TECHNISCHE UNIVERSITÄT DRESDEN

DRESDEN concept

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Insitute for Process Engineering and Environmental Technology

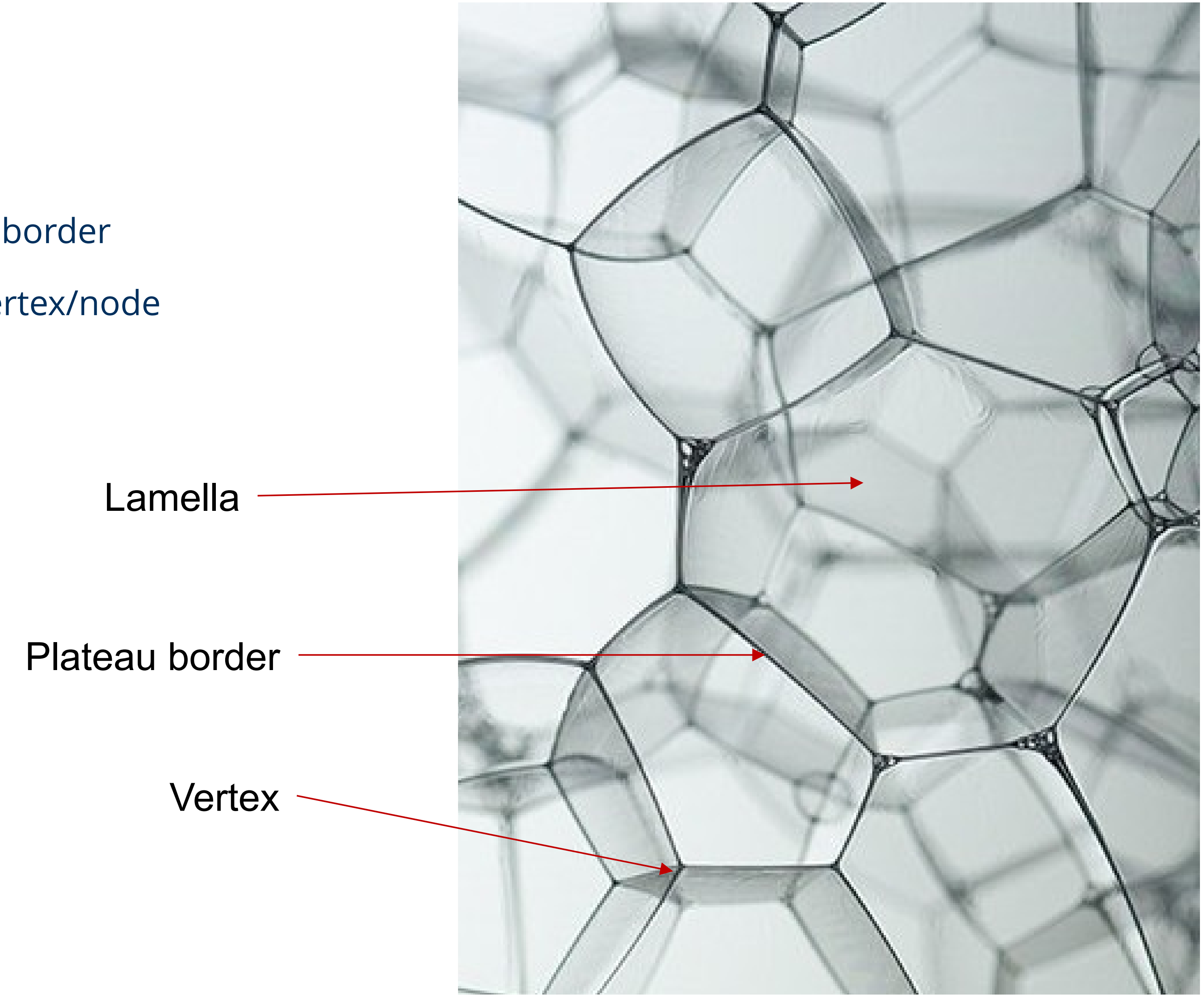
Lecture 7: Foam Drainage

Structure

- Review lecture 6
- Liquid fraction balance
- Darcy's Law
- Osmotic pressure
- Drainage equation

Structural elements

- Lamella/Film separates two bubbles
- Three Lamellas intersect in a **P**lateau border
- Four Plateau borders intersect in a vertex/node

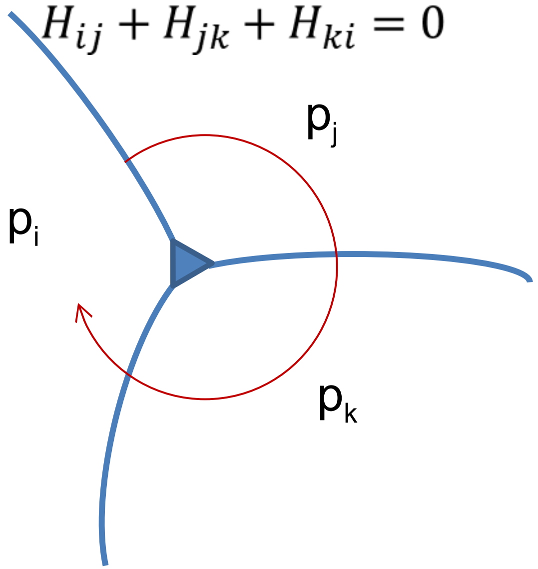


Plateau‘s Laws

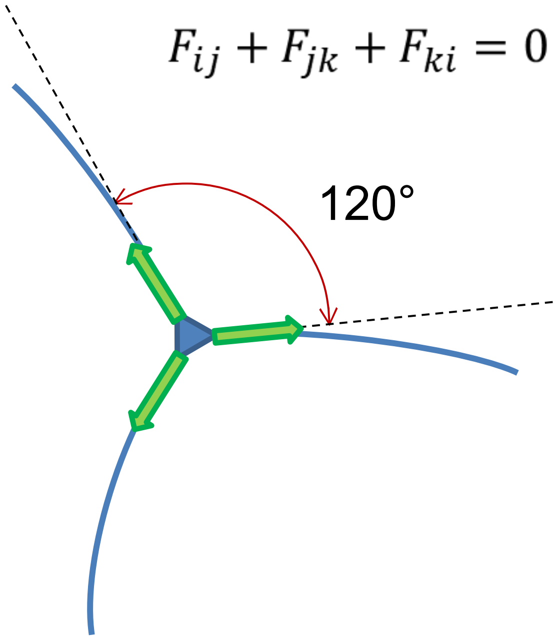
- Assumptions: negligible liquid fraction, mechanical equilibrium, constant surface tension, incompressible foam
- 1st law: Constant mean curvature of each film
- 2nd law: Three films meet in a Plateau border under 120° → force balance
- 3rd law: Four Plateau borders meet in a vertex with tetrahedral symmetry

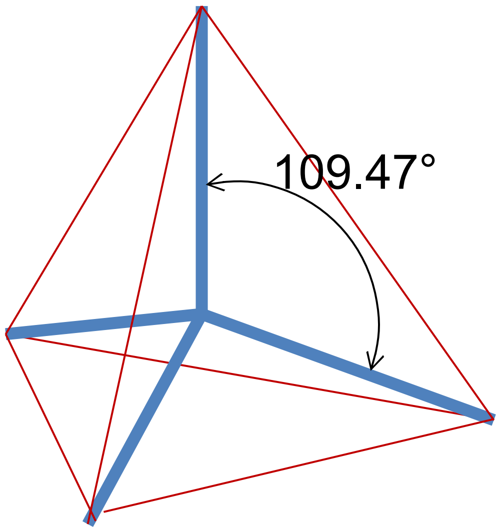
$$p_i - p_j = 2\gamma H_{ij} = const$$

$$H_{ij} + H_{jk} + H_{ki} = 0$$



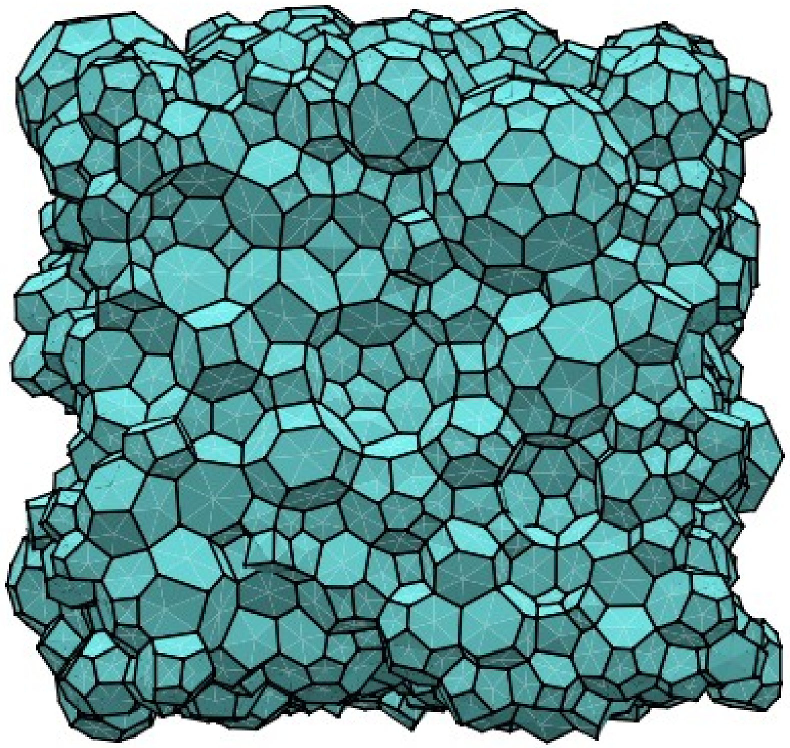
$$F_{ij} + F_{jk} + F_{ki} = 0$$

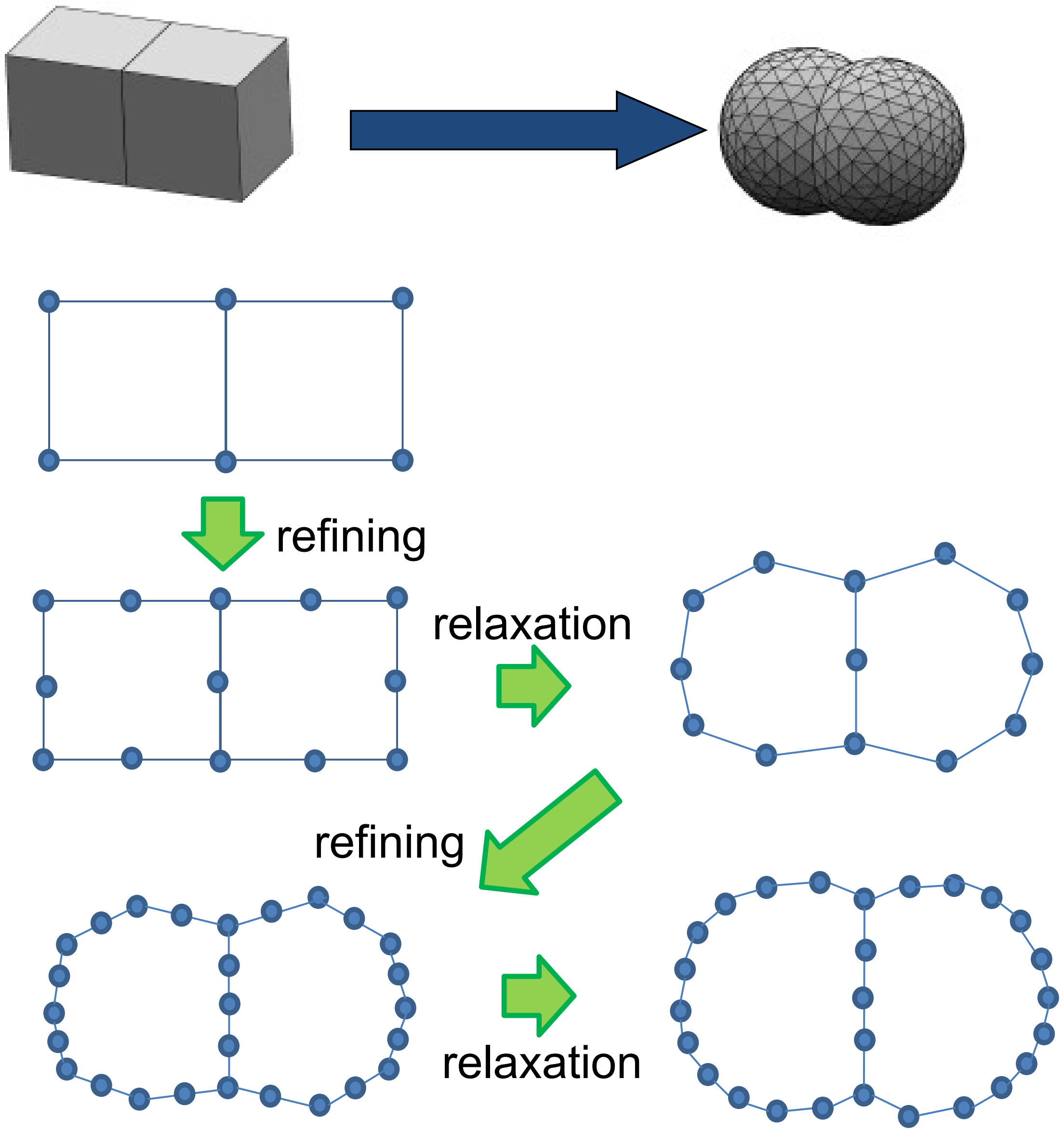




Surface optimization

- Define Topology, generate Mesh on surface
- → move Mesh towards local energy minimum

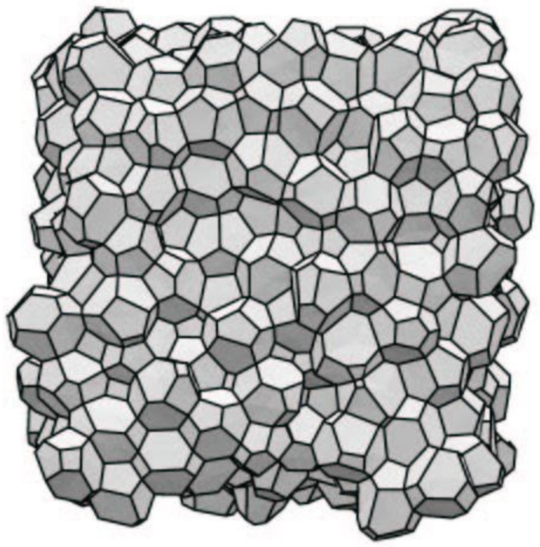




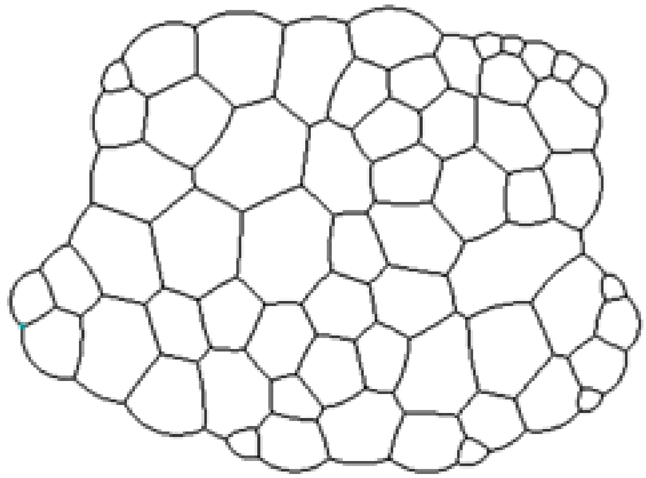
Euler's formula for foam

- Applies to any space filling tiling, i.e. foam
- Applies also in 2D (faces=bubbles)
- χ_{euler} is a small integer
- χ_{euler} might vary, due to boundary conditions

