Physics Beyond the Standard Model — Exercise Sheet 1

Summer term 2019 Prof. Dominik Stöckinger, Dr. Hyejung Stöckinger-Kim (IKTP)

1. **P- and T-violation: the electric dipole moment** An electric field can be generated e.g. by a static, charged capacitor. A magnetic field can be generated e.g. by a circular current.

   a) What happens to an electric and magnetic field under spatial reflection $(x, y, z) \to (-x, -y, -z)$ (parity P)?

   b) What happens to an electric and magnetic field under time reversal $t \to -t$ (T)?

   c) What happens to angular momentum or spin under parity or time reversal?

   d) For an elementary particle, the only distinguished direction is the one of the spin $\vec{s}$. Discuss the different implications of a discovery of an elementary particle's magnetic dipole moment $\vec{\mu}$ or electric dipole moment $\vec{d}$. You might also use the effective interaction Hamiltonians

$$H_{\text{MDM}} = -|\vec{\mu}|\frac{\vec{s}\cdot\vec{B}}{|\vec{s}|},\ H_{\text{EDM}} = -|\vec{d}|\frac{\vec{s}\cdot\vec{E}}{|\vec{s}|}.$$

2. **Baryogenesis: a very simple model**

   Assume the following: in the very early universe, very heavy particles of type X and corresponding antiparticles $\bar{X}$ were created (e.g. particles related to some Grand Unified gauge theory). Initially, they might have been in equilibrium with the rest, but at some point their interactions became too weak, so they effectively became sterile, interaction-less heavy relics. They were rather stable, with a lifetime of a millisecond or so. Then, after this time, when the rest of the universe had cooled down to a temperature of, say, 100 GeV, they decayed.

   a) At the time of the decay, is Sakharov's condition of deviation from thermal equilibrium fulfilled? Why?

   b) Some possible decay modes might be:

$$X \to p + \bar{n}, X \to p + e^- + \bar{n}, X \to \pi^+\nu_e, X \to \bar{p} + e^+, X \to n + Z$$

   and corresponding decay modes might exist for $\bar{X}$. Maybe you can invent further decay modes. Choose some values of branching fractions which lead to a non-vanishing baryon-antibaryon asymmetry of the universe. Explain how the Sakharov criteria are fulfilled for your choice.

3. **Higgs Triplet Model: Higgs and gauge boson masses**

   This exercise touches one of the "miracles" of the SM: the relation between the W and Z boson masses following from an accidental (?) "custodial symmetry". In the Standard Model there is a Higgs doublet Φ with hypercharge $Y_\Phi = 1/2$. The gauge covariant derivative is given by

$$D^\mu\Phi = (\partial^\mu + ig_W T^a W^{a\mu} + ig_Y Y_\Phi B^\mu)\Phi,$$

where T^a denotes the $\mathrm{SU}(2)_L$ generators in the doublet representation, i.e. $T^a = \frac{\sigma^a}{2}$. The relevant term in the Lagrangian is

$$\mathcal{L}_{\mathrm{kin}} = (D^\mu \Phi)^\dagger (D_\mu \Phi).$$

Further, in the Standard Model one defines the $W^\pm$, Z bosons and the photon as linear combinations of $W^{a\mu}$ ($a = 1, 2, 3$) and B^μ, but this is not necessary here — you can keep working with $W^{a\mu}$ ($a = 1, 2, 3$) and B^μ for the purpose of this exercise.

Consider an alternative to the Standard Model, where the Higgs doublet is replaced by a Higgs triplet ϕ^i, $i = 1, 2, 3$, which transforms under $\mathrm{SU}(2)_L$ in the adjoint representation (i.e. the generators are $(T^a_{\mathrm{adj}})_{ij} = -i\epsilon_{aij}$). The covariant derivative and Lagrangian term has the same form as above, with obvious replacements.

a) Consider a Higgs triplet with hypercharge $Y_\phi = 0$ and vacuum expectation value $\langle(\phi_i)\rangle = (0, 0, v)^T$. Compute the tree-level gauge boson masses: for this computation evaluate the Lagrangian term above with the replacement $\phi \to \langle\phi\rangle$, write the result as a mass matrix of the form

$$(W^{1\mu}\ W^{2\mu}\ W^{3\mu}\ B^\mu)\mathcal{M}^2 \begin{pmatrix} W^1_\mu \\ W^2_\mu \\ W^3_\mu \\ B_\mu \end{pmatrix}$$

and specify the eigenvalues of this matrix.

b) Repeat the exercise for a Higgs triplet with hypercharge $Y_\phi = 1$ and vacuum expectation value $\langle(\phi_i)\rangle = (v, -iv, 0)^T/\sqrt{2}$. (Why this form? Because then the vacuum preserves QED gauge invariance: $Q\langle(\phi_i)\rangle = (T^3_{\mathrm{adj}} + Y_\phi)\langle(\phi_i)\rangle = 0$.)

c) Repeat the exercise for the ordinary SM with one Higgs doublet. Here, the mass matrix has an extra symmetry: its $_{11}$, $_{22}$, and $_{33}$ entries are equal — reflecting what is called "custodial symmetry".