}_{\mu\nu} = \bar{v}(p_2, \sigma_2)\,[-ieQ\gamma_\mu]\, \frac{i}{\not{q}_2 - m_e}\, [-ieQ\gamma_\nu]\, u(p_1, \sigma_1)$$

Idee "$\partial_\mu j^\mu = 0$" $\longrightarrow$ "$k_\mu \mathcal{M}^\mu = 0$"

Beh: $k_1^\mu \mathcal{M}_{\mu\nu} = 0$

$$k_1^\mu \mathcal{M}^{(1)}_{\mu\nu} = \bar{v}\,[-ieQ\gamma_\nu]\, \frac{i}{\not{q}_1 - m_e}\, [-ieQ\not{k}_1]\, \boxed{u}$$

$$= \bar{v}\,[-ieQ\gamma_\nu]\,[-eQ] \qquad u$$

$$k_1^\mu \mathcal{M}^{(2)}_{\mu\nu} = \bar{v}(p)\,[-ieQ\not{k}_1]\, \frac{i}{\not{q}_2 - m_e}\, [-ieQ\gamma_\nu]\, u$$

$, k_1 = q_2 + p_2$

$k_1 = \cancel{p}_1 - \cancel{q}_1$

$= (\cancel{p}_1 - m_e) - (\cancel{q}_1 - m_e)$

$$\boxed{(\cancel{p}_1 - m_e)\, u(p_1) = 0}$$

$k_1 = \cancel{q}_2 + \cancel{p}_2$

$$= \bar{v}\,[eQ]\,[-ieQ\gamma v]\,u$$

$$\Longrightarrow \quad k_1^{\mu}\left(\mathcal{M}^{(1)}_{\mu\nu} + \mathcal{M}^{(2)}_{\mu\nu}\right) = 0 \qquad \text{q.e.d.}$$

Pol. vektoren:

$\vec{k} \sim \vec{e}_z$:
$k^\mu \sim (1,0,0,1)$

$$\varepsilon^\mu(\vec{k},1) = (0,1,0,0) \quad \text{x-Richtg}$$

$$\varepsilon^\mu(\vec{k},2) = (0,0,1,0) \quad \text{y-Richt.}$$

$\longrightarrow$ phys., transv. Pol.
(phys. beob./erlaubt)

$$\varepsilon^\mu(\vec{k},3) = (0,0,0,1) \quad \text{z-Richt.}$$

$$\varepsilon^\mu(\vec{k},0) = (1,0,0,0) \quad \text{t-artig}$$

$\vec{k} \sim \vec{e}_k$: nimm R : $\vec{e}_k = R\,\vec{e}_z$

$$\varepsilon^\mu(\vec{k},1) = (0, R\,\vec{e}_x)$$

$$\varepsilon^\mu(\vec{k},2) = (0, R\,\vec{e}_y)$$

$$\varepsilon^\mu(\vec{k},3) = (0, \vec{e}_k) \quad \text{(long.)}$$

$$\varepsilon^\mu(\vec{k},0) = (1,0,0,0)$$

$\boxed{\text{transf. nicht kovariant unter Lorentztransformationen!}}$

$\boxed{\text{Frage:}}$

System 1 $\longrightarrow$ $\boxed{T_{fi} = \mathcal{M}_{\mu\nu}\,\varepsilon^{*\mu}\varepsilon^{*\nu}}$

Lorentz Λ

|| ???

System 2 $\longrightarrow$ $\boxed{T'_{fi} = \mathcal{M}'_{\mu\nu}\,\varepsilon'^{*\mu}\varepsilon'^{*\nu}}$

$$= (\not{q}_2 - m_e)$$

$$+ (\not{p}_2 + m_e) \to 0$$

(↓) System 2 ⟶ $\boxed{T'_{fi} = \mathcal{M}'_{\mu\nu}\, \varepsilon'^{*\mu} \varepsilon'^{*\nu}}$

kPar $\mathcal{M}'_{\mu\nu} = \Lambda_\mu{}^\rho\, \Lambda_\nu{}^\sigma\, \mathcal{M}_{\rho\sigma}$

aber $\varepsilon'^\mu = \Lambda^\mu{}_\nu\, \varepsilon^\nu + a k'^\mu$

Auflösung des Paradoxon:

Beh.: $\boxed{\varepsilon'^\mu = \Lambda^\mu{}_\nu\, \varepsilon^\nu + a k'^\mu}$

Bew.: N.B. Eigenschaften des phys. transversalen ε'^μ ??

→ Basis eines 2-dim. Unterraumes

→ 0-Komponente = 0 *

→ räuml. Komp. $\perp$ $\vec{k}'$

$\cap$ ⇒ orthogonal zum 4-Vektor k'^μ *

$$\varepsilon'^\mu k'_\mu = \varepsilon'^0 k'^0 - \vec{\varepsilon}' \vec{k}' = 0 - 0 = 0$$

Betrachte $\Lambda^\mu{}_\nu \varepsilon^\nu$:

$$\varepsilon^\nu k_\nu = 0$$

$$\Rightarrow (\Lambda \varepsilon)^\mu (\Lambda k)_\mu = 0$$

$$\Rightarrow (\Lambda \varepsilon)^\mu k'_\mu = 0$$

Betrachte $\Lambda^\mu{}_\nu \varepsilon^\nu + a k'^\mu = v^\mu$

$$v^\mu k'_\mu = 0$$

$k'^0 \neq 0$ $\Rightarrow \exists a : v^0 = 0$

---

zurück zur Frage:

System 2: $T' = (\Lambda \Lambda \mathcal{M})\, \varepsilon'^{*\mu} \varepsilon'^{*\nu}$

system 2 : $T'_{fi} = (\Lambda\Lambda\mathcal{M})_{\mu\nu}\, \varepsilon_1'^{*\mu}\, \varepsilon_2'^{*\nu}$

$$\varepsilon_1'^{*\mu} = (\Lambda\varepsilon_1)^\mu + a_1 k_1'^\mu$$
$$\varepsilon_2'^{*\nu} = (\Lambda\varepsilon_2)^\nu + a_2 k_2'^\nu$$