$$-N_{bubbles} + N_{vertices} - N_{edges} + N_{faces} = \chi_{euler}$$

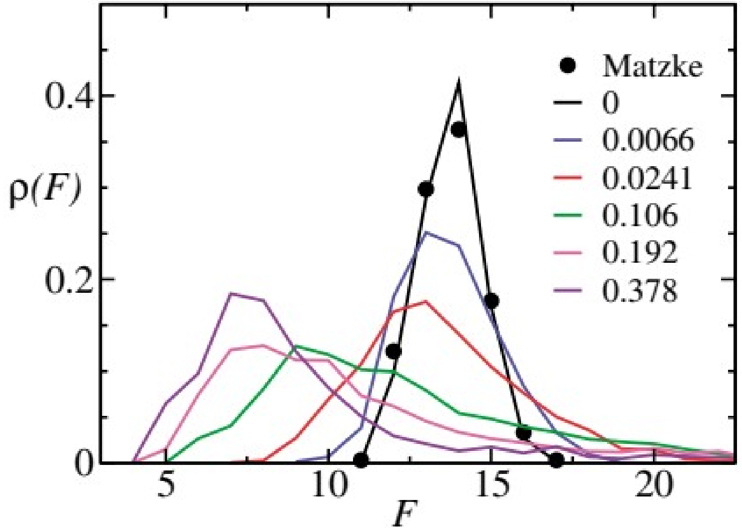


$$N_{bubbles} - N_{edges} + N_{vertices} = \chi_{euler}$$

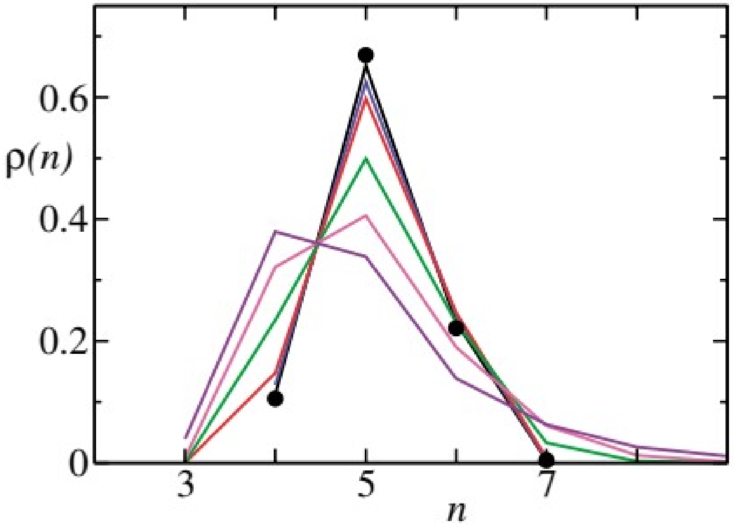


Numbers of Neighbors

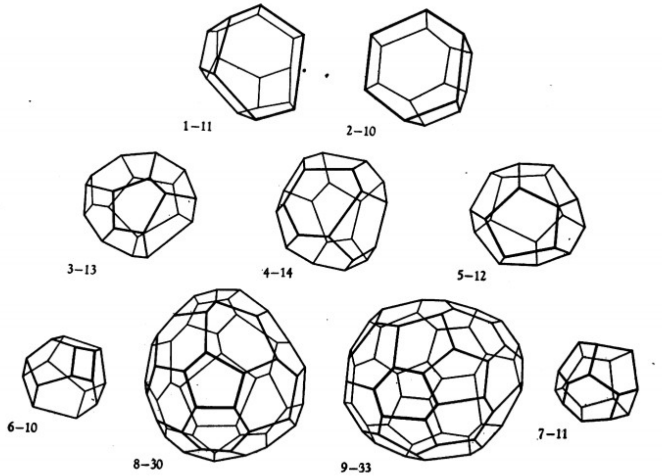
- Measurement by Matzke (1945) with microscope (binocular stereoscope)
- Confirmation with Surface Evolver simulation
- Approx 14 neighbours, approx. 5 edges per face

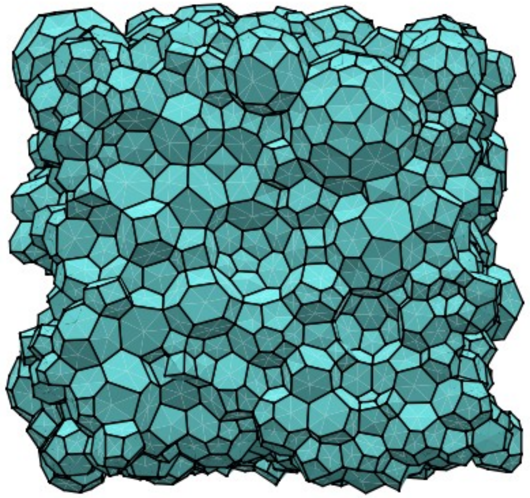


Number of Neighbours, faces



Number of Edges per face





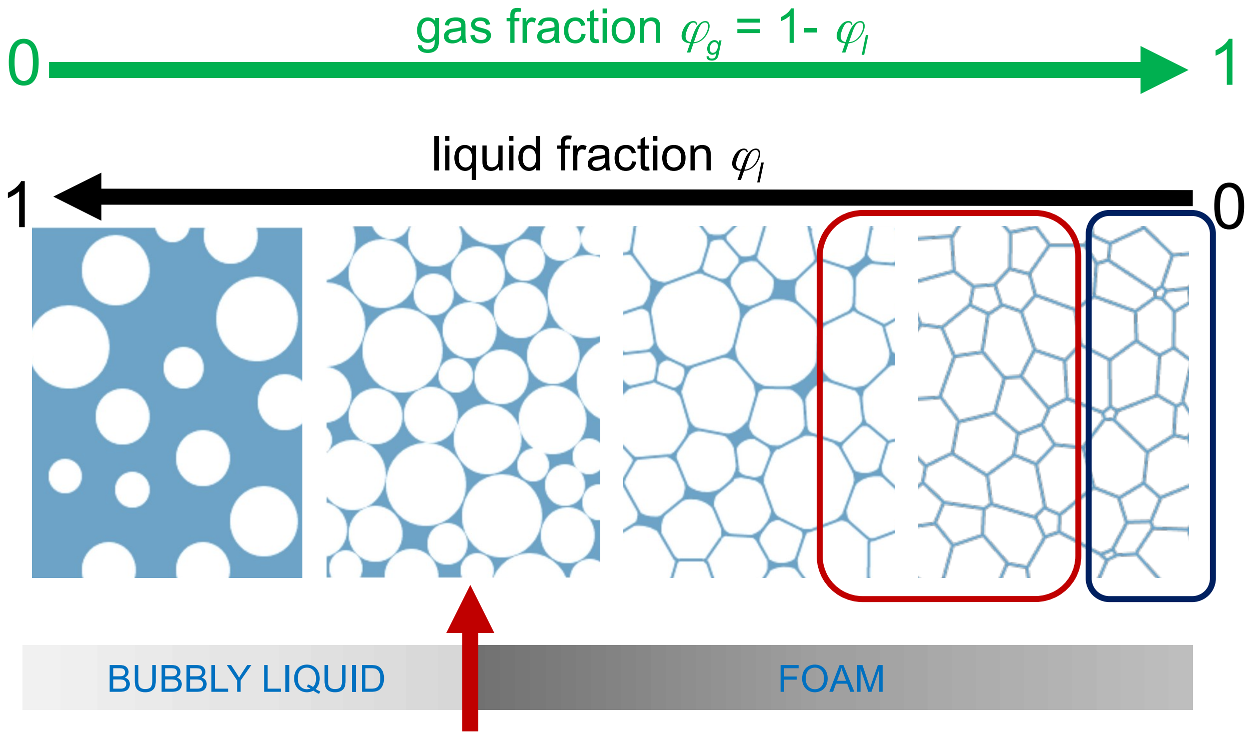
[Kraynik et al., PRL 93, 2004]

$$-N_{bubbles} + N_{vertices} - N_{edges} + N_{faces} = \chi_{euler}$$

$$-1 \quad + \frac{5 \times 14}{4} - \frac{5 \times 14}{3} + \frac{14}{2} = 0.16 \approx 0$$

Jamming limit

- Liquid fraction: Ratio of liquid volume and foam volume
- Jamming point at ≈36% liquid fraction
- Below that point: foam

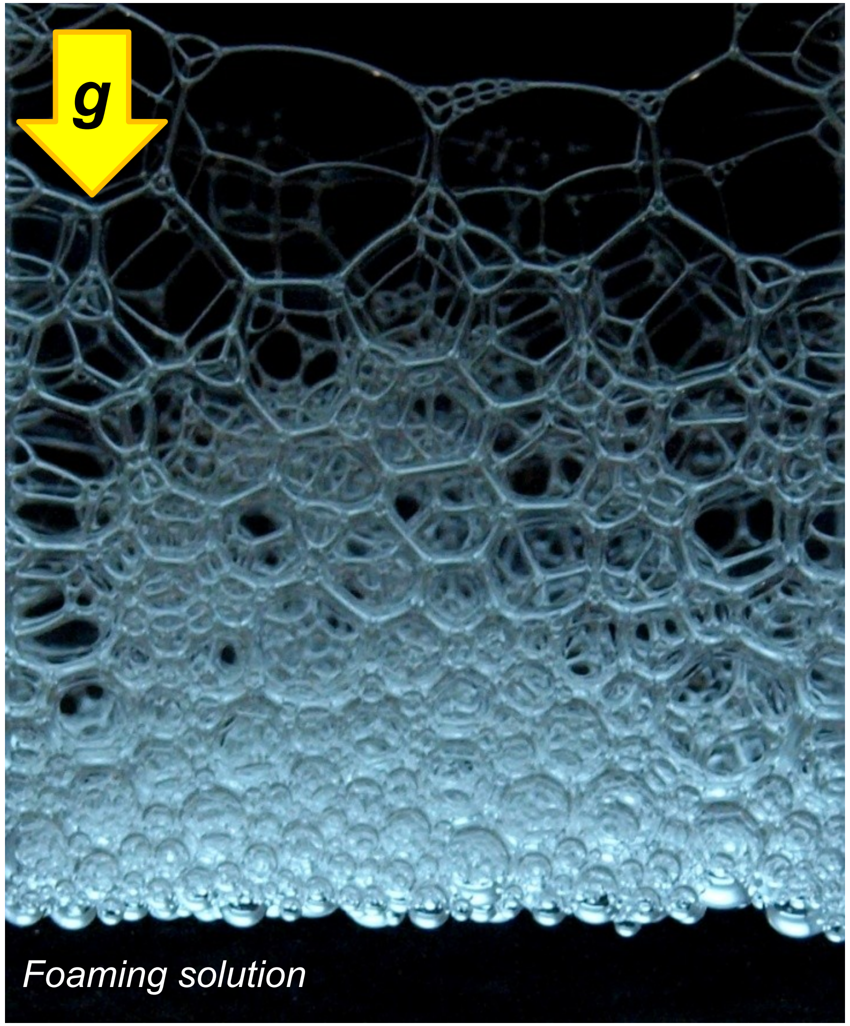


Critical liquid fraction $\varphi_c \approx 0.36$

Jamming point – Bubbles are in contact with each other

Jamming limit

- Coexistence of wet and dry foam
- Drainage influences liquid fraction



DRY LIMIT

DRY FOAM

$\varphi \approx$ 1% … 5%

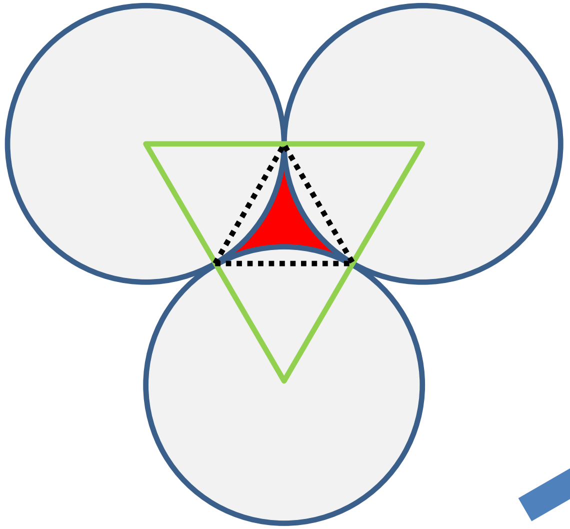
WET FOAM

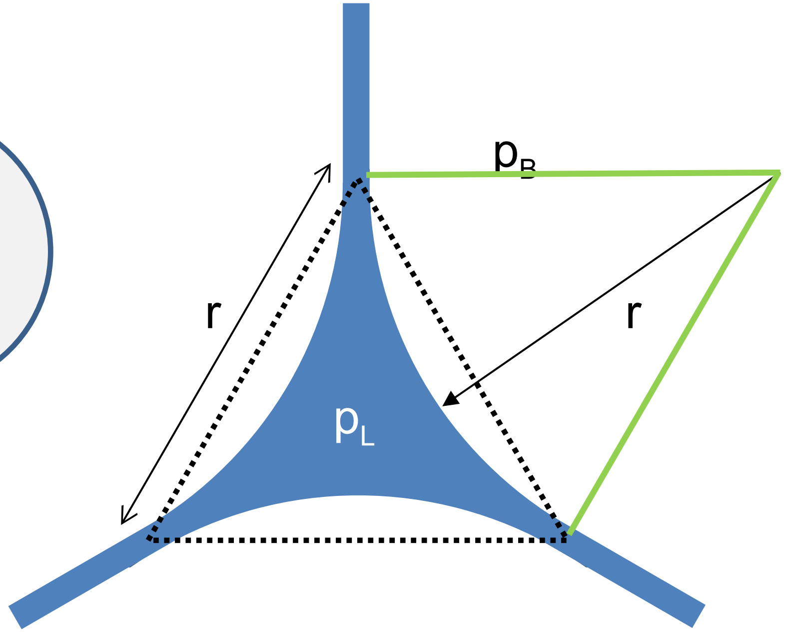
WET LIMIT

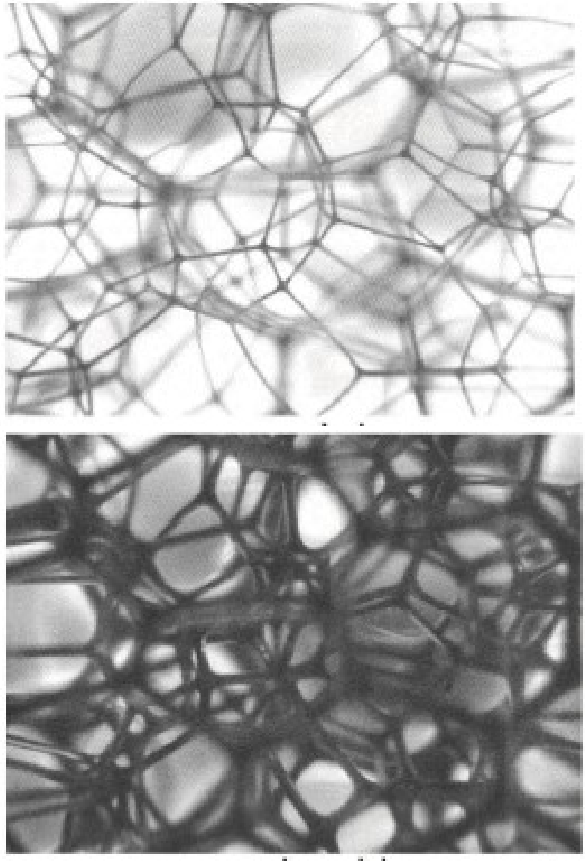
$\varphi \approx 0.36$

Plateau border geometry

- Size of Plateau border ≈ r
- Radius of curvature ≈ r



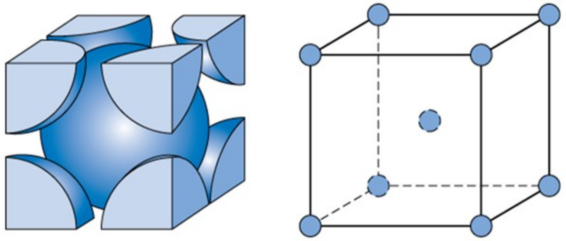




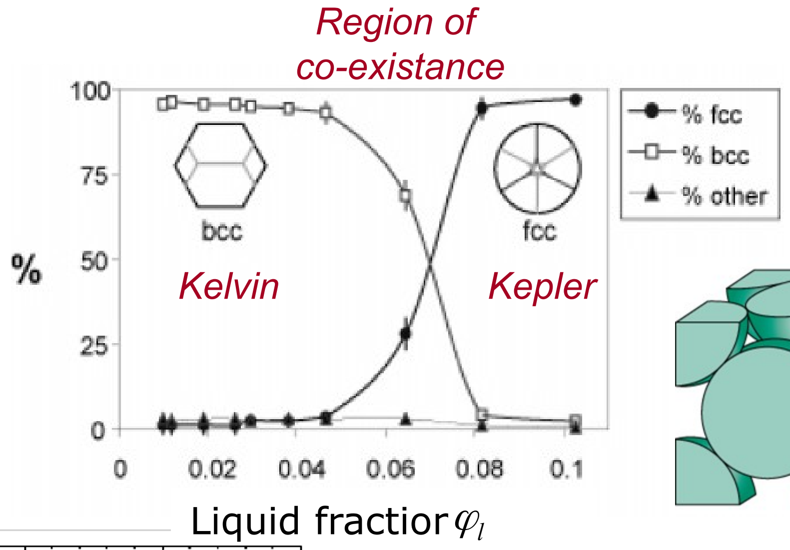
$$\varphi_l = \frac{n_{Pb}\, l_{Pb} A_{Pb}}{V_B} = \frac{\frac{14 \;\times 5}{3} \quad 0.8\, R_e \left(\sqrt{3} - \frac{\pi}{2}\right) r^2}{\frac{4}{3}\pi {R_e}^3} \approx 0.72\, \frac{r^2}{{R_e}^2}$$

