Physics Beyond the Standard Model — Exercise Sheet 4

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1. **Analysis of the general vacuum of the 2HDM**

   Literature: J.L. Diaz-Cruz, A. Méndez, Nucl. Phys. B380 39 (1992)

   After a proper $SU(2)\times U(1)$ rotation we obtain the most general form of the VEV of the two Higgs-doublets, Φ_1 and Φ_2

   $$\langle\Phi_1\rangle_{\mathrm{V}}^{\mathrm{T}} = (0, v_1), \qquad \langle\Phi_2\rangle_{\mathrm{V}}^{\mathrm{T}} = (u, v_2 e^{i\xi}) \tag{1}$$

   where v_1, v_2, u and ξ are real ($v_1 > 0$). We consider the CP-conserving and soft Z_2 breaking potential

   $$\begin{aligned} V^{\mathrm{2HDM}} &= m_{11}^2\Phi_1^\dagger\Phi_1 + m_{22}^2\Phi_2^\dagger\Phi_2 + m_{12}^2(\Phi_1^\dagger\Phi_2 + \Phi_2^\dagger\Phi_1) \\ &+ \lambda_1(\Phi_1^\dagger\Phi_1)^2 + \lambda_2(\Phi_2^\dagger\Phi_2)^2 + \lambda_3(\Phi_1^\dagger\Phi_1)(\Phi_2^\dagger\Phi_2) \\ &+ \lambda_4(\Phi_1^\dagger\Phi_2)(\Phi_2^\dagger\Phi_1) + \frac{1}{2}\lambda_5((\Phi_1^\dagger\Phi_2)^2 + (\Phi_2^\dagger\Phi_1)^2)\,, \end{aligned} \tag{2}$$

   and write the Higgs doublets with 8 real scalar fields

   $$\Phi_1 = \begin{pmatrix} \phi_1 + i\phi_2 \\ v_1 + \phi_3 + i\phi_4 \end{pmatrix}, \qquad \Phi_2 = \begin{pmatrix} u + \phi_5 + i\phi_6 \\ v_2 e^{i\xi} + \phi_7 + i\phi_8 \end{pmatrix}, \tag{3}$$

   where $\langle\phi_i\rangle_{\mathrm{V}} = 0$.

   (a) Verify the conditions for the potential extrema obtained from

   $$\left.\frac{\partial V}{\partial \phi_i}\right|_{\phi_k=0} = 0\,. \tag{4}$$

   i. $u(m_{12}^2 + (\lambda_4+\lambda_5)v_1 v_2\cos\xi) = 0$
   ii. $u v_1 v_2 \sin\xi(\lambda_4 - \lambda_5) = 0$
   iii. $m_{11}^2 v_1 + m_{12}^2 v_2\cos\xi + 2\lambda_1 v_1^3 + \lambda_3 v_1(u^2+v_2^2) + v_1 v_2^2(\lambda_4 + \lambda_5(\cos^2\xi - \sin^2\xi)) = 0$
   iv. $v_2\sin\xi(m_{12}^2 + 2\lambda_5 v_1 v_2\cos\xi) = 0$
   v. $u(m_{22}^2 + 2\lambda_2(u^2+v_2^2) + \lambda_3 v_1^2 = 0$
   vi. $m_{22}^2 v_2\cos\xi + m_{12}^2 v_1 + 2\lambda_2 v_2(u^2+v_2^2)\cos\xi + \lambda_{345}v_1^2 v_2\cos\xi = 0$
   vii. $v_2\sin\xi(m_{22}^2 + 2\lambda_2(u^2+v_2^2) + (\lambda_3+\lambda_4-\lambda_5)v_1^2) = 0$

   (b) Find the conditions for v_1, v_2, u, and ξ satisfying the equations above and discuss the cases for $m_{12}^2 = 0$ and $m_{12}^2 \neq 0$.

   (c) What is the symmetry of the vacuum if $u = 0$ and if $u \neq 0$? (*i.e.* which linear combination(s) of the four generators T^a and Y (with $a = 1, 2, 3$) annihilates the vacuum $\langle\Phi_{1,2}\rangle_{\mathrm{V}}$). What is the consequence for the gauge boson masses?

2. **FCNC-free Yukawa Lagrangian**

Show that no Higgs-mediated tree-level FCNCs appear if the Yukawa Lagrangian has the form

$$\mathcal{L}_{\rm Yuk} = Y^d_{ij}\bar{Q}^0{}_i\mathcal{F}_d(\{\Phi_k\})d^0_{j\rm R} + Y^u_{ij}\bar{Q}^0{}_i\mathcal{F}_u(\{\Phi_k\})u^0_{j\rm R} + Y^l_{ij}\bar{L}^0{}_i\mathcal{F}_l(\{\Phi_k\})e^0_{j\rm R} + \text{H.c.} \qquad (5)$$

where $\mathcal{F}_{u,d,l}(\{\Phi_k\})$ are functions of Higgs fields $\{\Phi_k\}$, which are $SU(2)_{\rm L}$ doublets and possess the following properties

(a) independent of the fermionic flavour indices i, j

(b) $\mathcal{F}_u$ transforming like an $SU(2)_{\rm L}$ doublet with hypercharge $Y = -1/2$ and $\mathcal{F}_d$ and $\mathcal{F}_l$ with hypercharge $Y = 1/2$

Here i, j are generation (flavour) indices. Q^0 and L^0 are $SU(2)_{\rm L}$ quark and lepton doublet weak eigenstates. $d^0_{\rm R}$, $u^0_{\rm R}$ and $e^0_{\rm R}$ are $SU(2)_{\rm L}$ singlet weak eigenstates. The matrices Y^f_{ij} are in general non-diagonal.

Task1: Refer to Standard Model books and the introduction of the CKM matrix and quark and lepton mass eigenstates and the diagonalization of the Yukawa matrices. Applying the same procedure here, answer whether Higgs couplings to quarks and leptons contain flavour changing neutral currents.

Task2: Give some examples of $\mathcal{F}_{u,d,l}(\{\Phi_k\})$. Each example consitutes a separate two-Higgs doublet model.