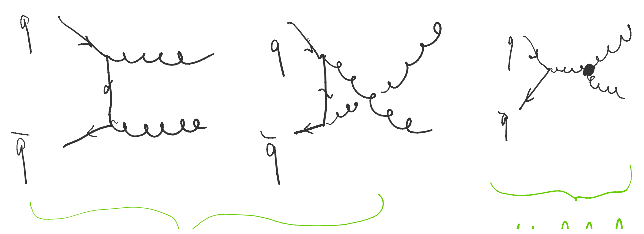
$$k_1^\mu \mathcal{M}_{\mu\nu} = 0 \qquad \text{Ward}$$
$$k_2^\nu \mathcal{M}_{\mu\nu} = 0$$

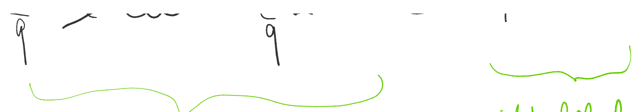
$$T'_{fi} = (\Lambda\Lambda\mathcal{M})_{\mu\nu} (\Lambda\varepsilon_1)^\mu (\Lambda\varepsilon_2)^\nu = T_{fi}$$

$\boxed{\text{Lorentzinvariant!}}$

Problem war: Lorentz inv. nicht offensichtlich, da phys.-transversale Pol.-vektoren nicht kovariant trafo.
Lösung: Eich → BRS → Ward-Identitäten $\mathcal{M}_{\mu\nu}$
⟹ Störterme fallen weg, Lorentzinv. gilt ☺

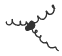
Q C D - Prozeß

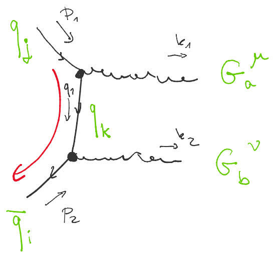




abelsch — nicht abelsch

- Ward Identität komplizierter / schwächer
- nicht abelsches Diagramm nötig, und:

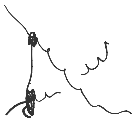
  Vertex  muß dieselbe Eichkopp. haben wie .



$$= \bar v(p_2,\sigma_2)\left[-ig\gamma_\nu T^b_{ik}\right] \cdot \frac{i}{\not q_1 - m}\left[-ig\gamma_\mu T^a_{kj}\right] \cdot u(p_1,\sigma_1)$$

$$= (T^b T^a)_{ij} \cdot \text{"QED"}\,\mathcal{M}^{(1)}$$

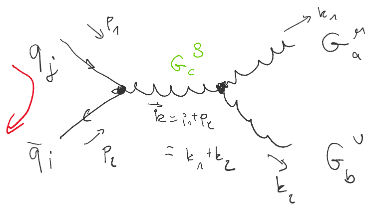
$$= \mathcal{M}^{(1)}_{\mu\nu}$$



$$= \mathcal{M}^{(2)}_{\mu\nu} \qquad \sim g^2$$

$$= (T^a T^b)_{ij} \quad \text{"QED"}\,\mathcal{M}^{(2)}$$

$$\boxed{k_1^\mu \, \mathcal{M}^{(1+2)}_{\mu\nu} \neq 0}$$



$$= \bar v(p_2,\sigma_2)\left[-ig\gamma_\rho T^c_{ij}\right]u(p_1,\sigma_1)$$

$$\frac{-i}{k^2} \, *$$

$$\sim (g\, g_3) \leftarrow g_3 = g \qquad (-g f_{abc})\left[g^{\mu\nu}(-k_1+k_2)^\rho + g^{\nu\rho}(-k_2-k)^\mu + g^{\rho\mu}(k+k_1)^\nu\right]$$

$$k_1^\mu \left(\mathcal{M}^{(1+2+3)}_{\mu\nu}\right) = ?$$

$$k_1^\mu \mathcal{M}^{(3)}_{\mu\nu} = + g^2 f_{abc} T^c_{ij} \frac{\left[\bar v \gamma_\rho u\right]}{(k_1+k_2)^2}\left[k_{1\nu}(k_2-k_1)^\rho + g^{\nu\rho}(-2k_2-k_1)\cdot k_1 + k_1^\rho(2k_1+k_2)_\nu\right]$$

NB $k_1^2 = 0 = k_2^2$

$$= g^2 f_{abc} T^c_{ij} \frac{\left[\bar v \gamma_\rho u\right]}{2k_1k_2}\left[k_{1\nu}(k_2-k_1)^\rho + g^{\nu\rho}(-2k_1k_2) + k_1^\rho(2k_1+k_2)_\nu\right]$$

$$\| = g^2 f_{abc} T^c_{ij} \left[\bar v \gamma_\rho u\right]\left\{\frac{k_{1\nu}k_{1\rho} + k_{1\nu}k_2^\rho + k_1^\rho k_{2\nu}}{2k_1k_2} - g_\nu^{\ \rho}\right\} \|$$

Vereinfachung $m_q = 0$

$$k_1^\mu \, \mathcal{M}^{(1+2)}_{\mu\nu} = -ig^2 \, \bar v \gamma_\nu u \left(-(T^b T^a)_{ij}\right.$$

$$\boxed{k_1^\mu \mathcal{M}^{(1+2)}_{\mu\nu}} = -ig^2\, \bar v \gamma_\nu u \left(-\left(T^b T^a\right)_{ij} + \left(T^a T^b\right)_{ij} \right)$$

$$= -ig^2 \bar v \gamma_\nu u \; i f_{abc} T^c_{ij}$$

$$= g^2 f_{abc}\, \bar v \gamma_\nu u \; T^c_{ij}$$

QED: $k_1^\mu \mathcal{M}^{(1+2)}_{\mu\nu} = 0 \to$ hier: $\sim f_{abc}$

$$\Rightarrow \boxed{k_1^\mu \mathcal{M}^{(1+2+3)}_{\mu\nu} \neq 0}$$

$$k_1^\mu \mathcal{M}^{(1+2+3)}_{\mu\nu} = g^2 f_{abc} T^c_{ij}\; \bar v \gamma_\rho u \left\{ g^\rho{}_\nu - g^\rho{}_\nu + \frac{k_{1\nu} k_1^\rho + k_{1\nu} k_2^\rho + k_1^\rho k_{2\nu}}{2 k_1 k_2} \right\}$$

$$= g^2 f_{abc} T^c_{ij}\; \bar v \gamma_\rho u \; \frac{\cancel{k_{1\nu} k_1^\rho} + \cancel{k_{1\nu} k_2^\rho} + k_1^\rho k_{2\nu}}{2 k_1 k_2}$$

