# Physics Beyond the Standard Model — Exercise Sheet 7

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1. **MSSM Higgs/Higgsino sector, Peccei-Quinn symmetry (3 points):**

   The superpotential of the minimal supersymmetric standard model (MSSM) with one generation of quarks and leptons is given by

   $$W_{\text{MSSM}} = y_d H_d \cdot QD + y_u H_u \cdot QU + y_e H_d \cdot LE - \mu H_d \cdot H_u. \tag{1}$$

   For details see Sheet 6.

   a) Consider the superpotential in the limit $\mu = 0$. Show that in this limit the entire supersymmetric part of the MSSM Lagrangian has a global $\text{U}(1)_{\text{PQ}}$ symmetry[1] under which

   $$H_u \to e^{i\alpha} H_u \qquad H_d \to e^{i\alpha} H_d \qquad D \to e^{-i\alpha} D \qquad \ldots \tag{2}$$

   (And provide transformations of all other superfields such that this transformation is indeed a symmetry!)

   b) Consider the soft SUSY breaking Lagrangian of the MSSM. Show that it is invariant under the same $\text{U}(1)_{\text{PQ}}$ symmetry in the limit $B\mu = 0$.

   c) In the vacuum the two Higgs doublets have vacuum expectation values

   $$H_d = \begin{pmatrix} v_d \\ 0 \end{pmatrix}, \qquad H_u = \begin{pmatrix} 0 \\ v_u \end{pmatrix}. \tag{3}$$

   Is the $\text{U}(1)_{\text{PQ}}$ symmetry spontaneously broken? Note that if Yes, you would expect a massless Goldstone boson. For $\mu, B\mu \neq 0$ this would imply that one Higgs mass eigenvalue is proportional to $\mu, B\mu$.

2. **Proton decay and R-parity violation (2 points):**

   Write down a Feynman diagram which could give rise to proton decay in the MSSM with the following R-parity violating terms:

   a) Additional term $UDD$ in the superpotential — what could be the final state of the proton decay?

   b) Additional term $LQD$ in the superpotential — what could be the final state of the proton decay?

[1] Such a symmetry is commonly called Peccei-Quinn symmetry.

3. **Flavour violating soft SUSY breaking parameters (2 points):**

   Assume that the MSSM Lagrangian contains the soft SUSY breaking term

$$\mathcal{L}_{\text{soft}} = \ldots + (M_Q^2)_{32}\,(\tilde{t}_L^\dagger,\, \tilde{b}_L^\dagger)\begin{pmatrix}\tilde{c}_L\\ \tilde{s}_L\end{pmatrix} + h.c. \tag{4}$$

   and treat this $(M_Q^2)_{32}$ term as a small interaction (i.e. giving rise to bilinear Feynman rules). Show that there exists a Feynman diagram which gives contributes to the process

$$b \to s\gamma \tag{5}$$

   at the one-loop level proportional to $eg_s^2(M_Q^2)_{32}$, where $g_s$ is the strong gauge coupling.

4. **MSSM Higgs potential (3 points):**

   The quartic terms of the MSSM Higgs potential are exclusively given by the $D$-terms for the $\mathrm{SU}(2)_L\times\mathrm{U}(1)_Y$ gauge groups (why?):

$$\mathcal{L}^D_{\text{Higgspot.}} = -\frac{g_W^2}{2}\left(H_d^\dagger T^a H_d + H_u^\dagger T^a H_u\right)^2 - \frac{g_Y^2}{2}\left(Y_{H_d} H_d^\dagger H_d + Y_{H_u} H_u^\dagger H_u\right)^2 \tag{6}$$

   where $T^a = \sigma^2/2$, the square implies a sum over $a$, and $Y_{H_u} = 1/2$, $Y_{H_d} = -1/2$. Note the following relations for doublets $\phi$, $\phi_i$:

$$\phi^\dagger\sigma^a\phi = -(\phi^C)^\dagger\sigma^a\phi^C, \qquad (\phi_1^\dagger\sigma^a\phi_1)\,(\phi_2^\dagger\sigma^a\phi_2) = 2\phi_1^\dagger\phi_2\,\phi_2^\dagger\phi_1 - \phi_1^\dagger\phi_1\,\phi_2^\dagger\phi_2. \tag{7}$$

   a) Identify $H_d^C = \Phi_1$ and $H_u = \Phi_2$ to map the MSSM Higgs sector to the one of the 2HDM, see e.g. Sheet 4. Argue first that the coefficients $\lambda_{1,2,3,4}$ of the general 2HDM Higgs potential from Sheet 4, eq. (2), are all proportional to $g^2_{W,Y}$. Argue also without calculation that $\lambda_5 = 0$.

   b) Determine the values of $\lambda_{1,2,3,4,5}$. Hint: some sample results are $\lambda_5 = 0$, $\lambda_1 = \lambda_2 \propto (g_W^2 + g_Y^2)$, $\lambda_4 \propto g_W^2$.

   Note: this calculation is the basis for the prediction that the lightest CP-even Higgs boson in the MSSM has a tree-level mass $\leq M_Z$, a result of utmost phenomenological interest.