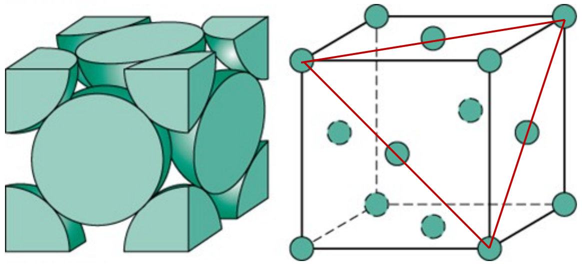
Kelvin vs. Keppler

- Energy minimization yields different structure

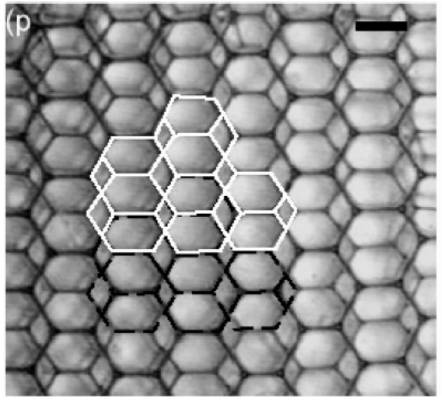


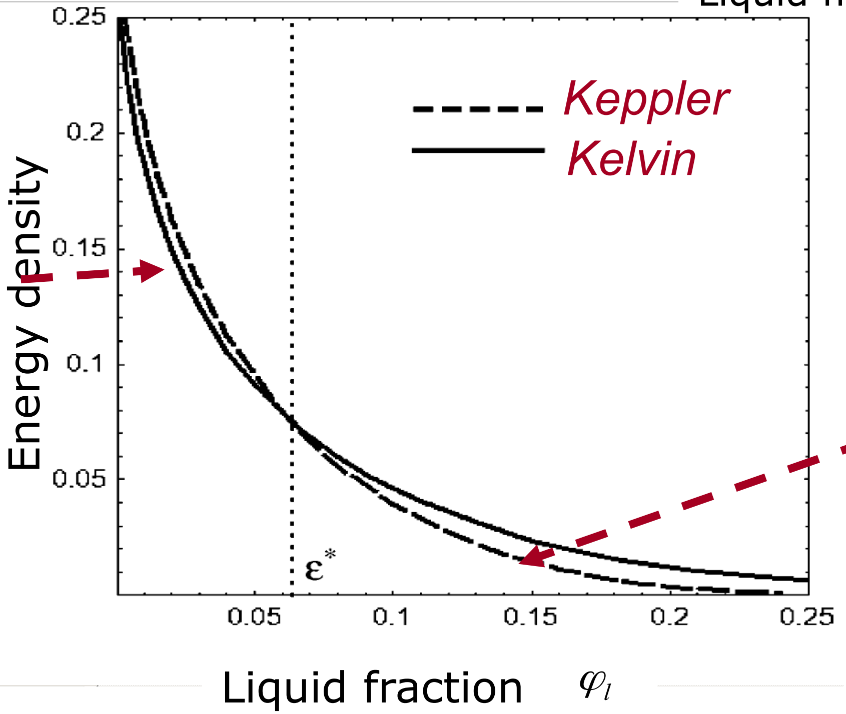
Kelvin

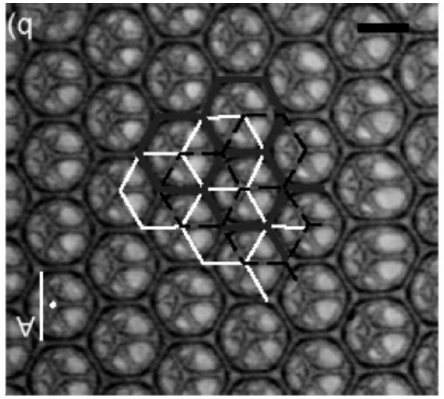




Keppler



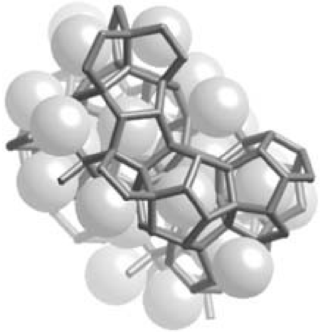




[Hohler et al., Langmuir, 2007]

Weaire-Phelan structure

- Even lower Energy per Volume ratio
- Combination of two Bubble shapes



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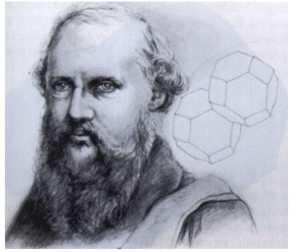
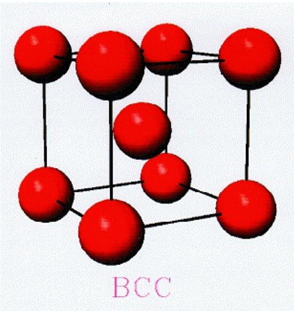
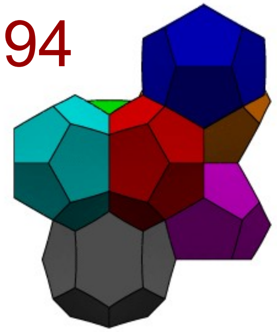
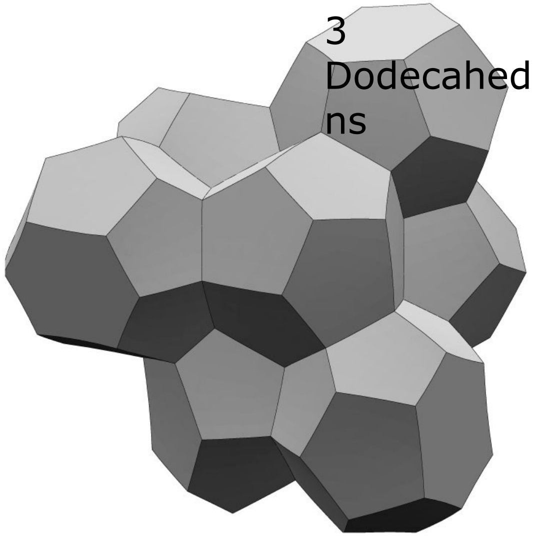




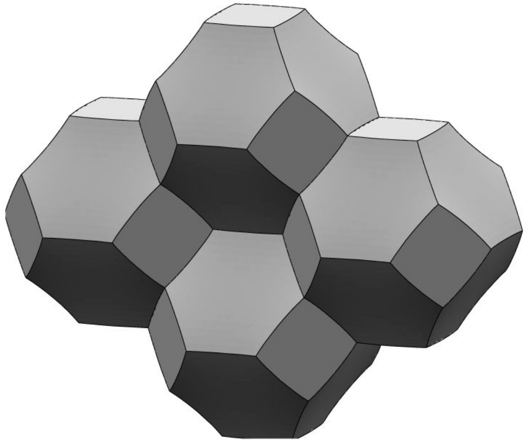
1994



1 Tetrakaidecahedron
3 Dodecahedrons



- 0.3 %



Kelvin structure

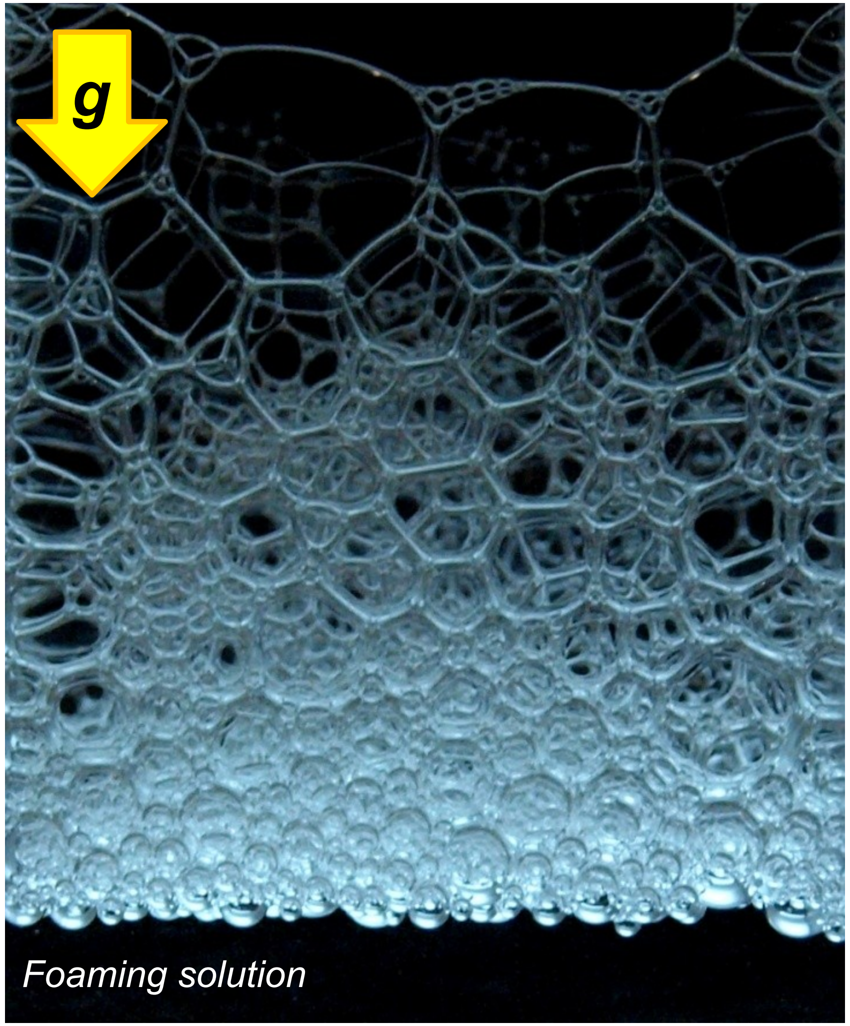
Weaire-Phelan structure

Structure

- Review lecture 6
- Liquid fraction balance
- Darcy’s Law
- Osmotic pressure
- Drainage equation

What is drainage?

- Experiment…



Literature?

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Flow in Foams and Flowing Foams

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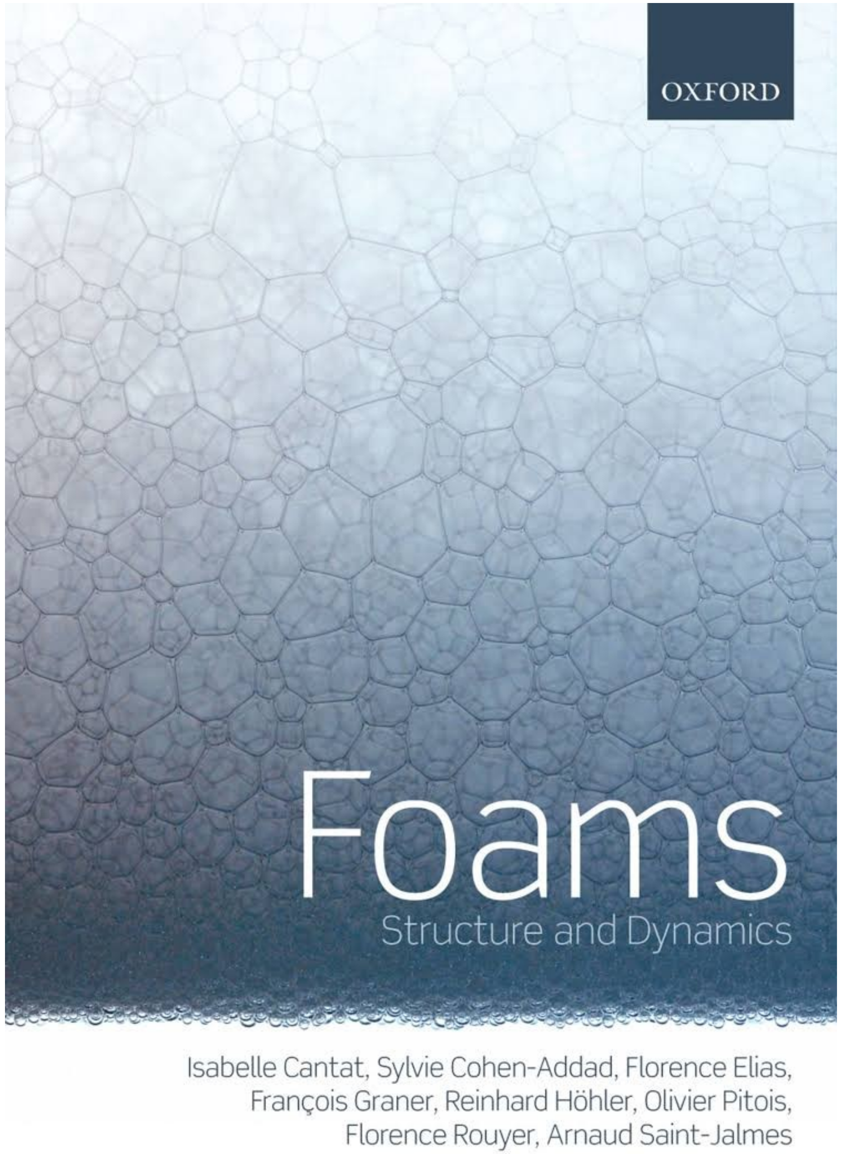
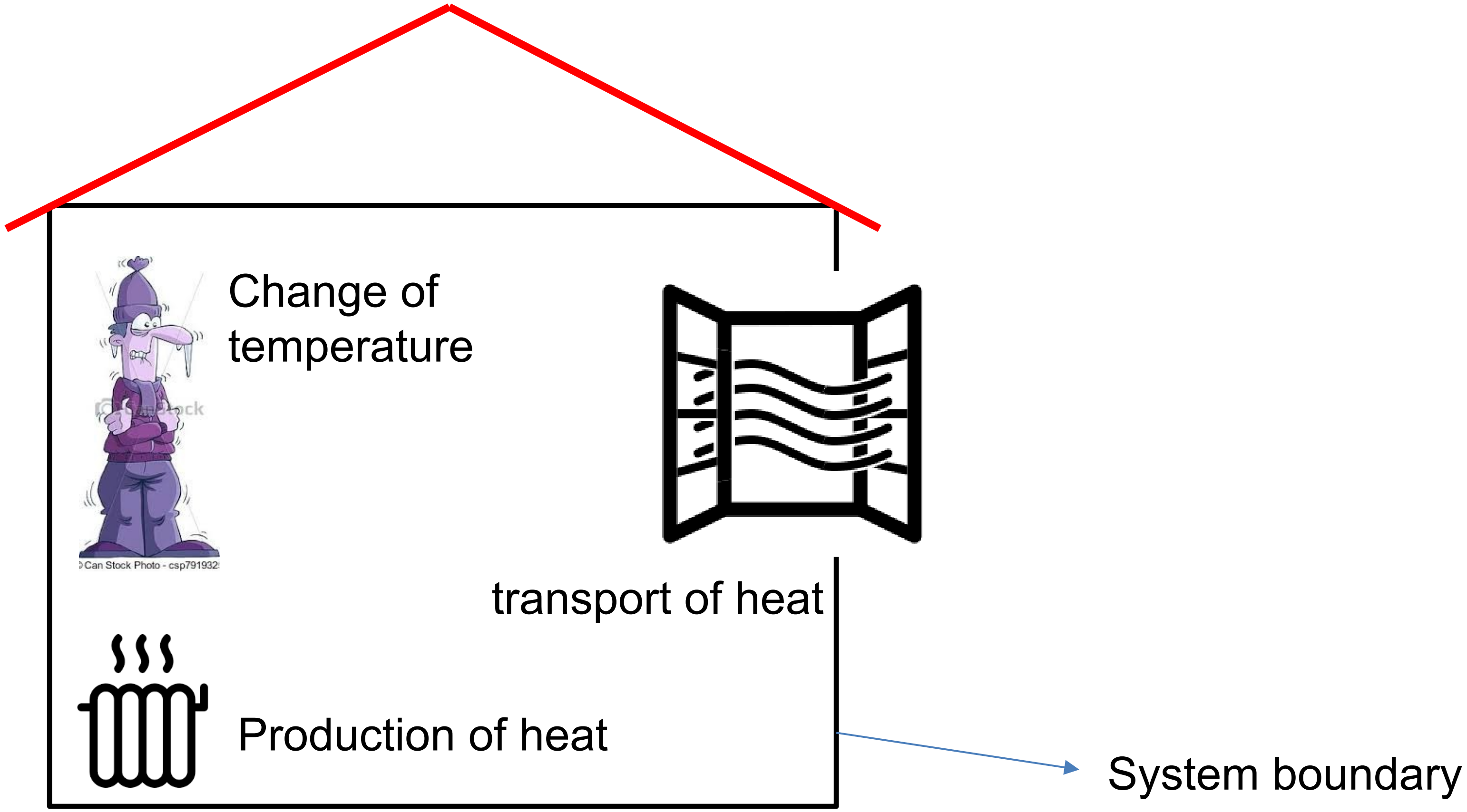