Wunsch:

$$\boxed{\sim k_{2\nu}}$$

$$= \quad k_{1\nu}\left(k_1^\rho + k_2^\rho\right) = k_{1\nu}\left(p_1^\rho + p_2^\rho\right) \to 0$$

$$\boxed{k_1^\mu \mathcal{M}^{(1+2+3)}_{\mu\nu} = g^2 f_{abc} T^c_{ij}\; \bar v \not{k}_1 u \; \frac{k_{2\nu}}{2 k_1 k_2}}$$

$$\boxed{\Rightarrow k_1^\mu \mathcal{M}_{\mu\nu}\, \varepsilon^{*\nu}(k_2,\lambda_2) = \dots\dots \underbrace{k_{2\nu}\, \varepsilon^{*\nu}(k_2,\lambda_2)}_{=0} = 0}$$

Lorentzinvarianz: untere Gl. ausreichend

$\Rightarrow$ Lorentzinv. gilt, wie in QED.

Universalität der Eichkoppl.

$\Updownarrow$

Ward-/ Slavnov-Taylor-Id.

$\Rightarrow$ Lorentzinv.

$\Rightarrow$ Eichunabh. (ξ)

$\Rightarrow$ Unitarität der S-Matrix

$$\mathcal{L} = \frac{1}{2}(\partial^\mu\phi)(\partial_\mu\phi) \underbrace{- \frac{m^2}{2}\phi^2 - \frac{g}{4!}\phi^4}_{-V(\phi)}$$

später: $\mathcal{L}_{ct}$ $\quad \delta Z,\ \delta m^2,\ \delta g$

tree:

$$\underline{V(\phi) = \frac{m^2}{2}\phi^2 + \frac{g}{4!}\phi^4}$$

Frage: $\underline{V_{\text{tree}+1L}(\phi)\ ?}$

Frage: $V_{\text{tree}+1L}(\phi) < 0$

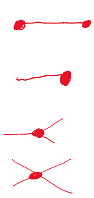
Antwort: $$\boxed{-\frac{\partial V}{\partial \phi}\Big|_{\phi_0} = \frac{\delta \Gamma}{\delta \phi}\Big|_{\phi_0} = \frac{\delta \Gamma^{(\phi_0)}}{\delta \phi}\Big|_0 = \sum \text{Tadpolediagr. d. Theorie mit } \mathcal{L}^{(\phi_0)}(\phi) = \mathcal{L}(\phi+\phi_0)}$$

Verschobene Theorie

$$\mathcal{L}^{(\phi_0)}(\phi) = \frac{1}{2}(\partial^\mu\phi)(\partial_\mu\phi) - \frac{m^2}{2}(\phi+\phi_0)^2 - \frac{g}{4!}(\phi+\phi_0)^4$$

$$= \frac{1}{2}(\partial^\mu\phi)(\partial_\mu\phi) - \text{const}$$
$$- \phi\left[m^2\phi_0 + \frac{g}{3!}\phi_0^3\right]$$
$$- \phi^2\left[\frac{m^2}{2} + \frac{g}{4}\phi_0^2\right]$$
$$- \phi^3\left[\frac{g}{3!}\phi_0\right]$$
$$- \phi^4\left[g/4!\right]$$

Feynmanregeln:



$$= \frac{i}{p^2 - M^2}, \qquad \boxed{M^2 = m^2 + \frac{g}{2}\phi_0^2}$$

$$= -it \qquad \boxed{t = m^2\phi_0 + \frac{g}{3!}\phi_0^3}$$

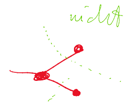
$$= -i g_3 \qquad \boxed{g_3 = g\,\phi_0}$$

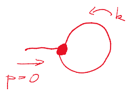
$$= -i g$$

1-Punkt-Funktion / "Tadpolediagramme"

tree:  $= -it$

nicht 1PI



1-Loop: 

Symmetriefaktor

$$= -\frac{1}{2}\, i g_3 \cdot \int \frac{d^D k}{(2\pi)^D}\mu^{4-D} \frac{i}{k^2 - M^2}$$
$$= \frac{g_3}{2}\frac{i}{16\pi^2} A_0(M^2)$$

ϕ_0 !

$D = 4 - 2\epsilon$

$$A_0(M^2) = -M^2\left(\frac{M^2}{4\pi\mu^2}\right)^{-\epsilon}\Gamma(\epsilon-1) = \frac{M^2}{\epsilon} + M^2\left(1 - \ln\frac{M^2}{\mu^2_{\overline{MS}}}\right) + \ldots$$

$$\left[\mu^2 = \mu^2_{\overline{MS}}\frac{e^{\gamma_E}}{4\pi}\right]$$

$$= \frac{i}{16\pi^2}\frac{g\,\phi_0}{2}\left(m^2 + \frac{g}{2}\phi_0^2\right)^{1-\epsilon} f(\epsilon)$$
$$= \frac{i}{16\pi^2}\frac{1}{2}\frac{1}{2-\epsilon}\frac{\partial}{\partial\phi_0}\left(m^2 + \frac{g}{2}\phi_0^2\right)^{2-\epsilon} f(\epsilon)$$
$$= \frac{\partial}{\partial\phi_0}\frac{1}{2}\frac{1}{2-\epsilon}\frac{i}{16\pi^2}\cdot A_0(M^2)\,M^2$$

$$\Longrightarrow \boxed{\begin{aligned} V^{1\text{Loop}}(\phi_0) &= -\frac{1}{2}\frac{1}{2-\epsilon}\frac{1}{16\pi^2}A_0(M^2)\,M^2 \\ V^{\text{tree}}(\phi_0) &= \frac{m^2}{2}\phi_0^2 + \frac{g}{4!}\phi_0^4 \end{aligned}}$$

$$V^{1\text{Loop}}(\phi_0) = \ldots \sim \left(m^2 + \frac{g}{2}\phi_0^2\right)^2 \ln\left(m^2 + \frac{g}{2}\phi_0^2\right)$$
$$\sim \phi_0^4 \ln(\phi_0)$$

