Physics Beyond the Standard Model — Exercise Sheet 2

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1. **The Standard Model extended with one real scalar singlet field**

   Literature: Pruna, Robens, "The Higgs Singlet extension parameter space in the light of the LHC discovery", arxiv:1303.1150.

   We consider the standard model (SM) extended with one real scalar singlet field $\mathcal{S}$. This singlet-extended SM contains the SM Higgs doublet field Φ and the new singlet $\mathcal{S}$, which can be parametrised in unitary gauge as

$$\Phi(x) = \begin{pmatrix} 0 \\ (\phi(x)+v)/\sqrt{2} \end{pmatrix}, \tag{1}$$

$$\mathcal{S}(x) = \frac{s(x)+v_s}{\sqrt{2}}. \tag{2}$$

   The Lagrangians of the ordinary SM and the singlet-extended SM are given as

$$\mathcal{L}_{\text{SM}} = \mathcal{L}_{\text{gauge,q,l,H kin}} + \mathcal{L}_{\text{Yuk}} - V_{\text{SM}}(\Phi), \tag{3}$$

$$\mathcal{L}_{\mathcal{S}} = \mathcal{L}_{\text{gauge,q,l,H kin}} + \mathcal{L}_{\text{Yuk}} + \mathcal{L}_{\mathcal{S}\text{ kin}} - V(\Phi,\mathcal{S}) \tag{4}$$

   where several parts are identical, and where

$$\mathcal{L}_{\mathcal{S}\text{ kin}} = \frac{1}{2}(\partial^\mu \mathcal{S})(\partial_\mu \mathcal{S}), \tag{5}$$

$$V(\Phi,\mathcal{S}) = -m^2|\Phi|^2 - \mu^2\mathcal{S}^2 + \lambda_1|\Phi|^4 + \lambda_2\mathcal{S}^4 + \lambda_3|\Phi|^2\mathcal{S}^2, \tag{6}$$

   so that $\mathcal{L}_{\text{SM}} = \mathcal{L}_{\mathcal{S}}|_{\mathcal{S}=0}$. The Lagrangian obeys a Z_2-symmetry — it is invariant under $\mathcal{S} \to -\mathcal{S}$.

   (a) Rewrite the potential $V(\Phi,\mathcal{S})$ in terms of the ϕ and s fields, until you find a form

$$V(\Phi,\mathcal{S}) = \text{const.} + t_\phi\phi + t_s s + \text{terms of higher powers in } \phi, s. \tag{7}$$

   Give an expression for the "tadpole" parameters t_ϕ, t_s. What is the solution of the minimization conditions $\partial V/\partial\phi|_{\phi=s=0} = \partial V/\partial s|_{\phi=s=0} = 0$? (You can express the solution as equations for m^2 and μ^2.)

   (b) We define $\mathcal{M}^2$ as the mass mixing matrix in the Higgs sector, i.e. we have by definition

$$V(\Phi,\mathcal{S}) = \text{const.} + t_\phi\phi + t_s s + \frac{1}{2}(\phi,s)\mathcal{M}^2\begin{pmatrix}\phi \\ s\end{pmatrix} + \text{terms of even higher powers in } \phi, s. \tag{8}$$

   Show that the mass matrix takes the following value, if the minimization conditions from above are satisfied:

$$\mathcal{M}^2 = \begin{pmatrix} 2\lambda_1 v^2 & \lambda_3 v v_s \\ \lambda_3 v v_s & 2\lambda_2 v_s^2 \end{pmatrix} \tag{9}$$

We then diagonalize the matrix $\mathcal{M}^2$ by choosing a suitable angle α and define new fields $h(x)$, $H(x)$ such that

$$\begin{pmatrix} m_h^2 & 0 \\ 0 & m_H^2 \end{pmatrix} = \begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix} \mathcal{M}^2 \begin{pmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{pmatrix}, \tag{10}$$

$$\begin{pmatrix} h \\ H \end{pmatrix} = \begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix} \begin{pmatrix} \phi \\ s \end{pmatrix}. \tag{11}$$

The new fields are called "mass eigenstate fields".