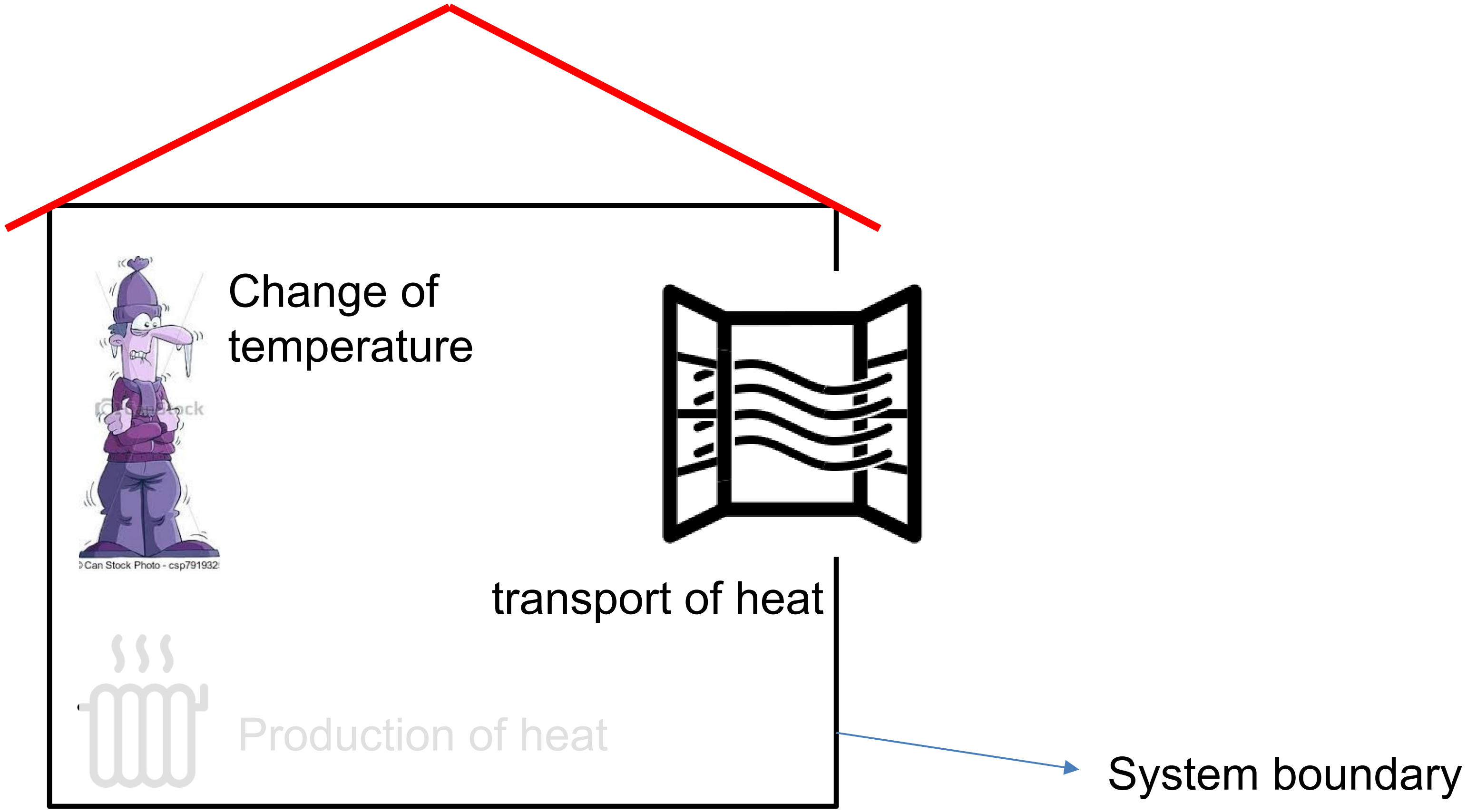
Continuity Equations

- Result from conservation laws (e.g. conservation of mass, momentum, energy)
- Change of X = Transport of X + Production of X



Continuity equation of mass

- Mass can not be generated or annihilated (neglect relativity!)
- Change of X = Transport of X + ~~Production of X~~



Continuity equation of mass

- Differential form – describes a point

$$\frac{\partial \rho}{\partial t} + \vec{\nabla} \cdot (\rho \vec{v}) = 0$$

- Integral form – describes a system with boundaries

  - For non-moving system boundary:

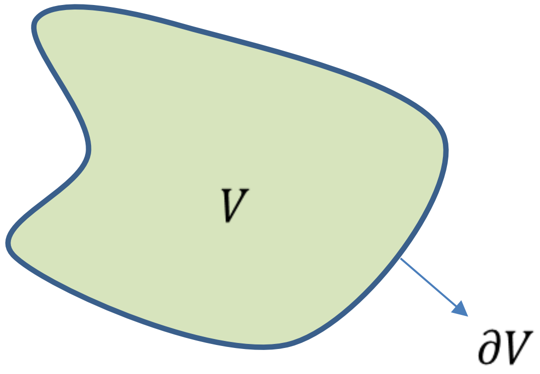
$$\int_V \frac{\partial \rho}{\partial t} dV \quad + \quad \oint_{\partial V} (\rho \vec{v})\, \overrightarrow{dS} = 0$$

Density change within the system

Flux of mass over the system boundary

Gauss's theorem:

$$\oint_{\partial V} (\rho \vec{v})\, \overrightarrow{dS} = \int_V \; \vec{\nabla}$$



Continuity equation of liquid mass in foam

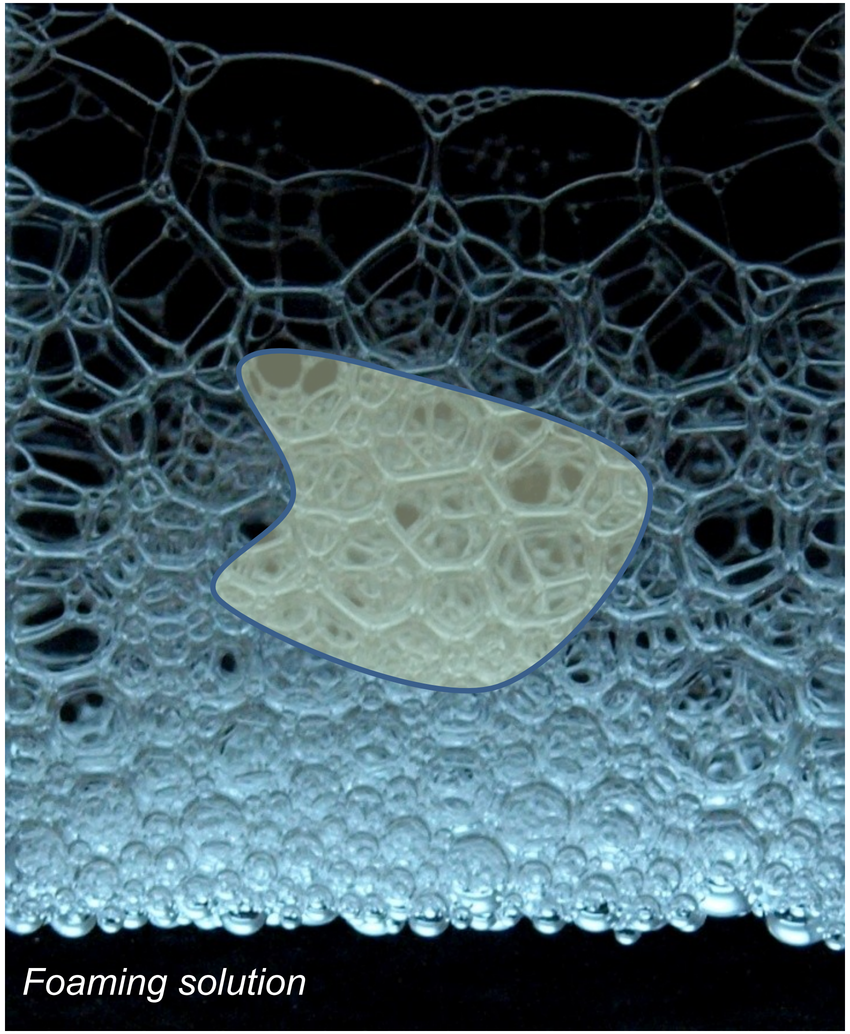
- Liquid density is constant, foam density results from liquid fraction

$$\rho_{foam} = \rho_{liquid}\,\varphi_l$$

- → Conservation of liquid fraction:

$$\frac{\partial \varphi_l}{\partial t} + \vec{\nabla} \cdot (\varphi_l\, \vec{v}) = 0$$

$$\int_V \frac{\partial \varphi_l}{\partial t} dV \;+\; \oint_{\partial V} (\varphi_l\, \vec{v})\, \overrightarrow{dS} = 0$$



Foam drainage equation

- Complex differential equation
- Describes change of local liquid fraction
- Includes gravitationally driven transport
- Includes capillary driven transport

$$\frac{\partial \varphi_l}{\partial t} + \vec{\nabla} \cdot (\varphi_l \, \vec{v}) = 0$$

gravity

surface tension

$$\frac{\partial \varphi_l}{\partial t} + \frac{\rho}{\mu} \vec{\nabla} \cdot (\alpha \vec{g}) - \frac{\gamma}{2\, \delta_b R_e \, \mu} \vec{\nabla} \cdot \left(\frac{\alpha}{{\varphi_l}^{3/2}} \vec{\nabla} \varphi_l \right) = 0$$

Structure

- Review lecture 6
- Liquid fraction balance
- Darcy’s Law
- Osmotic pressure
- Drainage equation

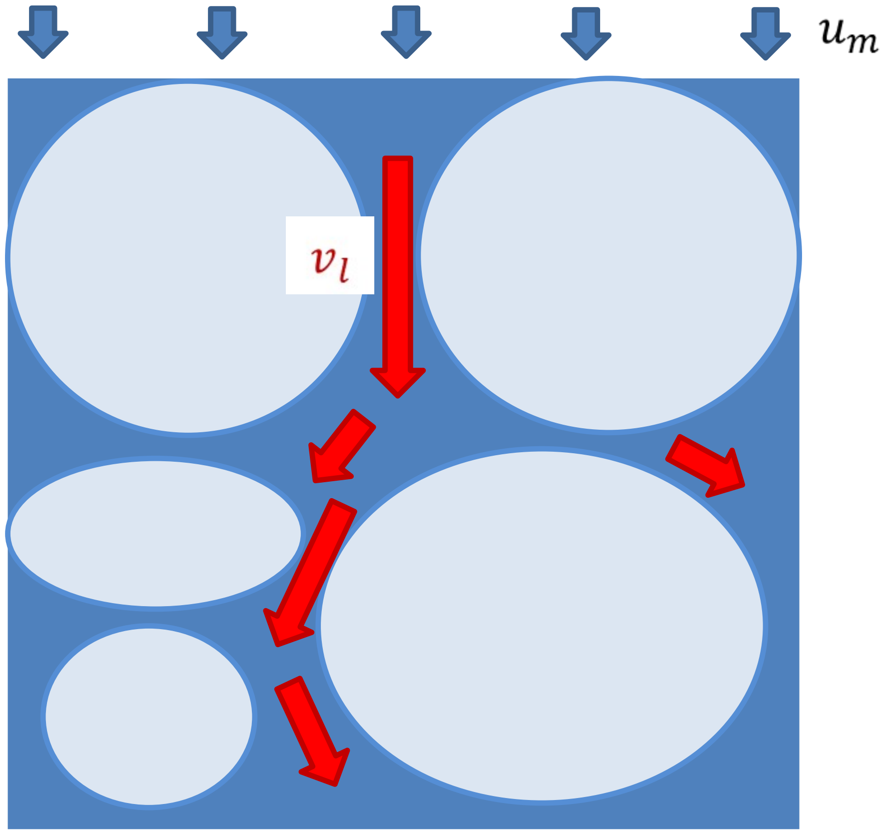
Superficial velocity

- Superficial velocity u_m results from volumetric flow

$$\frac{Q}{A} = u_m$$

- Superficial velocity does not equal local velocity v_l!
- Superficial velocity and average local velocity are linked by the liquid fraction:

$$u_m = \langle v_l \rangle \, \varphi_l$$



Origin of Darcy‘s Law

- Geology, drainage of water in soil

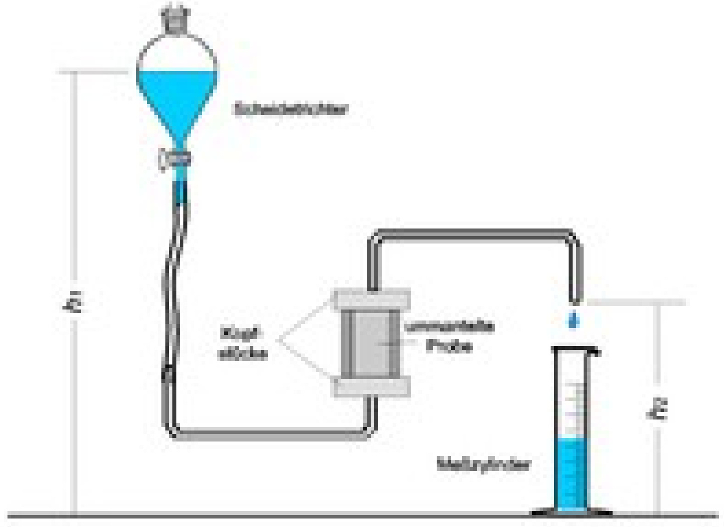
$$\frac{Q}{A} = \frac{\alpha}{\mu}\frac{dp}{dx}$$

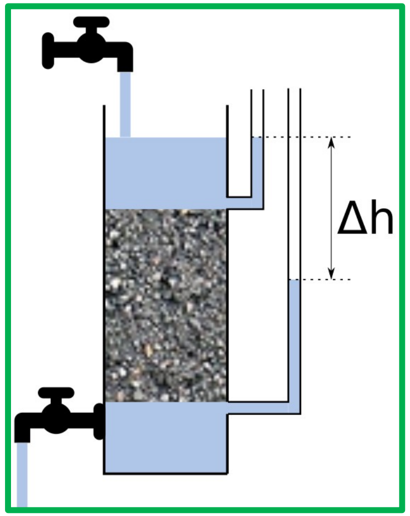
- Units:

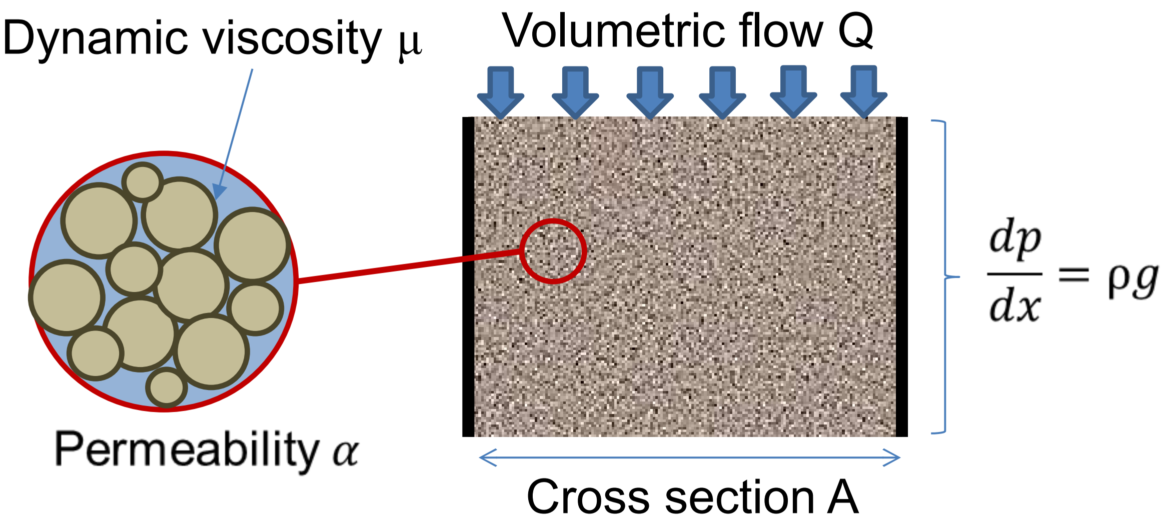
$$\frac{m^3/s}{m^2} = \frac{m^2}{Pa\,s}\frac{Pa}{m}$$