Renormierung:

$\mathcal{L}_{ct}$: aus $\mathcal{L}_{cl}$ durch $g \to g + \delta g$
$m^2 \to m^2 + \delta m^2$

$\mathcal{L}_{ct}$: aus $\mathcal{L}_{cl}$ durch $g \to g + \delta g$

$m^2 \to m^2 + \delta m^2$

$\phi \to \sqrt{Z}\,\phi$

aus QFT : 1-Loop $\sqrt{Z} = 1$

$\curvearrowright$ $V^{tree} \longrightarrow V^{tree} + V^{ct}$

$$V^{ct} = \frac{\delta m^2}{2}\phi_0^2 + \frac{\delta g}{4!}\phi_0^4$$

Damit $V^{tot}_{\leq 1Loop} = V^{tree + ct + 1Loop}$

ist klar, dass Theorie endlich $\Longrightarrow$ $\frac{1}{\epsilon}$-Pole müssen wegfallen

$$V^{1Loop} = -\frac{1}{2}\frac{1}{2-\epsilon}\frac{1}{16\pi^2}A_0(M^2)M^2$$

$$\overset{1/\epsilon}{=} -\frac{1}{4}\frac{1}{16\pi^2}\frac{1}{\epsilon}M^2M^2$$

$$= -\frac{1}{4}\frac{1}{16\pi^2}\frac{1}{\epsilon}\left(m^2 + \frac{g}{2}\phi_0^2\right)^2$$

$$= -\frac{1}{4}\frac{1}{16\pi^2}\frac{1}{\epsilon}\left(m^4 + g m^2\phi_0^2 + \frac{g^2}{4}\phi_0^4\right)$$

Forderung : $V^{1Loop+ct} = \text{finite}$ (bis auf Konstante)

$$\Leftrightarrow \quad \frac{\delta m^2}{2} - \frac{1}{4}\frac{1}{16\pi^2}m^2 g\frac{1}{\epsilon} = 0$$

$$\frac{\delta g}{4!} - \frac{1}{4}\frac{1}{16\pi^2}\frac{g^2}{4}\frac{1}{\epsilon} = 0$$

$$\Leftrightarrow \quad \boxed{\begin{aligned} \delta m^2 &= m^2\frac{g}{32\pi^2}\frac{1}{\epsilon} + \text{finite} \\ \delta g &= \frac{3g^2}{32\pi^2}\frac{1}{\epsilon} + \text{finite}\end{aligned}}$$

Renormierungsschema $\Longleftrightarrow$ Wahl des endl. Anteils

$\overline{MS}$ - Schema $\Leftrightarrow$ "finite" $:= 0$

$\curvearrowright$ $V^{\overline{MS}-\text{Schema}}_{\leq 1Loop} = \ldots$

SM :

Näherung : nur Higgs + top-Quark

$$\mathcal{L}_{\text{"SM"}} = \frac{1}{2}(\partial^\mu\phi)(\partial_\mu\phi) - \underbrace{\frac{m^2}{2}\phi^2 - \frac{\lambda}{4!}\phi^4}$$

$\lambda = g$

$$+ y_t\,\phi\,\bar{\psi}\psi$$

ψ: top-Quark-Spinor

$$\mathcal{L}^{(\phi_0)}_{SM} = \frac{1}{2}(\partial^\mu\phi)(\partial_\mu\phi) - t\phi - \frac{M^2}{2}\phi^2 - \frac{g_3}{3!}\phi^3 - \frac{g}{4!}\phi^4$$

$$+ y_t(\phi+\phi_0)\bar{\psi}\psi$$

Feynmanregeln : wie bisher

$$\bullet\!\!\longrightarrow\!\!\bullet \; = \frac{i}{p^2 - M^2}$$

$$\xrightarrow{\text{top}} \; = \frac{i}{\not{p} - M_t}, \quad M_t = -y_t\phi_0$$

$$\longrightarrow\!\bullet \; = -it$$

$$-\!\!< \; = -ig_3$$

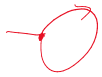
 $= -ig$

 $= i y_t$

$-i \frac{\partial}{\partial \phi_0} V^{1\text{Loop}}$: $\Leftrightarrow$ 

$=$  wie oben

 top-loop



$$= i y_t \,\mathrm{Tr}\left(\int \frac{d^D k}{(2\pi)^D} \frac{i}{\not{k} - M_t} \right) (-1)$$

$$= -i^2 y_t \,\mathrm{Tr} \left(\int \frac{d^D k}{(2\pi)^D} \frac{\not{k} + M_t}{k^2 - M_t^2} \right)$$

$$= \int \frac{d^D k}{(2\pi)^D} \frac{4 M_t y_t}{k^2 - M_t^2}$$

$$= 4 M_t y_t \frac{i}{16\pi^2} A_0(M_t^2)$$

$$= -4 \phi_0 y_t^2 \frac{i}{16\pi^2} A_0(\phi_0^2 y_t^2)$$

vorhin: $\frac{g_3}{2} \frac{i}{16\pi^2} A_0(M^2)$

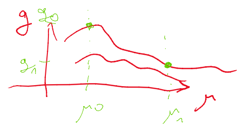
$\quad \llcorner\; m^2 + \frac{g}{2} \phi_0^2$

Renormierungsgruppe (zurück zur ϕ^4-Theorie)

$$\left(\frac{\partial}{\partial \ln \mu} + \beta \frac{\partial}{\partial g} + \ldots \cancel{+ \gamma \phi \frac{\partial}{\partial \phi}} \right) V(\phi) = 0$$

$\Leftrightarrow \quad V(\phi, \mu_0, g_0, \ldots) \overset{\text{bis}}{\underset{\text{auf}}{=}} V(\phi, \mu_1, g_1, \ldots)$

Umnormierung von ϕ



logs bleiben klein, wenn wir immer $\mu \approx \phi_0$ wählen.

