Physics Beyond the Standard Model — Exercise Sheet 5

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1. **Spinor and Superfield Calculations** Carry out as many of the calculations below as you like! Focus in particular on parts (c) and (e).

   (a) Rewrite the QED-Lagrangian

   $$\mathcal{L}_{\text{QED}} = -\tfrac{1}{4}F_{\mu\nu}F^{\mu\nu} + \bar{\Psi}\left(i\not{D} - m\right)\Psi \tag{1}$$

   in terms of two-component Weyl-spinors

   $$\Psi = \begin{pmatrix} \Psi_{L\alpha} \\ \bar{\Psi}_R^{\dot{\alpha}} \end{pmatrix} \quad \text{and} \quad \bar{\Psi} = \begin{pmatrix} \Psi_R^{\alpha} & \bar{\Psi}_{L\dot{\alpha}} \end{pmatrix}.$$

   (b) Using

   $$\epsilon_{\alpha\beta} = -\epsilon_{\beta\alpha} \quad , \qquad \epsilon_{12} = 1 \tag{2}$$

   $$\epsilon^{\alpha\beta}\,\epsilon_{\beta\gamma} = \delta^{\alpha}_{\ \gamma} \quad , \qquad \epsilon^{\alpha\beta} = \epsilon_{\beta\alpha} \tag{3}$$

   $$\psi^{\alpha} = \epsilon^{\alpha\beta}\psi_{\beta} \tag{4}$$

   $$\psi\chi = \psi^{\alpha}\chi_{\alpha} = -\chi_{\alpha}\psi^{\alpha} = \chi^{\alpha}\psi_{\alpha} = \chi\psi \tag{5}$$

   show the following identities:

   $$\theta^{\alpha}\,\theta_{\beta} = \tfrac{1}{2}\,\delta^{\alpha}_{\ \beta}\,\theta\theta \tag{6}$$

   $$\bar{\theta}_{\dot{\alpha}}\,\bar{\theta}^{\dot{\beta}} = \tfrac{1}{2}\,\delta_{\dot{\alpha}}^{\ \dot{\beta}}\,\bar{\theta}\bar{\theta} \tag{7}$$

   $$\partial_{\alpha}\theta\theta = 2\,\theta_{\alpha} \tag{8}$$

   $$\bar{\partial}^{\dot{\alpha}}\bar{\theta}\bar{\theta} = 2\,\bar{\theta}^{\dot{\alpha}} \tag{9}$$

   $$\theta\sigma^{\mu}\bar{\psi} = -\bar{\psi}\bar{\sigma}^{\mu}\theta \tag{10}$$

   $$\psi\theta\,\theta_{\alpha} = -\tfrac{1}{2}\,\theta\theta\,\psi_{\alpha} \tag{11}$$

   $$\bar{\psi}\bar{\theta}\,\bar{\theta}^{\dot{\alpha}} = -\tfrac{1}{2}\,\bar{\theta}\bar{\theta}\,\bar{\psi}^{\dot{\alpha}}. \tag{12}$$

   (c) Show $\left\{Q_{\alpha}, \bar{Q}_{\dot{\beta}}\right\} = 2\sigma^{\mu}_{\alpha\dot{\beta}}P_{\mu}$ using

   $$P^{\mu} = i\partial^{\mu} \tag{13}$$

   $$Q_{\alpha} = i(\partial_{\alpha} + i_{\alpha}(\sigma^{\mu}\bar{\theta})\partial_{\mu}) \tag{14}$$

   $$\bar{Q}_{\dot{\alpha}} = i(-\bar{\partial}_{\dot{\alpha}} - i(\theta\sigma^{\mu})_{\dot{\alpha}}\partial_{\mu}). \tag{15}$$

   (d) Show that the following Lagrangian $\mathcal{L} = \bar{\psi}i\bar{\sigma}^{\mu}\partial_{\mu}\psi + |\partial_{\mu}\phi|^2 + |F|^2$ is invariant (up to total derivatives) under the supersymmetry transformation

   $$\delta\phi = \sqrt{2}\,\epsilon\psi, \qquad \delta\psi^{\alpha} = \sqrt{2}\,\epsilon^{\alpha}\,F - \sqrt{2}\,i(\bar{\epsilon}\bar{\sigma}^{\mu})^{\alpha}\partial_{\mu}\phi, \qquad \delta F = -i\sqrt{2}\bar{\epsilon}\bar{\sigma}^{\mu}\partial_{\mu}\psi$$

(e) Chiral superfields are superfields with a particularly simple structure. They are used to describe e.g. the electron-selectron or Higgs-Higgsino multiplets. Two chiral superfields $\Phi_{L,R}$ are defined as follows:

$$\Phi_{L,R}(x,\theta,\bar{\theta}) \;=\; \exp(-i\theta\sigma^\mu\bar{\theta}\partial_\mu)\,\phi_{L,R}(x,\theta) \tag{16}$$

with

$$\phi_{L,R}(x,\theta) \;=\; A(x)+\sqrt{2}\theta\psi(x)+\theta\theta F(x)\,. \tag{17}$$

i. Compute the $\theta\theta$-part of the product $\Phi_L\Phi_R$, which is defined to be

$$\int \mathrm{d}^2\theta\;\Phi_L\Phi_R|_{\bar{\theta}=0}$$

This is the electron mass term in SUSY QED.

ii. Compute Φ_L by evaluating the exponential as a power series.

iii. Calculate the $\theta\theta\bar{\theta}\bar{\theta}$-part of the product $\Phi_l^\dagger\Phi_L$, defined to be

$$\int \mathrm{d}^4\theta\;\;\Phi_L^\dagger\Phi_L \;\;\equiv\;\; \int \mathrm{d}^2\theta\mathrm{d}^2\bar{\theta}\;\;\Phi_L^\dagger\Phi_L\,.$$

Show that the result is equal to the Lagrangian given in part (d).

2. **Type X Yukawa interaction in the 2HDM**
In the type X Yukawa interaction we impose the Z_2 transformation: $\Phi_1 \to -\Phi_1$, $e^0_{j\mathrm{R}} \to -e^0_{j\mathrm{R}}$, and $\mathcal{S} \to \mathcal{S}$ for all the other fields. The Z_2-invariant Yukawa interaction term can be written as

$$-\mathcal{L}^{\mathrm{X}}_{\mathrm{Y}} = \bar{Q}^0_i\Delta^u_{ij}\Phi^c_2 u^0_{j\mathrm{R}} + \bar{Q}^0_i\Delta^d_{ij}\Phi_2 d^0_{j\mathrm{R}} + \bar{L}^0_i\Gamma^d_{ij}\Phi_1 e^0_{j\mathrm{R}}\,. \tag{18}$$

(a) Referring to the lecture notes, diagonalize the mass matrices and calculate the Yukawa interaction couplings of the fermions to the neutral and charged Higgs fields.

(b) As a phenomenological example we show here a Barr-Zee type Feynman diagram contributing to the muon anomalous magnetic moment. For which fermion f in the inner loop is the Feynman diagram enhanced by $\tan^2\beta$? (Note that $\tan\beta$ can be realistically as large as ~ 60.)