- Permeability k is a feature of the packing
- For water in soil (polydisperse grains) estimated from effective grain diameter:

$$\alpha \approx 0.00116\ {d_{eff}}^2$$







Permeability of foam

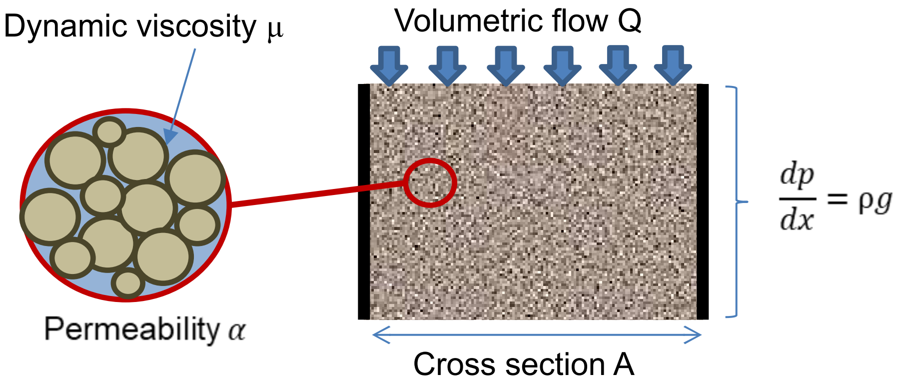
- Darcy's law is valid for foam $\frac{Q}{A} = \frac{\alpha}{\mu}\frac{dp}{dx}$

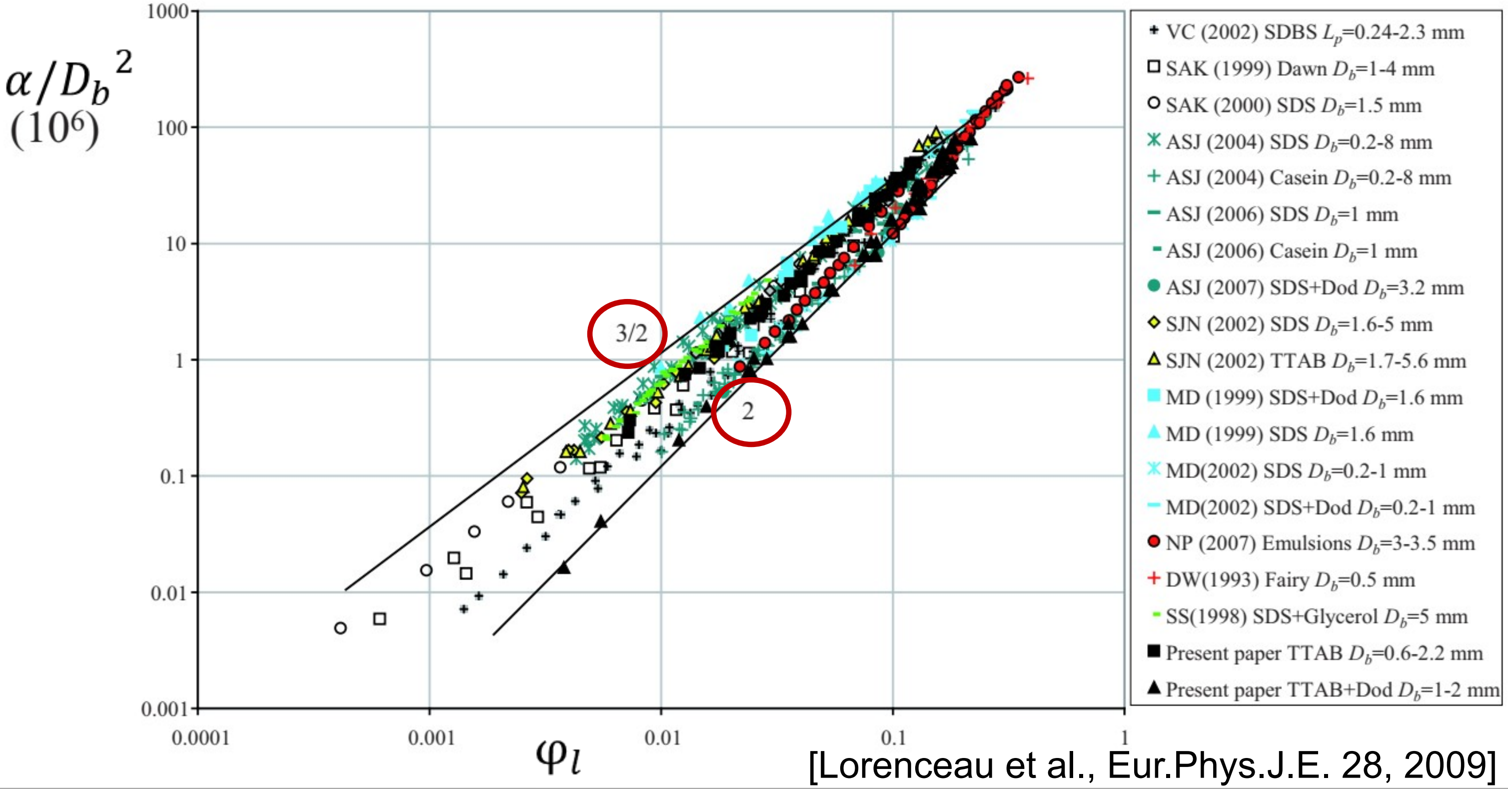
- BUT (!) Permeability depends on liquid fraction

- Exponent depends on surfactant

- → SDS:
  $\alpha \sim \varphi_l^{3/2} D_b^{2}$

- → Casein:
  $\alpha \sim \varphi_l^{2} D_b^{2}$





[Lorenceau et al., Eur.Phys.J.E. 28, 2009]

Flow inside the Plateau borders

- Liquid is concentrated in Plateau borders
- Plateau borders form a network (of pipes)
- Poiseuille-flow:

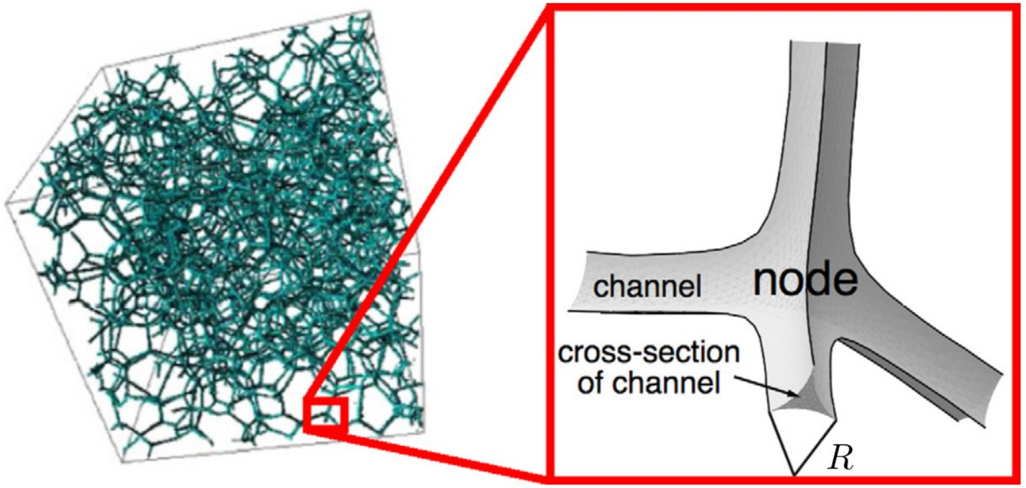
$$Q = \frac{8}{\pi}\frac{d_{tube}{}^4}{\mu}\frac{dp}{dx} \Longleftrightarrow \frac{Q}{A} = \frac{\alpha}{\mu}\frac{dp}{dx}$$

$$\frac{Q}{R_e{}^2} \sim \frac{1}{R_e{}^2}\frac{r^4}{\mu}\frac{dp}{dx}$$

$$\left(\varphi_l \approx 0.72\,\frac{r^2}{R_e{}^2}\right)^2$$

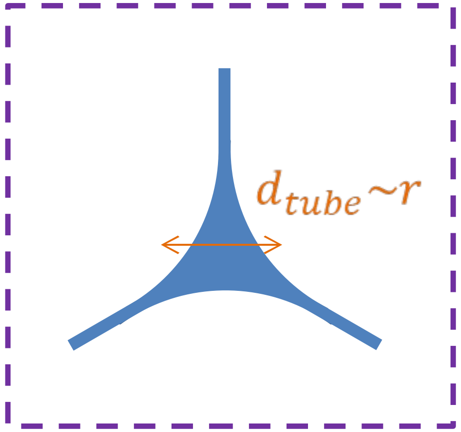
$$\frac{Q}{R_e{}^2} \sim \frac{\varphi_l{}^2 R_e{}^2}{\mu}\frac{dp}{dx}$$

$$\alpha \sim \varphi_l{}^2 R_e{}^2$$



[Dollet & Raufaste 2015]

$A \sim R_e{}^2$



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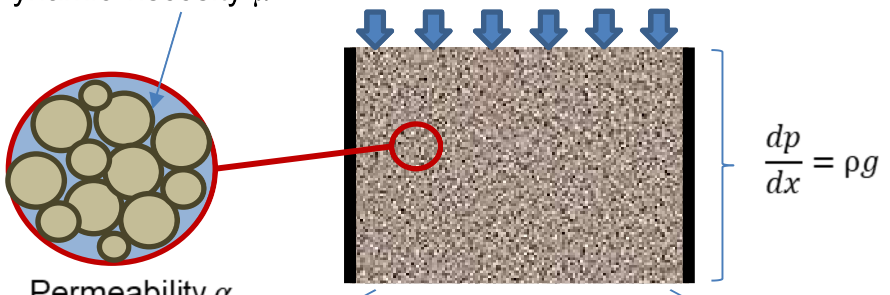

- BUT (!) Permeability depends on liquid fraction
- Exponent depends on surfactant
- → SDS:
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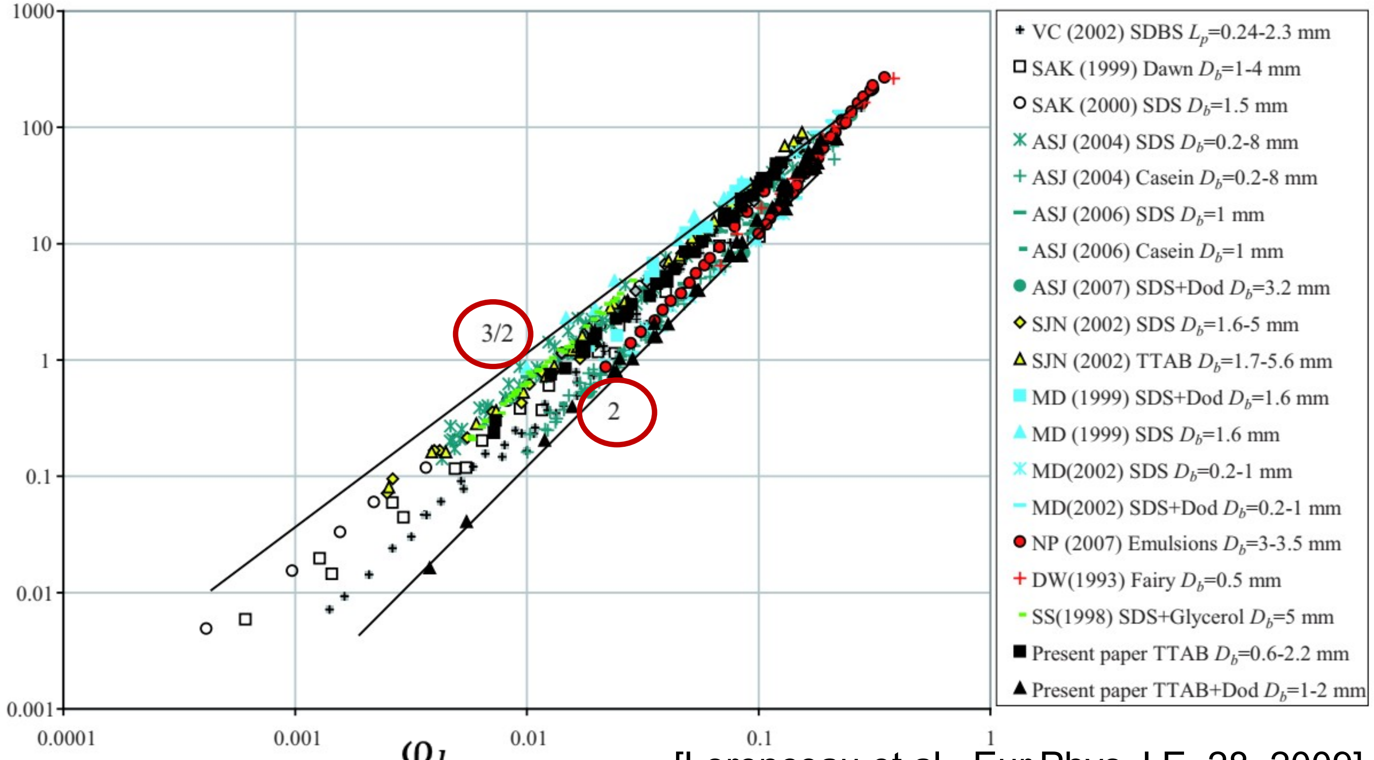
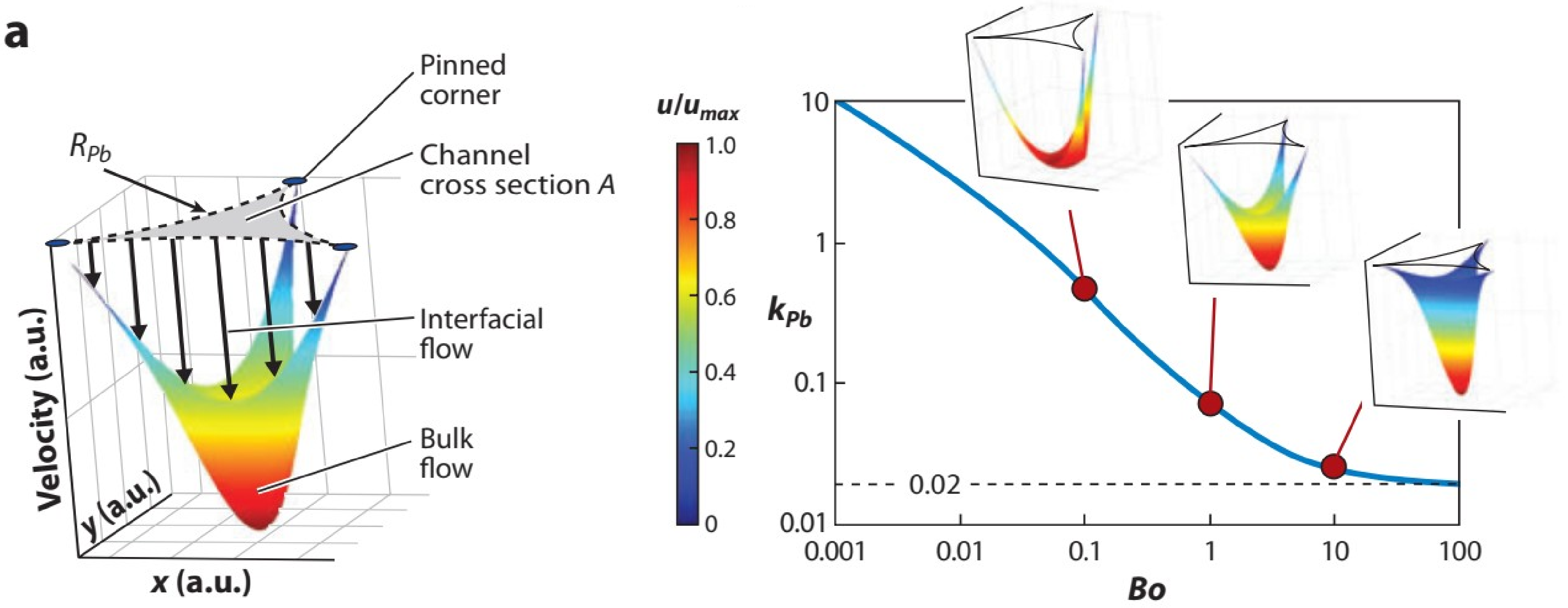


[Lorenceau et al., Eur.Phys.J.E. 28, 2009]