$\Rightarrow$ Kopplungen in V ändern sich als Funktion von $\mu \approx \phi_0$

$\Rightarrow$ Wichtig im SM: RG-Trajektorien studieren!

Bisher: $V^{SM}_{\overline{MS}} = -\frac{\lambda}{4} \phi^4 + \frac{1}{16\pi^2} \phi^4 \left[\lambda^2 \ln \frac{\phi^2}{\mu^2} - y_t^4 \ln \frac{\phi^2}{\mu^2} \right]$

μ = fixiert, $\phi \to \infty$

Besser: $\mu \approx \phi$, z.B. $\mu = \phi$ und $\lambda(\mu)$, $y_t(\mu)$ gemäß RG-Trajektorie

$$V^{SM}_{\overline{MS}} = -\frac{\lambda(\phi)}{4} \phi^4 + \quad \sim 0$$

gemäß RG-Trajektorie

$$V^{SM}_{\overline{MS}} = \sim \frac{\lambda(\phi)}{4}\phi^4 + \quad \sim 0$$

Bestimme β in der ϕ^4-Theorie!

$$\Big(\underbrace{\frac{\partial}{\partial \ln\mu}}_{\mathcal{O}(\hbar)} + \underbrace{\beta}_{\mathcal{O}(\hbar)}\underbrace{\frac{\partial}{\partial g}\Big)V(\phi)}_{\mathcal{O}(\hbar^0)} = 0$$

auf 1-Loop, ϕ^4-Anteil (m^2-Terme, γ-Terme...)

$$V = \frac{g}{4}\phi^4 + \frac{1}{16\pi^2}\frac{1}{4}M^4\left(L - \frac{3}{2}\right)$$

$$\left(M^2 = \frac{g}{2}\phi^2\right.$$

$$= \ln M^2/\mu^2 = \ln M^2 - 2\ln\mu$$

$$\curvearrowright \quad \frac{\partial}{\partial \ln\mu}V(\phi) = \frac{1}{16\pi^2}\,\frac{1}{4}M^4\cdot(-2)$$

$$\frac{\partial}{\partial g}V(\phi) = \frac{1}{4}\phi^4 + \ldots\ldots$$

$$\Rightarrow \quad \beta \quad = \beta\,\frac{1}{4}\phi^4$$

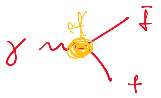
EWPO

(Pseudo-) Observablen

$$\cos\vartheta_W = \frac{M_W^{pol}}{M_Z^{pol}}$$

$\leftrightarrow$ Pole der Propagatoren $\Longrightarrow$ $\cos\vartheta_W \equiv c_W$

$$\tan\vartheta_W = \frac{g_1}{g_2} \qquad e = g_1\cos\vartheta_W = \frac{g_1 g_2}{\sqrt{g_1^2+g_2^2}}$$



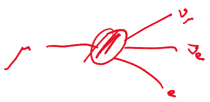
$= -ieQ_f^{\,eff}\,\gamma^\mu \rightarrow e^f_{eff}$

limes $E_\gamma \to 0$, f in Ruhe

$= -\frac{ie}{\sqrt{2}\,s_\theta}\,\gamma^\mu\,\frac{1-\gamma_5}{2} \rightarrow \sin\vartheta^{eff}_{W,f,\,W\text{-}WW}$

passe das limes

$= -\frac{ie}{\sqrt{2}\,s_\theta c_\theta}\,\gamma^\mu\left[I_{3f} - 2Q_f s_\theta^2 - I_{3f}\gamma_5\right] \rightarrow \sin\vartheta^{eff}_{W,\,f\text{-}Z}$



$= G_\mu = \frac{e^2}{4\sqrt{2}M_W^2 s_\theta^2} \Rightarrow G_\mu$

---

tree-level Vorhersage des SM

$$e_{eff,f} = e$$

$$\sin\vartheta_W^{eff} = s_W = \sin\vartheta^{eff}_{Z,ZZ} = \sin\vartheta_W \equiv \frac{g_1}{\sqrt{g_1^2+g_2^2}}$$

$$G_\mu = \frac{e^2}{4\sqrt{2}M_W^2 s_\theta^2}$$

i. a.

Bsp: $s_W^2 \left(= 1 - \frac{M_W^2}{M_Z^2}\right)^{Pol} = 0{,}2226$

$\sin\theta^{lept}_{eff, Z_{ff}} = 0{,}23153\,(16)$

Technisch implementieren!

Def. SM auf Loopniveau durch Renormierungsschema

$\Leftrightarrow$ endl. Anteile des Ren. konst.

$\Leftrightarrow$ phys. Bedeutung der renorm. Größen

On-Shell Schema im EWSM

- wähle als unabh. Parameter:

$$e,\ M_W,\ M_Z,\ M_h$$

- Renorm.-bed. so, dass:

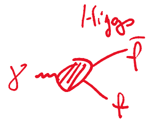
$$M_{W,Z,h} \equiv \text{Polmassen} = \text{Polstellen der Prop.}$$

$$e = e_{eff,f} \quad \text{bei } E_\gamma \to 0 \quad \text{fermion = Ruhe}$$

$$\Longleftrightarrow \quad = -i\hat{\Sigma}_{W,Z}(p^2 = M_{W,Z}^2) \overset{!}{=} 0$$

$$\left(\overset{1\text{Loop}}{\Longrightarrow} \quad \delta M_{W,Z}^2 = \Sigma_{W,Z}(p^2 = M_{W,Z}^2)\right)$$

