Physics Beyond the Standard Model — Exercise Sheet 6

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1. **MSSM Superpotential:** The superpotential of the minimal supersymmetric standard model (MSSM) with one generation of quarks and leptons is given by

$$W_{\text{MSSM}} = y_d H_d \cdot QD + y_u H_u \cdot QU + y_e H_d \cdot LE - \mu H_d \cdot H_u. \tag{1}$$

The appearing chiral superfields are the two Higgs doublets H_u, H_d, the left-handed quark doublet Q, the left-handed lepton doublet L, the right-handed quark and lepton singlets U, D, E. The notation of the component fields and further details are given in Tab. 1.

The Yukawa coupling parameters are $y_{d,u,l}$. Like in exercise 2, the SU(2)-invariant dot product of SU(2) doublets is defined by $H_d \cdot Q = H_d^1 Q^2 - H_d^2 Q^1 = \epsilon_{ij} H_d^i Q^j$ etc.

a) Write down the corresponding Lagrangian for the terms involving fermions. Show that you get appropriate mass terms for quarks and leptons if you assume the vacuum expectation values (VEVs)

$$H_d = \begin{pmatrix} v_d \\ 0 \end{pmatrix}, \qquad H_u = \begin{pmatrix} 0 \\ v_u \end{pmatrix} \tag{2}$$

(Note these VEVs are annihilated by electric charge $Q = T^3 + Y$)

superfield	components	$SU(3)_C \times SU(2)_L \times U(1)_Y$
Φ	$A, \quad \psi$	
V	$\lambda, \quad A_\mu$	
Q	$\tilde{q}_L = \binom{\tilde{u}_L}{\tilde{d}_L}, \quad q_L = \binom{u_L}{d_L}$	$(3, 2, 1/6)$
U	$\tilde{u}_R^\dagger, \quad u_R$	$(3^*, 1, -2/3)$
D	$\tilde{d}_R^\dagger, \quad d_R$	$(3^*, 1, 1/3)$
L	$\tilde{l}_L = \binom{\tilde{\nu}_L}{\tilde{e}_L}, \quad l_L = \binom{\nu_L}{e_L}$	$(1, 2, -1/2)$
E	$\tilde{e}_R^\dagger, \quad e_R$	$(1, 1, 1)$
H_d	$H_d, \quad \psi_{H_d}$	$(1, 2, -1/2)$
H_u	$H_u, \quad \psi_{H_u}$	$(1, 2, 1/2)$
v'	$\lambda', \quad B_\mu$	$(1, 1, 0)$
V^a	$\lambda^a, \quad W_\mu^a$	$(1, 3, 0)$
V_s^a	$\lambda_s^a, \quad G_\mu^a$	$(8, 1, 0)$

Table 1: MSSM superfields, their components and charges. The first two lines show the generic notation given in the lecture. Note that we use the same notation for the Higgs superfields and Higgs scalar components. $SU(3)_C$-generators: 3: $T_s^a = \frac{\lambda^a}{2}$, 3^*: $T_s^a = -\frac{\lambda^*}{2} = -\frac{\lambda^T}{2}$ (with Gell-Mann matrices λ^a, $a = 1 \ldots 8$). $SU(2)_L$-generators: 2: $T^a = \frac{\sigma^a}{2}$. $SU(3)_C$ octet and $SU(2)_L$ triplet are adjoint representations given by structure constants.

b) Sketch the Feynman rules for the interactions between fermions and scalars (there are interactions involving Higgs scalars and interactions involving Higgsinos). What are possible decay modes of a Higgsino $\tilde{H}_d$ and of a Higgsino $\tilde{H}_u$?

c) Write down all the solutions of the equations of motion for all F-auxiliary fields. Then write down the Lagrangian for 3- and 4-scalar interactions (which results after eliminating the F-auxiliary fields). Sketch the Feynman rules.

d) Now replace the Higgs scalars by their VEVs given above, and determine all mass terms for stops (i.e. here: all terms bilinear in the $\tilde{u}_{L,R}$). Show that you get a non-diagonal mass matrix in the basis $\binom{\tilde{u}_L}{\tilde{u}_R}$.

2. **Spontaneous SUSY breaking:**

a) Show that the scalar potential of a SUSY theory based on chiral superfields is given by

$$V_{\text{scalar}} = \sum_i |F_i|^2,$$

where the F's can be replaced by the respective solutions of their equations of motion.

b) In the vacuum, all spinor fields must vanish and all scalars must take constant values. In the lecture it was shown that spontaneous SUSY breaking is equivalent to positive vacuum energy density, i.e. a positive value of V_{scalar}. Show therefore, for a theory based on chiral superfields:

$$\text{spontaneous SUSY breaking} \Leftrightarrow \text{impossible to choose } F_i \equiv 0 \ \forall i$$

c) A famous simple model is the O'Raifeartaigh model, based on three chiral superfields and the superpotential

$$W = \lambda\Phi_0 + m\Phi_1\Phi_2 + g\Phi_0\Phi_1\Phi_1, \qquad \text{with } m^2 > 2\lambda g$$

Show that the scalar potential is minimized for $A_1 = A_2 = 0$, and that its value at the minimum is λ^2. What is the influence of A_0?

d) Determine the mass terms, i.e. the terms in the Lagrangian bilinear in the fields. What are the mass eigenvalues of scalars and spinors? (Hint: the result contains values 0, m, $2m$, $m^2 \pm 2g\lambda$)