Flow inside the Plateau borders

- Assumption: no-slip boundary at the Plateau border interfaces → depends on surfactant!
- Free-slip → no energy dissipation, no flow resistance



$$\frac{Q}{A_{Pb}} = \frac{k_{Pb}/A_{Pb}}{\mu}\frac{dp}{dx}$$

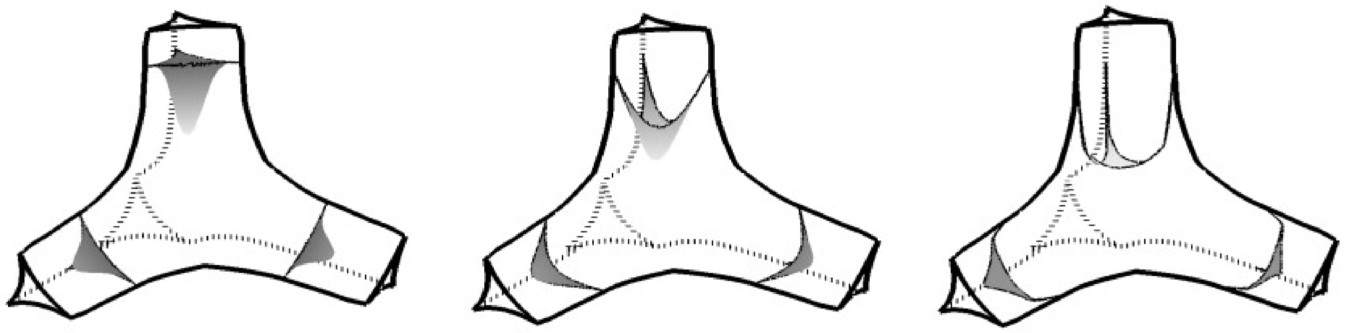
[Cohen-Addad et al. 2013]

$$Bo = \frac{\mu_s}{\mu\, r_{Pb}}$$

Dissipation inside the vertices

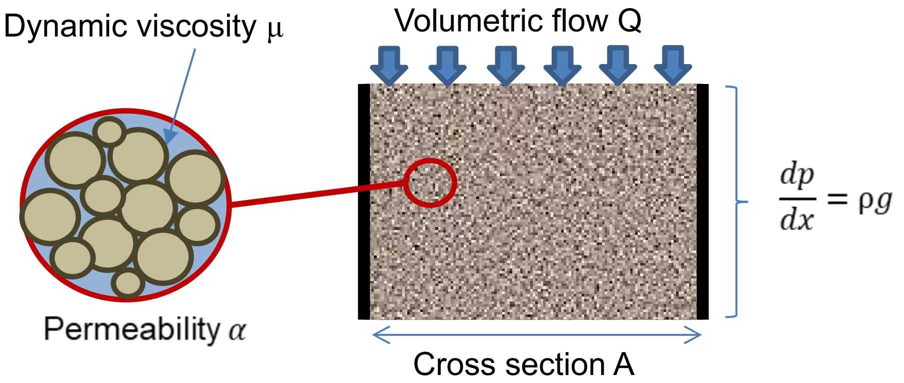
- Free-slip → Dissipation takes place in the nodes
- High liquid fractions → Dissipation takes (partially) place in the nodes
- No clear model, different experimental and numerical findings

$$\alpha \sim \varphi_l^{3/2} D_b^{\,2}$$



Permeability of foam

- Darcy's law is valid for foam $\frac{Q}{A} = \frac{\alpha}{\mu} \frac{dp}{dx}$



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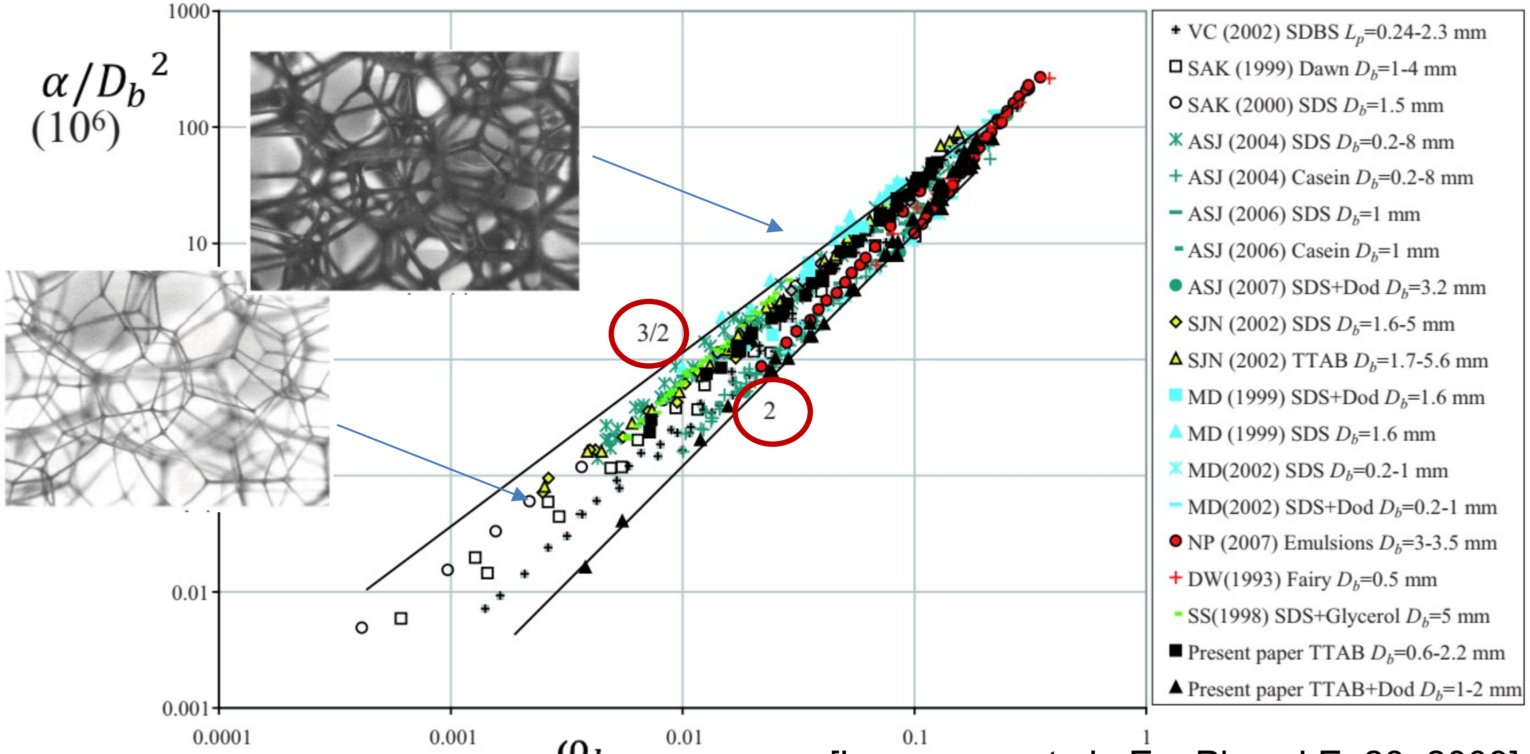


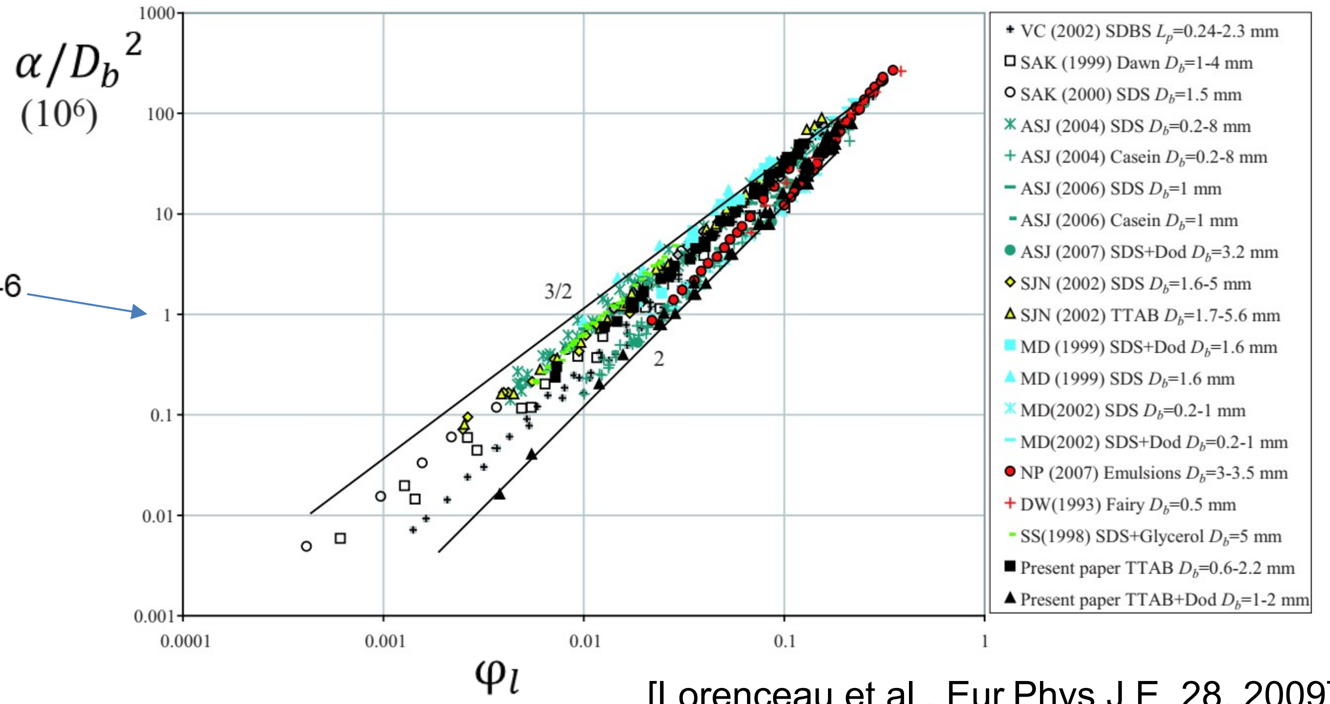
[Lorenceau et al., Eur.Phys.J.E. 28, 2009]

Exercise:

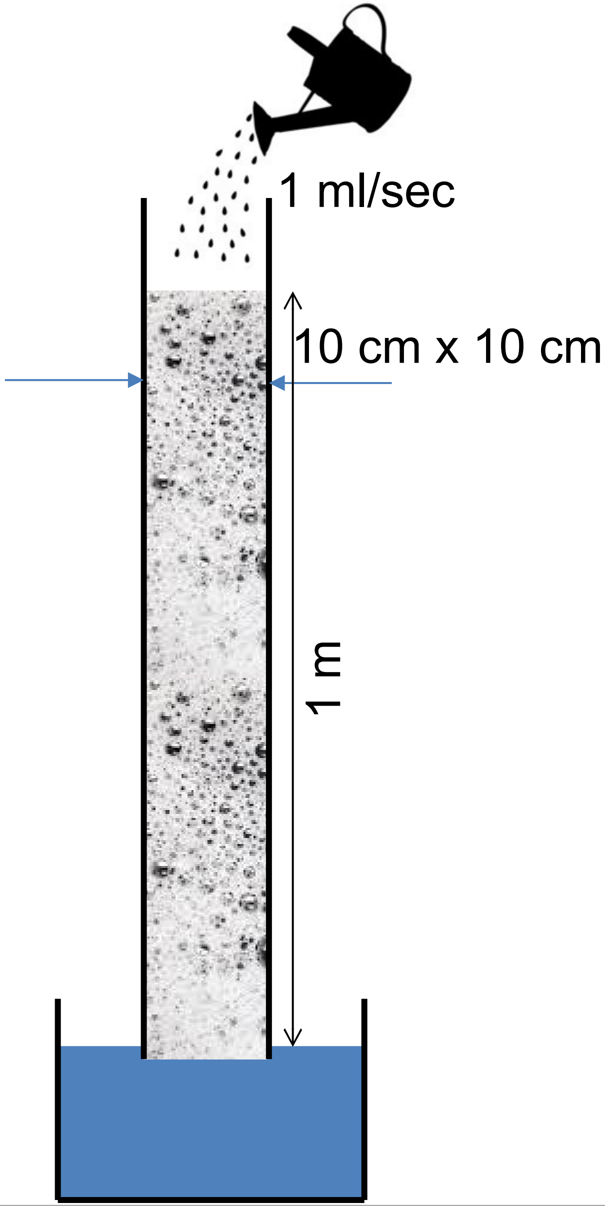
- Forced drainage setup (constant liquid flow is applied at the top)
- SDS foam, 5 mm bubble diameter
- → what is the resulting liquid fraction?

$$\frac{Q}{A} = \frac{\alpha}{\mu}\frac{dp}{dx}$$

$\alpha / D_b^2 = 10^{-6}$



[Lorenceau et al., Eur.Phys.J.E. 28, 2009]



Exercise:

- Forced drainage setup (constant liquid flow is applied at the top)
- SDS foam, 5 mm bubble diameter
- → what is the resulting liquid fraction?

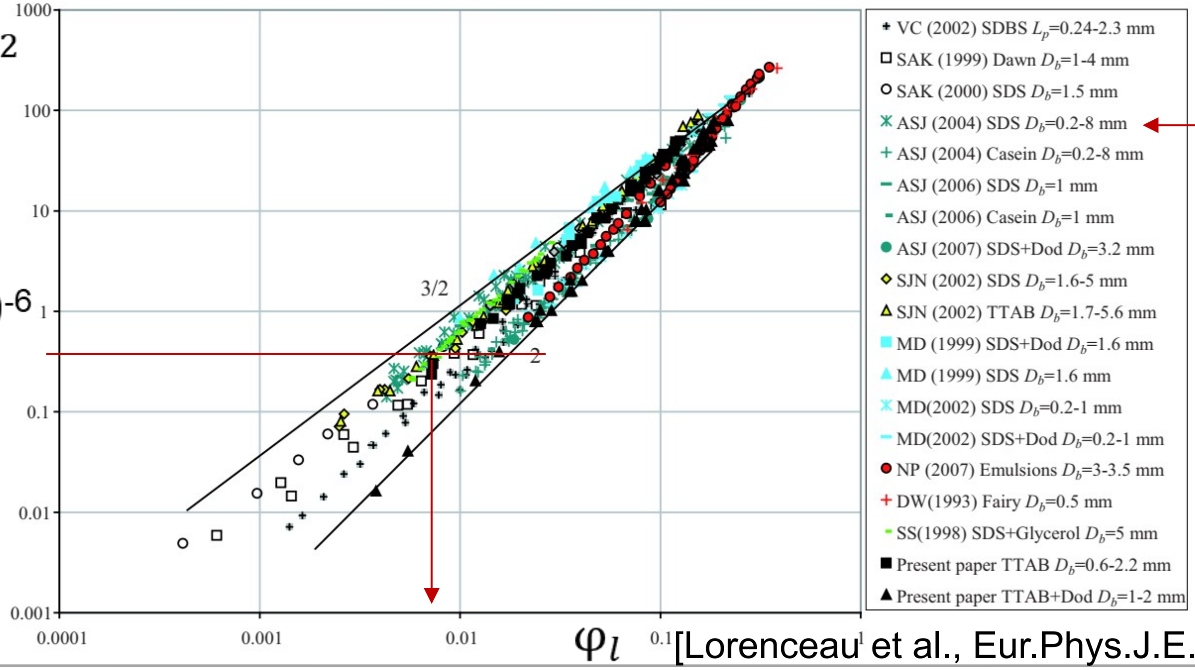
$$\frac{Q}{A} = \frac{\alpha}{\mu}\frac{dp}{dx}$$

$$\frac{10^{-6}}{10^{-2}} = \frac{\alpha/m^2}{10^{-3}}10^4 \rightarrow \alpha = 10^{-11}m^2$$

$$\frac{\alpha}{{D_b}^2} = 4 \times 10^{-7}$$

$\alpha/{D_b}^2$ (10⁶)

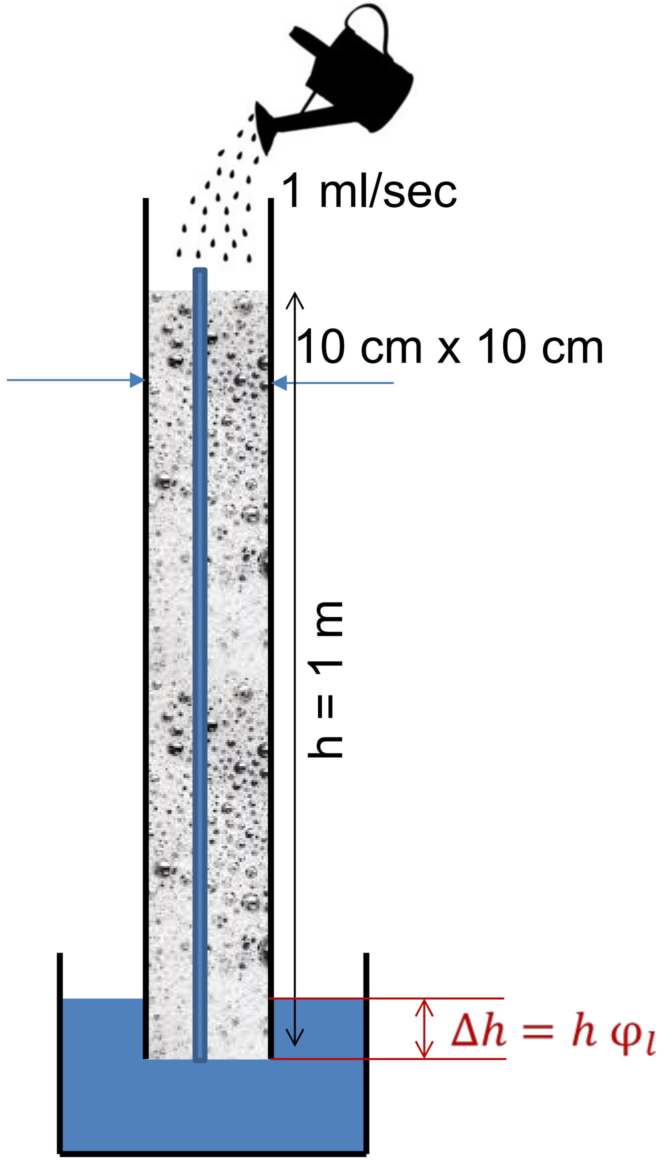
$\alpha/D_b^2 = 10^{-6}$



φ_l

[Lorenceau et al., Eur.Phys.J.E. 28, 2009]