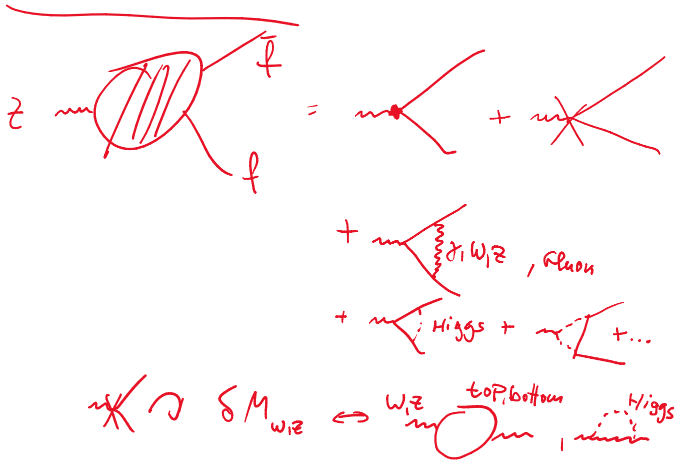
Higgs analog



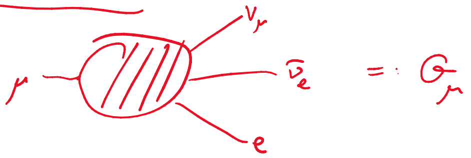
$q^2 = 0$, $p_f^2 = m_f^2$: $\quad -ieQ\,\hat{\Lambda}(q, p_f) \overset{!}{=} -ieQ\gamma_\mu$

Beispiel :

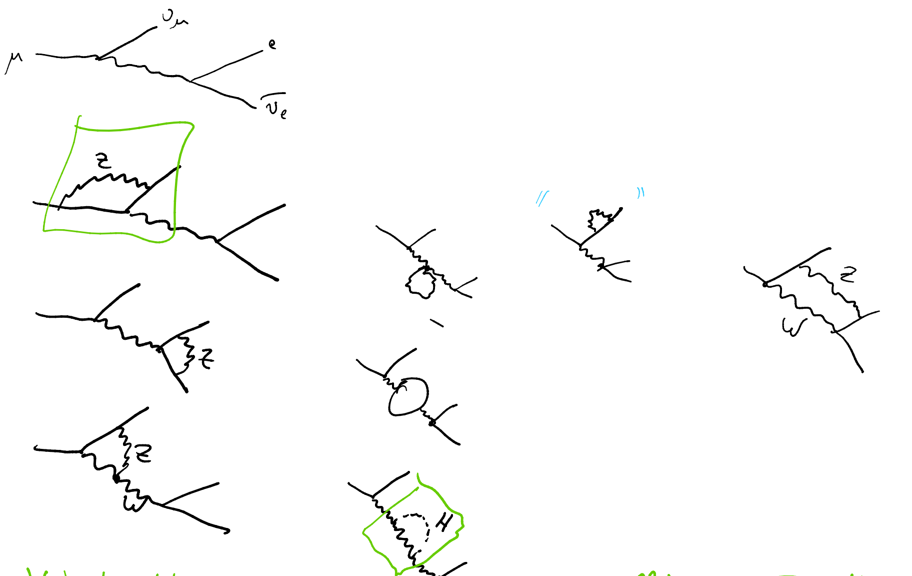
$\sin \theta^{eff}_{W,ZZ}$



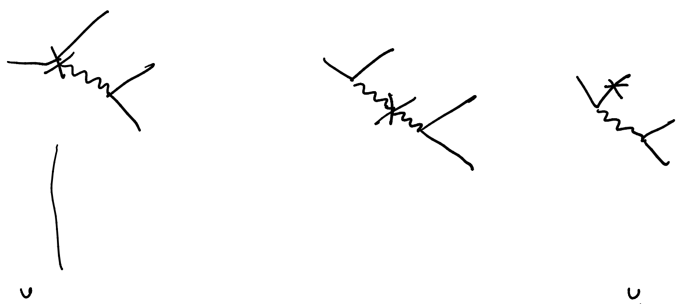
Myon-Zerfall



Aufgabe : Feynmandiagramme !



Vertexkorrekturen W-Selbstenergie f-Selbsten. Boxdiagr.



 $= -i\frac{e}{\sqrt{2}\, s_W}\gamma^\mu P_L \implies$  $= -i\frac{e}{\sqrt{2}\, s_W}\gamma^\mu P_L \cdot \left(\frac{\delta e}{e} - \frac{\delta s_W}{s_W} + \frac{1}{2}\delta Z_{\mu_L} + \frac{1}{2}\delta Z_{\nu_\mu} + \frac{1}{2}\delta Z_W\right)$

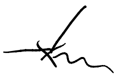
$e \to e + \delta e$

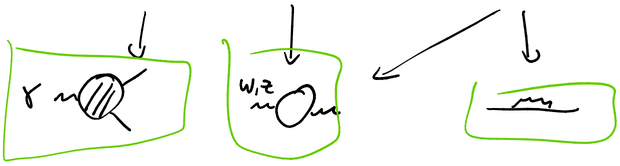
$M^2 \to M^2 + \delta M^2$

$\phi \to \sqrt{Z_\phi}\,\phi$

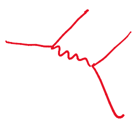
$$s_W^2 \equiv 1 - c_W^2 \equiv 1 - \frac{M_W^2}{M_Z^2}$$

$$\delta s_W^2 = \delta\left(1 - \frac{M_W^2}{M_Z^2}\right) = -\frac{\delta M_W^2}{M_Z^2} + \frac{M_W^2}{M_Z^4}\delta M_Z^2 = 2 s_W \delta s_W$$

 $\to \delta e, \; \delta M_{W,Z} \; ; \; \delta Z_{\mu,\nu,W}$



Myonzerfall

$G_\mu \;\hat{=}$  + Vertex + Box + Selbstenergie + CTe

---

• 

$$= \frac{-i g^{\mu\nu}}{q^2 - M_W^2 + \Sigma_W(q^2)}$$

• $e \to e + \delta e = e_{bare}$, $M^2 \to M^2 + \delta M^2 = M^2_{bare}$

$$\rightsquigarrow G_\mu^{CT} \;\hat{=}\; \frac{e^2}{4\sqrt{2}\, M_W^2\, s_W^2} \longrightarrow \frac{e_{bare}^2}{4\sqrt{2}\, M_{W,bare}^2\, s_{W,bare}^2}$$