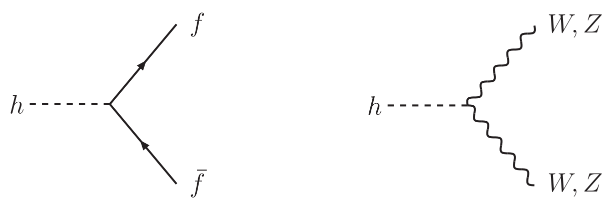
(c) The Yukawa terms (the interactions between SM-like Higgs and all the fermions) are given as

$$\mathcal{L}_{\text{Yuk}} = \sum_{\text{fermions} f} (v+\phi)\frac{-m_f}{v}\bar{f}f\,. \tag{12}$$

Furthermore, the Higgs kinetic terms with covariant derivatives give rise to the following Higgs–gauge boson interaction terms

$$\mathcal{L}_{\text{Higgs–gauge}} = \frac{(v+\phi)^2}{v^2}\left(\frac{M_Z^2}{2}Z^\mu Z_\mu + M_W^2 W^{+\mu}W_\mu^-\right)\,. \tag{13}$$

Rewrite these Lagrangians in terms of the mass eigenstate fields and determine what are the Feynman rules for the diagrams below:



You can express the results also in terms of the corresponding SM results.

(d) Now we discuss physics consequences for the case $m_h = m_{\text{Higgs at LHC}} = 125$ GeV and $m_H > m_h$. Hint: the result of the previous exercise is simply always $\cos\alpha\times$SM-value.

- What is the total production cross section $\sigma_{pp\to h}$ compared to the SM case?
- What are all partial decay widths (probabilities) $\Gamma_{h\to ii}$, where i is any SM particle, compared to the SM cases? How about the total decay rate $\Gamma_h = \sum_i \Gamma_{h\to ii}$ and the branching ratios?
- What kind of constraint on which singlet-extension parameter do you expect from the fact that all LHC measurements of the products $\sigma_{pp\to h} \times \text{BR}(h \to ii)$ agree with the SM-prediction to within approximately 10%?
- Consider the heavy Higgs H and assume it can decay only into SM particles (not into hh). What are the heavy Higgs Feynman rules? Which parameter can you constrain (from above or below?) from the non-discovery of any new Higgs bosons in searches for, e.g. $pp \to H \to \tau\tau$?

2. **General ρ parameter (at tree-level):**

   Consider an alternative to the Standard Model, where the Higgs doublet is replaced by a single Higgs multiplet ϕ^i, which transforms in some irreducible representation of $\mathrm{SU}(2)_L$ and which has the hypercharge Y_ϕ. The $\mathrm{SU}(2)_L$ representation is characterized by the eigenvalue of the Casimir operator $\vec{T}^2 = (T^1)^2 + (T^2)^2 + (T^3)^2 = T(T+1)$ with $T \in \{0, \frac{1}{2}, 1, \frac{3}{2}, \ldots\}$ (note that this is analogous to the case of angular momentum since the rotation group and SU(2) are locally isomorphic). The vacuum expectation value v is annihilated by $Q = T^3 + Y_\phi$. Define the tree-level ρ-parameter by

$$\rho = \frac{M_W^2}{M_Z^2 \cos\theta_W^2} \tag{14}$$

   and show, by computing the gauge boson masses, that at tree-level

$$\rho = \frac{T(T+1) - (Y_\phi)^2}{2(Y_\phi)^2} \tag{15}$$

   The calculation can proceed in the same way as in exercise 3/sheet 1. Check/evaluate the formula in the Standard Model and in the Higgs triplet models of that previous exercise.

3. **Leptoquark models:**

   Consider an extension of the SM by leptoquarks ϕ. Out of the 10 possible types of leptoquarks (see particle data group review on leptoquarks), we take the two options for interactions

$$\mathcal{L}_{LQ1} = \lambda_{IJ}\phi_1 \bar{d}^I P_L L^J + h.c. \tag{16}$$

$$\mathcal{L}_{LQ2} = \lambda'_{IJ}\bar{Q}^I P_R l^J \phi_2 + \lambda''_{IJ}\bar{u}^I P_L L^J \cdot \phi_2 + h.c. \tag{17}$$

   where L^I, Q^I are the SM doublets of left-handed leptons and quarks of generation I, and similar for the right-handed singlets l^I, u^I, d^I.

   a) What must the hypercharges of ϕ_1 and ϕ_2 be in order for these interactions to be gauge invariant? In case 2, check that both terms are gauge invariant for the same hypercharge assignment.

   b) Write down Feynman diagrams for the three interactions. In case 2, the leptoquark is an $\mathrm{SU}(2)_L$ doublet $\phi_2 = \binom{\phi_2^1}{\phi_2^2}$, and the dot product is defined as $L \cdot \phi_2 = \nu\phi_2^2 - l\phi_2^1$.

   c) In which case(s) can you obtain a leptoquark tree-level contribution to the process $b \to ce\bar{\nu}_e$? Which couplings are involved in the Feynman diagram?

   d) In which case(s) can you obtain a leptoquark one-loop contribution to $g-2$ of the muon, i.e. to the muon-muon-photon interaction? Which couplings are involved?