$\varphi_l \approx 0.8\%$



Wetting front

- Start with a column filled with dry foam – pour steady amount of liquid int
- Formation of a wetting front, moving downwards with constant velocity

$$\int_{CV} \frac{\partial \varphi_l}{\partial t} dV \quad + \quad \oint_{\partial CV} (\varphi_l \, \vec{v}) \, \overrightarrow{dS} = 0$$

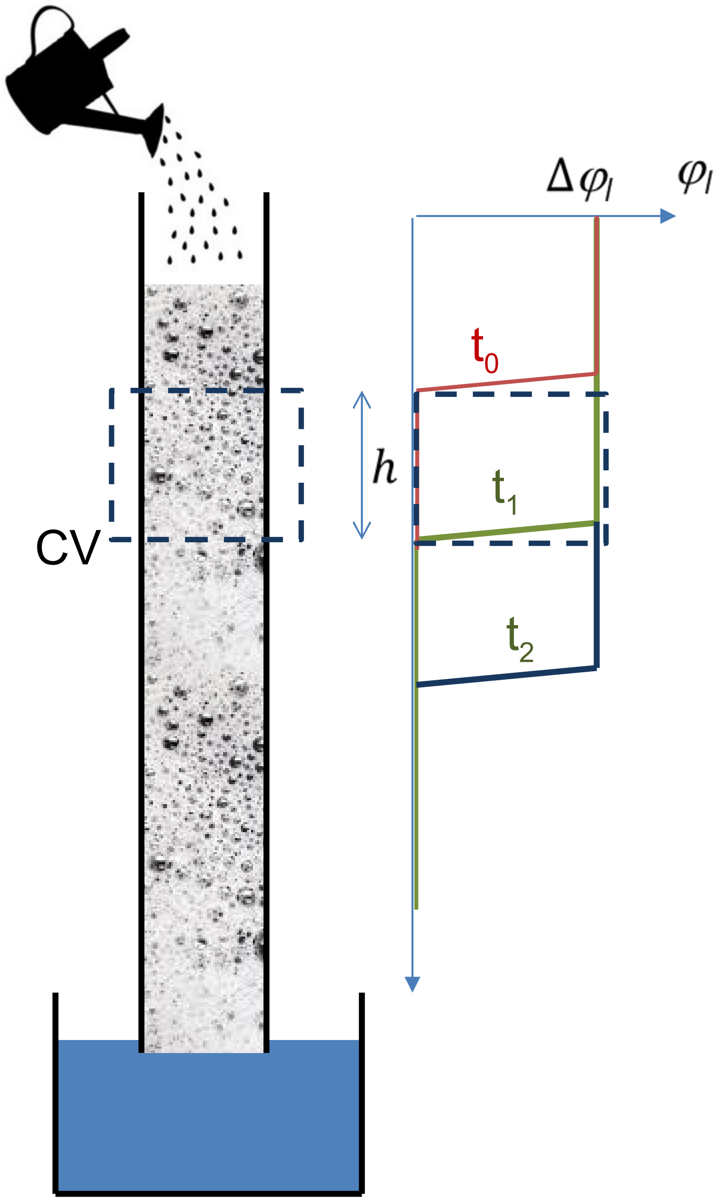
$$\int_{CV} \frac{\Delta \varphi_l}{\Delta t} dV \quad + \quad \oint_{\partial CV} (\vec{u}_m) \, \overrightarrow{dS} = 0$$

$$\frac{h}{\Delta t} A \, \Delta \varphi_l + u_m \mathrm{A} = 0$$

$$\frac{h}{\Delta t} \; A \, \Delta \varphi_l + u_m \mathrm{A} = 0$$

$$u_{front} = u_m \frac{1}{\Delta \varphi_l} = \frac{\alpha}{\mu} \rho g \frac{1}{\Delta \varphi_l}$$

$$\frac{Q}{A} = u_m = \frac{\alpha}{\mu} \frac{dp}{dx} = \frac{\alpha}{\mu} \rho g$$



Superficial velocity

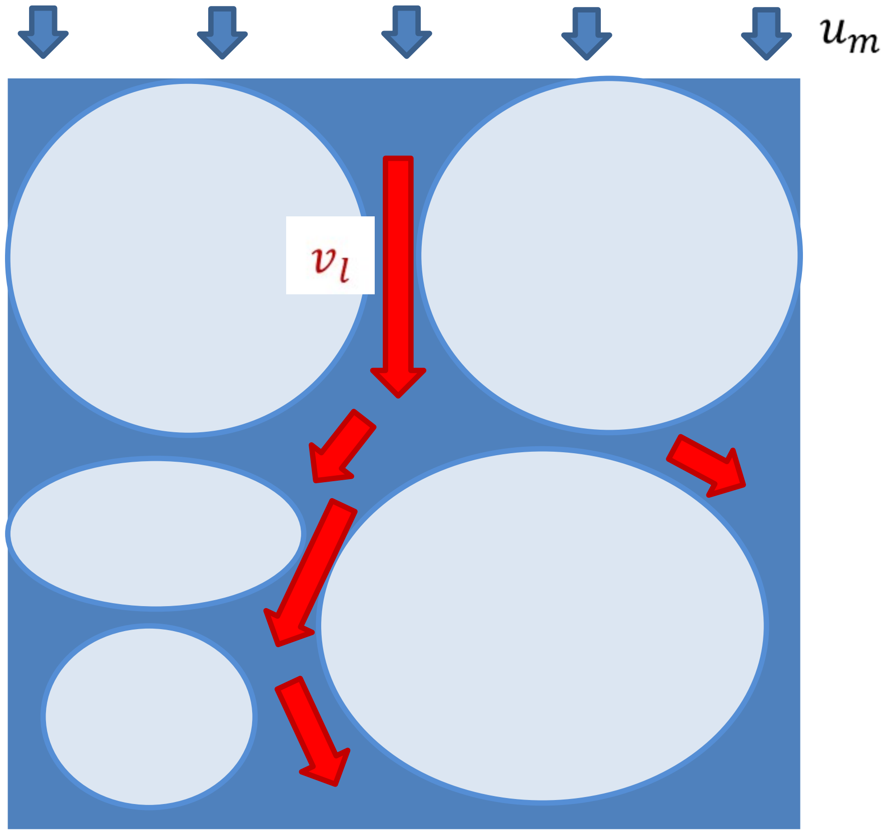
- Superficial velocity u_m results from volumetric flow

$$\frac{Q}{A} = u_m$$

- Superficial velocity does not equal local velocity v_l!
- Superficial velocity and average local velocity are linked by the liquid fraction:

$$u_m = \langle v_l \rangle \varphi_l$$

$$v_l = \frac{u_{m,l}}{\varphi} \qquad v_g = \frac{u_{m,g}}{(1-\varphi)}$$



Pneumatic foam theory

ON THE DRIFT-FLUX ANALYSIS OF FLOTATION AND FOAM FRACTIONATION PROCESSES

Paul Stevenson,[1]* Paul S. Fennell[2] and Kevin P. Galvin[1]

1. Centre for Advanced Particle Processing, University of Newcastle, Callaghan, New South Wales 2308, Australia
2. Department of Chemical Engineering and Chemical Technology, Imperial College, London SW7 2AZ, U.K.

- Superficial liquid drainage velocity from Darcy's law:

$$u_{m,l}=\frac{\alpha}{\mu}\frac{dp}{dx}=\frac{\alpha}{\mu}\rho g$$

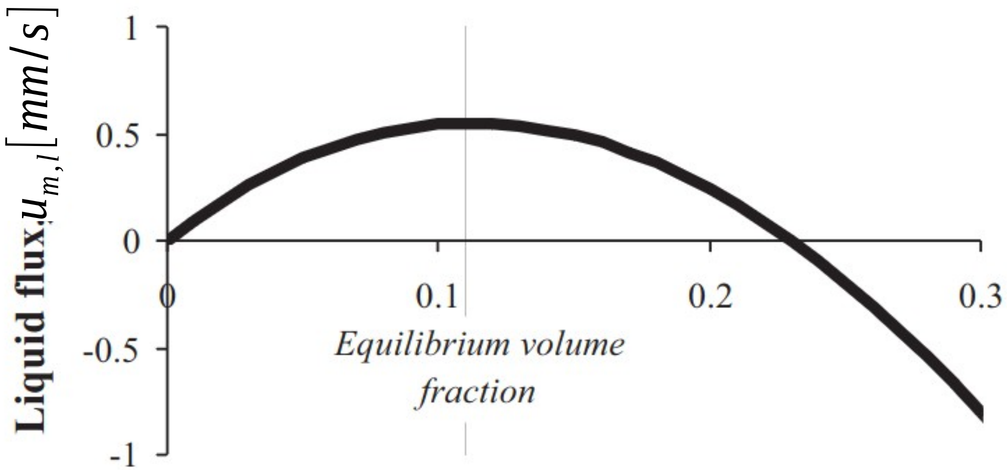
- Frozen foam

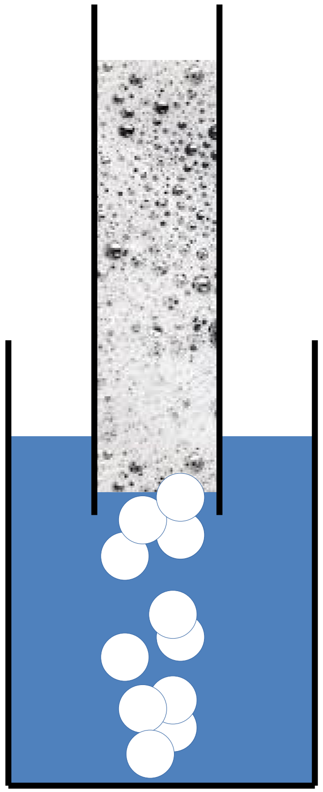
$$v_l=\frac{u_{m,l}}{\varphi} \qquad v_g=\frac{u_{m,g}}{(1-\varphi)}$$

$$v_l=v_g \rightarrow u_{m,l}=\frac{\varphi u_{m,g}}{1-\varphi}$$

- Frozen foam + drainage

$$u_{m,l}=\frac{\varphi u_{m,g}}{1-\varphi}-\frac{\alpha}{\mu}\rho g$$





Foam drainage equation

- Complex differential equation
- Describes change of local liquid fraction
- Includes gravitationally driven transport
- Includes capillary driven transport

$$\frac{\partial \varphi_l}{\partial t} + \vec{\nabla} \cdot (\varphi_l \vec{v}) = 0$$

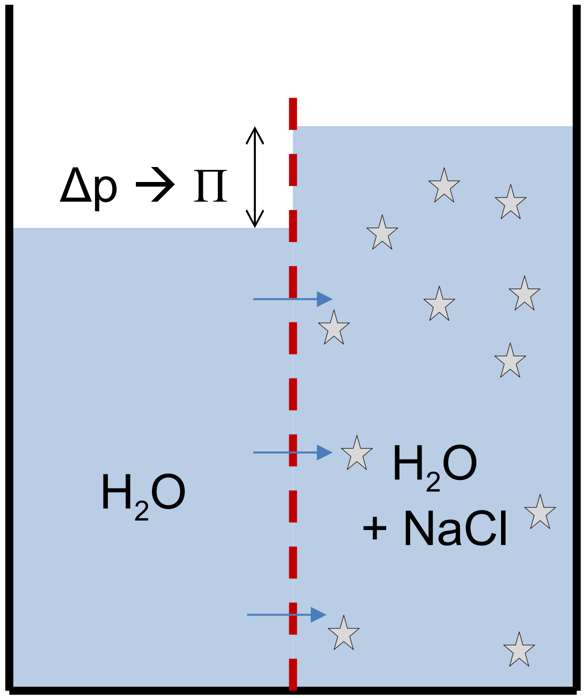
$$\frac{\partial \varphi_l}{\partial t} + \boxed{\frac{\rho}{\mu} \vec{\nabla} \cdot (\alpha \vec{g})} - \frac{\gamma}{2\, \delta_b R_e\, \mu} \vec{\nabla} \cdot \left(\frac{\alpha}{{\varphi_l}^{3/2}} \vec{\nabla} \varphi_l \right) = 0$$

Structure

- Review lecture 6
- Liquid fraction balance
- Darcy’s Law
- Osmotic pressure
- Drainage equation

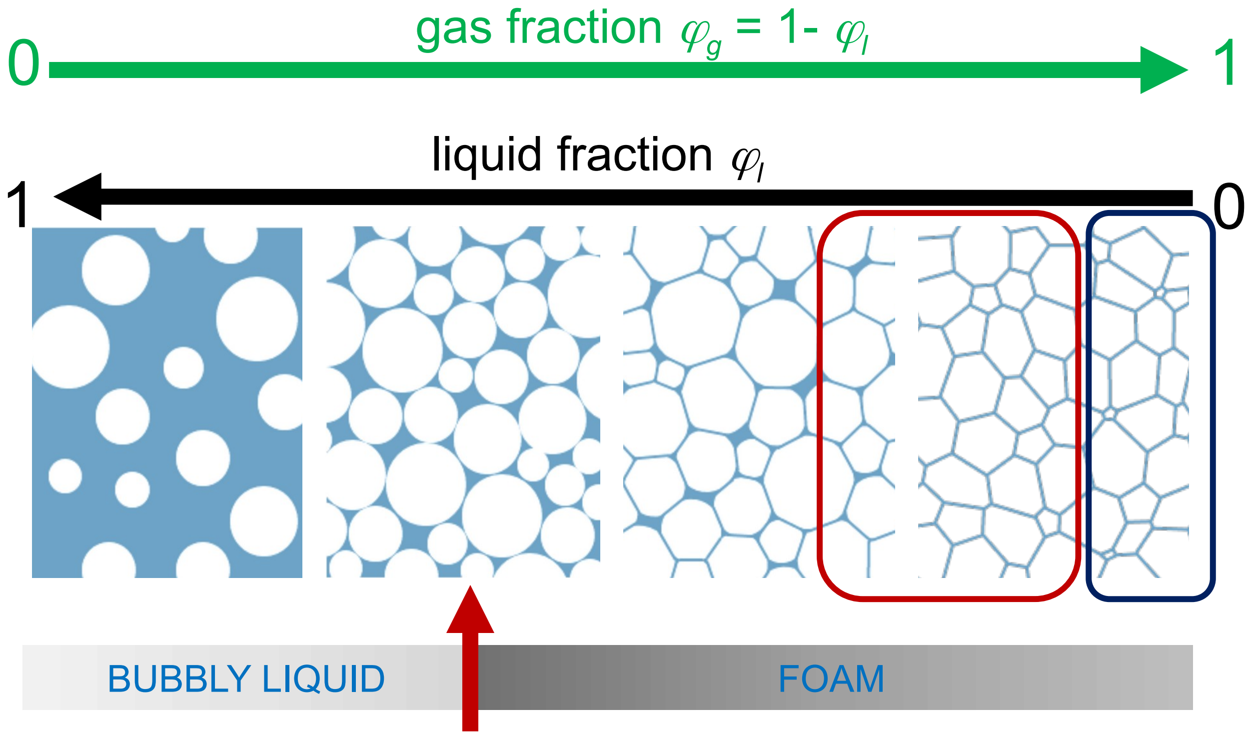
Osmotic pressure

- Transport of solvent through semi-permeable membrane
- Salt water "sucks in" pure water



Jamming limit

- Liquid fraction: Ratio of liquid volume and foam volume
- Jamming point at ≈36% liquid fraction
- Below that point: foam