• $\rightsquigarrow G_\mu = \frac{e_{bare}^2}{4\sqrt{2}\,(M^2_{W,bare} - \Sigma_W(0))\, s^2_{W,bare}}$ + Vertex + Box + Feldrenormierungs-CTe

---

Näherung: - größt mögliche Beiträge (vgl. typische Beiträge)

$$\int \frac{d^4k}{(2\pi)^4} \frac{1}{(k^2)^2} \frac{1}{(k^2-M^2)^2} = \frac{1}{16\pi^2} \frac{1}{M^4} \cdots$$

$\sim$ $\ast \;\ldots$

$$\frac{e^2}{M^2 s_w^2} \qquad \frac{e^2}{16\pi^2 s_w^2} = \frac{\alpha}{4\pi} \frac{1}{s_w^2} \sim 4\,‰$$

$\Longrightarrow$ betrachte nur speziell verstärkte Beiträge

---

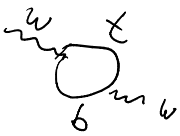
2 Mechanismen

(1) $\sim \dfrac{m_t^2}{M_W^2} \sim \dfrac{(172)^2}{(80)^2} \sim 4$

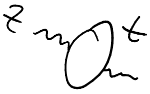
(2) $\sim \log \dfrac{m_{\text{leichte } f}}{M_Z} \longrightarrow \infty \quad (m_f \to 0)$

---

(1)



$\to \Sigma_W(q^2) \longrightarrow$ exp. ; $\longrightarrow \delta M_W^2$



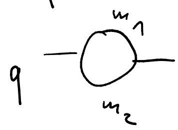
$\to \Sigma_Z(q^2) \to \delta M_Z^2$

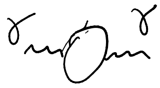
(2)

$B_0(q^2, m_1, m_2)$

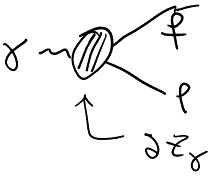
singulär, falls

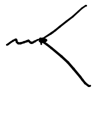
$q^2 = (m_1 + m_2)^2$





$\to \delta Z_\gamma$



$=$ 

$\xRightarrow{\text{o. Bew.}} \quad \dfrac{\delta e}{e} \cong -\frac{1}{2}\delta Z_\gamma \Longleftarrow$

δZ_γ enthält

$\log m_f^2$

δZ_γ enthält
$\log \frac{m_f^2}{\mu^2}$

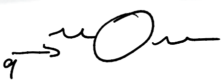
In dieser Näherung:

1) Vertex, Box, Feldren. $\longrightarrow$ vernachlässigbar

2) $e^2_{bare} = e^2\left(1+2\frac{\delta e}{e}\right) \simeq e^2\left(1-\delta Z_\gamma\right)$ log m_f leicht

3) $\left(M^2_{W,bare} - \Sigma_W(0)\right) s^2_{W,bare}$

$$\simeq \left(M_W^2 + \underbrace{\delta M_W^2}_{=\Sigma_W(M_W^2)} - \Sigma_W(0)\right) s_W^2 \left(1+2\frac{\delta s_W}{s_W}\right)$$

m_t^2-Anteile in  sind q^2-unabhängig

$\Rightarrow \Sigma_{W,Z}(q^2) \simeq \Sigma_{W,Z}(0)$

$$\boxed{G_\mu^{Näh.} = G_\mu^{tree}\left(\frac{1+\Delta\alpha}{1+2\frac{\delta s_W}{s_W}}\right)}$$

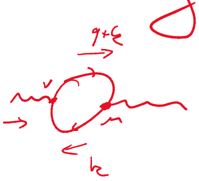
log m_f-Anteil $\leftarrow$ 6%

m_t^2-Anteil

m_t^2-, log m_f - Anteile universell, tauchen immer auf!

δe:

Berechnung in QED

$-i\Sigma^{\mu\nu}(q) =$ 

$$= -\mathrm{Tr}\left(\int_k (-ieQ)\gamma^\mu \frac{i}{\not q+\not k-m}(-ieQ)\gamma^\nu \frac{i}{\not k - m}\right)$$

$$= -\int \frac{d^Dk}{(2\pi)^D}\mu^{4-D}(eQ)^2 \frac{\mathrm{Tr}\left[\gamma^\mu(\not q+\not k+m)\gamma^\nu(\not k+m)\right]}{\left[(q+k)^2-m^2\right]\left[k^2-m^2\right]}$$

$$= -(eQ)^2\int_k \frac{m^2 4g^{\mu\nu} + 4\left((q+k)^\mu k^\nu + (q+k)^\nu k^\mu - (q+k)\cdot k\, g^{\mu\nu}\right)}{[\quad][\quad]}$$

$$= -(eQ)^2 \int_k \frac{m^2 4 g^{\mu\nu} + 4\left((q+k)^\mu k^\nu + (q+k)^\nu k^\mu - (q+k)\cdot k\, g^{\mu\nu}\right)}{[\;\;"\;\;][\;\;"\;\;]}$$

$$= -4(eQ)^2 \int_k \frac{m^2 g^{\mu\nu} + q^\mu k^\nu + q^\nu k^\mu + 2k^\mu k^\nu - g^{\mu\nu}(qk + k^2)}{[\;\;"\;\;][\;\;"\;\;]}$$

QED-Ward Id.

$$\Sigma^{\mu\nu}(q) \cdot q_\nu = 0$$

$$\Rightarrow \Sigma^{\mu\nu}(q) = \left(g^{\mu\nu} q^2 - q^\mu q^\nu\right) \underbrace{\Pi_\gamma(q^2)}$$

Lorentz inv.
"Vakuumpolaris"

Vereinfachen:

$$g_{\mu\nu} \Sigma^{\mu\nu}(q) = D q^2 - q^2 \Pi_\gamma(q^2)$$

$$\Rightarrow \Pi_\gamma(q^2) = \frac{1}{(D-1) q^2} \underbrace{\Sigma^\mu{}_\mu(q^2)}_{\text{reicht aus!}}$$

Hier:

$$-i\Sigma^\mu{}_\mu(q^2) = -4(eQ)^2 \int_k \frac{m^2 D + q\cdot k \cdot 2 + 2k^2 - D(qk + \ldots}{[\;\;"\;\;][\;\;"\;\;]}$$

$$= -4(eQ)^2 \int_k \frac{m^2 D + k^2(2-D) + q\cdot k\,(2 \ldots}{[\;\;"\;\;][\;\;"\;\;]}$$

$$= -4(eQ)^2 \int_k \frac{k^2(2-D) + q\cdot k\,(2-D) + m^2 D}{[(q+k)^2 - m^2][k^2 - m^2]}$$

Struktur: $\displaystyle \int \frac{k^2 + k q}{N_1 N_2} = \int \frac{\frac{1}{2} N_1 + \frac{1}{2} N_2 - \frac{1}{2} \ldots}{N_1 N_2}$

$$= \int \frac{1}{2}\frac{1}{N_2} + \frac{1}{2}\frac{1}{N_1} + \frac{-\frac{1}{2}q^2 + m^2}{N_1 N_2}$$

$$= \frac{1}{2}\frac{i}{16\pi^2} A_0(m) + \frac{1}{2}\frac{i}{16\pi^2} A_0(m) + \left(-\frac{1}{2}q^2 + m^2\right)\frac{i}{16\pi^2} B_0(q^2, \ldots$$

$$\Rightarrow -i\Sigma^\mu{}_\mu(q^2) = -4(eQ)^2 \frac{i}{16\pi^2} \left\{ (2-D)\left[\frac{2}{2} A_0(m) + \left(m^2 - \frac{q^2}{2}\right) B_0\right] + \ldots\right.$$

$$\Rightarrow \Pi_\gamma(q^2) \Rightarrow \Sigma^{\mu\nu}(q^2)$$

$$\Rightarrow \Pi \ldots \quad q^2 \to 0 \quad 4(eQ)^2 \ldots \quad q^2$$

$+k^2)$

$-D))$

q^2+m^2

$m,n)$

$Du^2 B_0\}$

T

$$\Rightarrow \Pi_\gamma(q^2) \xrightarrow{q^2\to 0} \frac{4(eQ)^2}{16\pi^2}\,\frac{1}{3}\Big\{\underbrace{B_0(0,m,m)}_{\left[\frac{1}{\epsilon}+\ln\frac{\mu^2}{m^2}\right]} + \frac{q^2}{5m^2}$$

$$\Rightarrow \frac{\delta e}{e} \sim \Pi_\gamma(0) \sim \frac{4}{16\pi^2}e^2Q^2 \ln\frac{\mu^2}{m^2}$$

Häufig $\Pi_\gamma(0)^{f\text{-Anteil}} =: \Delta\alpha(\mu) \sim \sum_f \frac{\alpha}{4\pi}$

$$\boxed{\begin{array}{l}\Delta\alpha = \text{extrem wichtige, universelle Größe}\\ \alpha \to \alpha(1+\Delta\alpha)\\ \Delta\alpha^{\text{alle leichten } f}(M_Z) = 0{,}05920\ (22)\\ \qquad\approx 6\,\%\text{-Effekt!}\end{array}}$$

$$\frac{\delta s_W^2}{s_W^2} = \frac{\delta\left(1-\frac{M_W^2}{M_Z^2}\right)}{1-\frac{M_W^2}{M_Z^2}} = \frac{1}{s_W^2}\left(-\frac{\delta M_W^2}{M_Z^2} + \frac{\delta M_Z^2}{M_Z^2}\right.$$

$$= \frac{1}{s_W^2}\frac{M_W^2}{M_Z^2}\left(-\frac{\delta M_W^2}{M_W^2} + \frac{\delta M_Z^2}{M_Z^2}\right)$$

$$= \underbrace{\frac{c_W^2}{s_W^2}}\left(\frac{\delta M_Z^2}{M_Z^2} - \frac{\delta M_W^2}{M_W^2}\right)$$

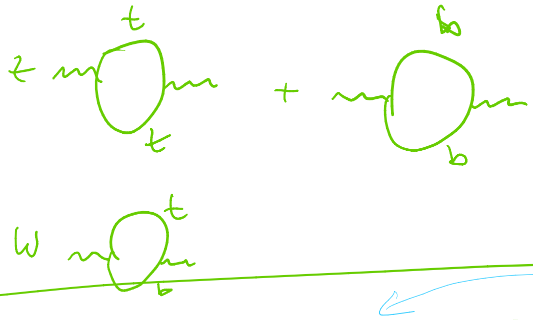
~ 3

$\ln \frac{\mu}{m_f}$

~ 3

$$\underbrace{-\frac{1}{M^2}\Sigma(0)}_{=:\Delta\rho}$$

zu berechnen:



$$\Delta\rho = \frac{3\, m_t^2\, e^2}{64\pi^2\, s_W^2\, M_W^2}$$

Farben der Quarks

$$\sim \frac{m_t^2}{M_W^2}$$

$\Delta\rho$ = universelle, wichtige Größe

$\sim m_t^2/M_W^2$

$\sim 1\,\%$

im μ-Zerfall $\Longrightarrow$ 3 %-Effekt

negativ

$\Longrightarrow$

$$G_\mu^{\text{SM-Vorhersage}} =: G_\mu^{\text{tree}}\left(1 + \Delta r\right)$$

Näh.

$$\approx G_\mu^{\text{tree}}\left(1 + \Delta\alpha - \frac{c_W^2}{s_W^2}\,\Delta\rho\right)$$

$$\approx G_\mu^{tree}\left(1+\Delta\alpha-\frac{c_W^2}{s_W^2}\Delta\rho\right)$$

$$(1+6\%-3\%)$$

$\Rightarrow$ 3 % - Korrektur

zur Relation zw.

$$\boxed{\mu\text{-Lebensdauer} \longleftrightarrow \alpha,\ M_W,\ M_Z}$$

nach Korrektur: Übereinstimmung mit Exp.!

nötig dafür: $m_t \approx 170$ GeV