Critical liquid fraction $\varphi_c \approx 0.36$

Jamming point – Bubbles are in contact with each other

Osmotic pressure in foam

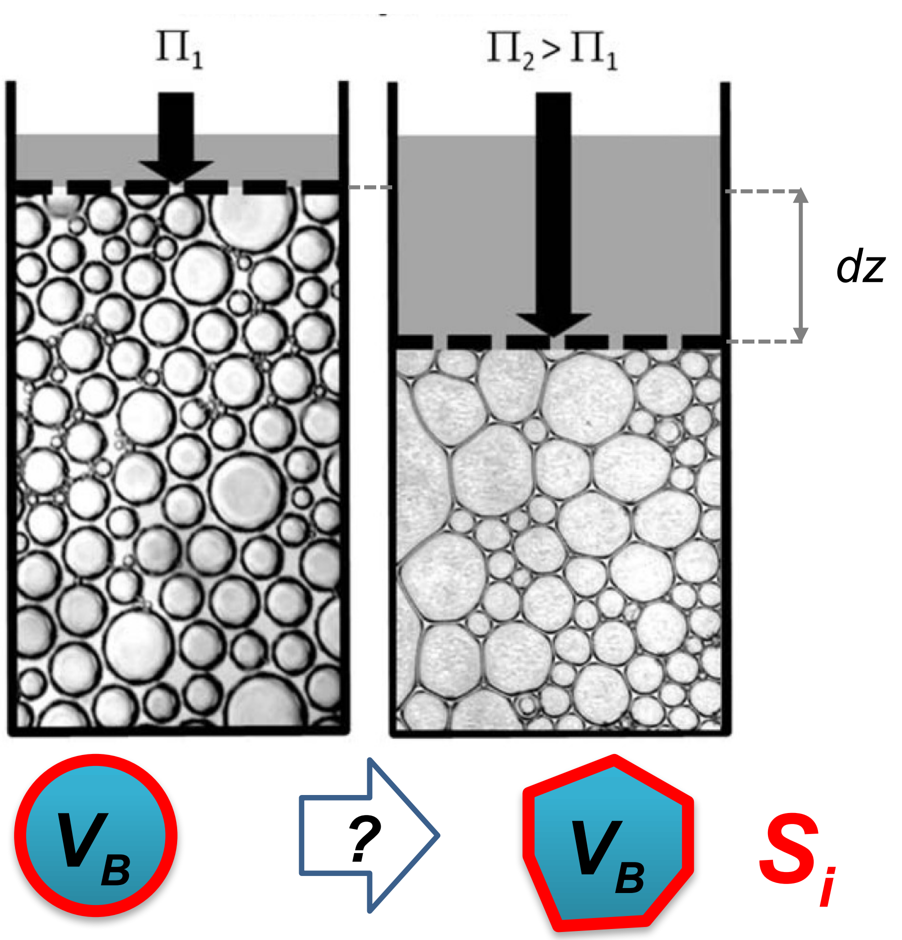
- Bubbles prefer to maintain spherical shape
- Dry foam "sucks in" water
- Mechanical work W required to remove water from foam

$$\mathrm{d}W = -F\,dz = -\Pi \underbrace{A\,dz}_{dV_l} = \gamma \sum_i dS_i$$

$$\Pi = -\gamma \sum_i \frac{dS_i}{dV_l}$$

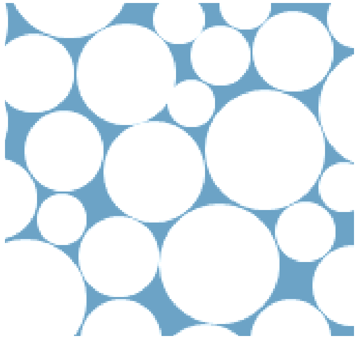
$$\varphi_l = \frac{V_l}{V_l + V_g} \qquad d\varphi_l = d\left(\frac{V_l}{V_l + V_g}\right) = \frac{(1-\varphi_l)^2}{V_g} dV_l$$

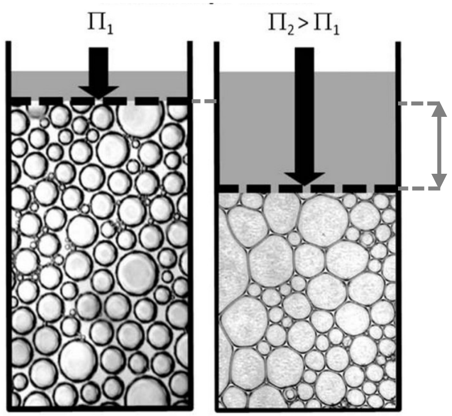
$$\Pi = -\gamma\,(1-\varphi_l)^2 \sum_i \frac{d(S_i/V_g)}{d\varphi_l}$$

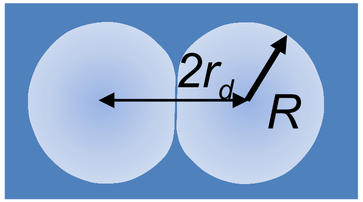


Osmotic pressure for wet foam

- Dense packing of deformable spheres
- On average z_c=6 neighbours per bubble
- Liquid fraction close to jamming point at $\varphi_c \approx 0.36$
- Interaction potential between “kissing” bubbles: $U(r_d)$
- Set into definition for osmotic pressure:







$$U(r) \sim \gamma R^2 \left[\left(\frac{R}{r_d} \right)^3 - 1 \right]^k$$

$$\mathrm{d}W = -F\,dz = -\Pi\,A\,dz = \gamma \sum_i dS_i$$

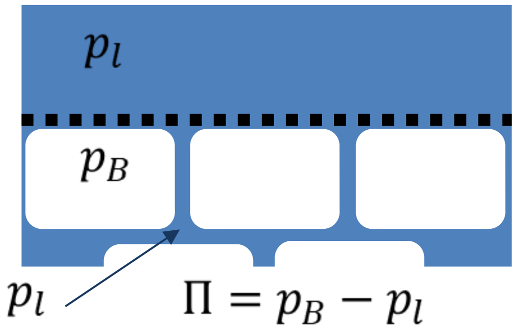
$$\frac{\mathrm{d}W}{dV_l} = -\Pi = z_c \frac{\mathrm{d}U(r_d)}{dr_d} \frac{\mathrm{d}r_d}{dV_l}$$

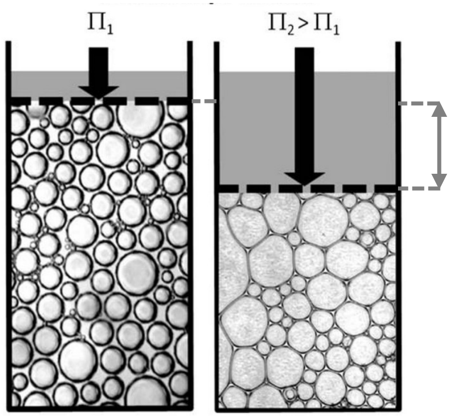
$$\Rightarrow \quad \Pi \sim \frac{\gamma}{R} k z_c (1 - \varphi_l)^2 (\varphi_l - \varphi_c)^{k-1} \xrightarrow{\varphi_l \approx} \boxed{\Pi \sim \frac{\gamma}{R} (\varphi_l - \varphi_c)^{k-1}}$$

[Mason et al. PRE 1997]

Osmotic pressure for dry foam

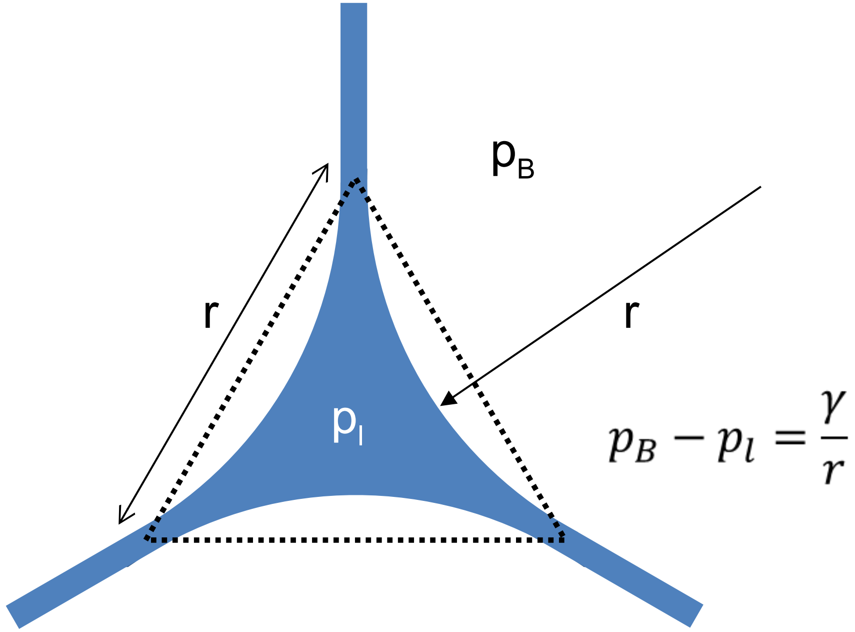
- Osmotic pressure equals capillary pressure p_B-p_l
- Capillary pressure balanced by Plateau border curvature
- Liquid fraction results from decoration theorem





$$\Pi = p_B - p_l = \frac{\gamma}{r} \qquad \varphi_l \approx 0.72\,\frac{r^2}{{R_e}^2}$$

$$\Pi = \frac{\gamma}{R_e}\frac{\sqrt{0.72}}{\sqrt{\varphi_l}} \rightarrow \boxed{\Pi \sim \frac{\gamma}{R_e}\frac{1}{\sqrt{\varphi_l}}}$$



Osmotic pressure for foam

- Combination of dry and wet limit:

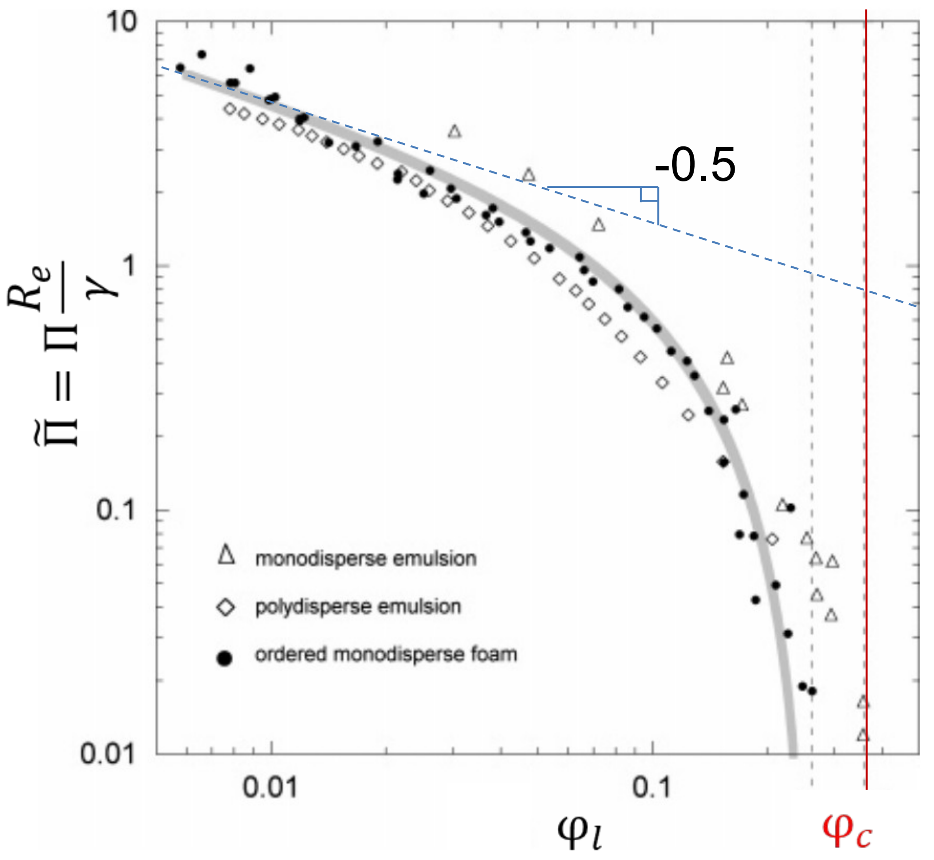
$$\boxed{\Pi \sim \frac{\gamma}{R_e} \frac{1}{\sqrt{\varphi_l}}}$$

$$\boxed{\Pi \sim \frac{\gamma}{R_e} (\varphi_l - \varphi_c)^{k-1}}$$

$$\Pi \sim \frac{\gamma}{R_e} \frac{(\varphi_l - \varphi_c)^{k-1}}{\sqrt{\varphi_l}}$$

- Empierical finding:

$$\Pi = 7.3 \frac{\gamma}{R_e} \frac{(\varphi_l - \varphi_c)^2}{\sqrt{\varphi_l}}$$



[Hoehler et al., 2008]

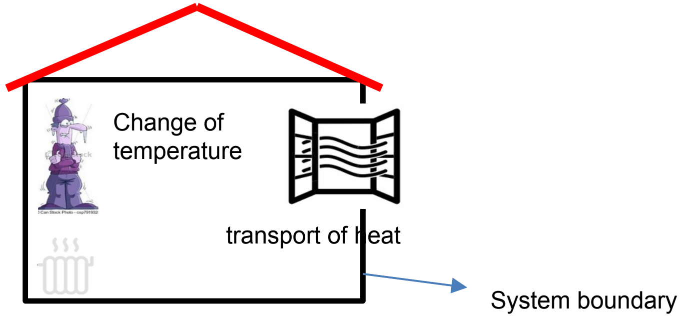
Structure

- Review lecture 6
- Liquid fraction balance
- Darcy's Law
- Osmotic pressure
- Drainage equation

Drainage equation

- Based on balance of liquid fraction

$$\frac{\partial \varphi_l}{\partial t} + \vec{\nabla} \cdot (\varphi_l\, \vec{v}) = 0 \qquad \frac{\partial \varphi_l}{\partial t} + \vec{\nabla} \cdot (\vec{u}_m) = 0$$



- Superficial velocity $\vec{u}_m$ includes gravity and osmotic pressure

$$\vec{u}_m = \frac{\alpha}{\mu} \vec{\nabla} p = \frac{\alpha}{\mu} \left(\rho\, \vec{g} + \vec{\nabla}\Pi\right) = \frac{\alpha}{\mu} \left(\rho\, \vec{g} + \vec{\nabla} \frac{\gamma}{r}\right)$$

For dry foam!!!

Substitute r

$$\varphi_l \approx 0.72 \frac{r^2}{{R_e}^2} \rightarrow \frac{1}{r} = \frac{\sqrt{0.72}}{R_e} \frac{1}{\sqrt{\varphi_l}}$$

$$\vec{u}_m = \frac{\alpha}{\mu} \vec{\nabla} p = \frac{\alpha}{\mu} \left(\rho\, \vec{g} + \frac{\gamma}{\delta_b R_e} \vec{\nabla} \frac{1}{\sqrt{\varphi_l}}\right) \qquad \delta_b = 1.73 = \frac{1}{\sqrt{0.33}}$$

$$\frac{\partial \varphi_l}{\partial t} + \frac{\rho}{\mu} \vec{\nabla} \cdot (\alpha \vec{g}) - \frac{\gamma}{2\, \delta_b R_e\, \mu} \vec{\nabla} \cdot \left(\frac{\alpha}{{\varphi_l}^{3/2}} \vec{\nabla} \varphi_l\right) = 0$$

Drainage resistance

- Drainage resistance depends on type of surfactant:

$$\frac{\partial \varphi_l}{\partial t} + \frac{\rho}{\mu}\vec{\nabla}\cdot(\alpha\vec{g}) - \frac{\gamma}{2\,\delta_b R_e\,\mu}\vec{\nabla}\cdot\left(\frac{\alpha}{\varphi_l{}^{3/2}}\vec{\nabla}\varphi_l\right) = 0$$

- Mobile interfaces – vertex-dominated dissipation: $\alpha \sim \varphi_l{}^{3/2} D_b{}^2 \quad \rightarrow \quad \alpha = C_v R_e{}^2 \varphi_l{}^{3/2}$

$$\frac{\partial \varphi_l}{\partial t} + \frac{C_v\,\rho\,g}{\mu}\frac{\partial}{\partial z}\left(R_e{}^2\varphi_l{}^{3/2}\right) - \frac{C_v\gamma}{2\,\delta_b R_e\mu}\vec{\nabla}\cdot\left(R_e{}^2\,\underline{\qquad}\,\vec{\nabla}\varphi_l\right) = 0$$

- Immobile interfaces – Plateau border dominated dissipation: $\alpha \sim \varphi_l{}^2 D_b{}^2 \quad \rightarrow \quad \alpha = C_P R_e{}^2 \varphi_l{}^2$

$$\frac{\partial \varphi_l}{\partial t} + \frac{C_v\,\rho\,g}{\mu}\frac{\partial}{\partial z}\left(R_e{}^2\varphi_l{}^2\right) - \frac{C_v\gamma}{2\,\delta_b R_e\mu}\vec{\nabla}\cdot\left(R_e{}^2\underline{\varphi_l{}^{1/2}}\vec{\nabla}\varphi_l\right) = 0$$

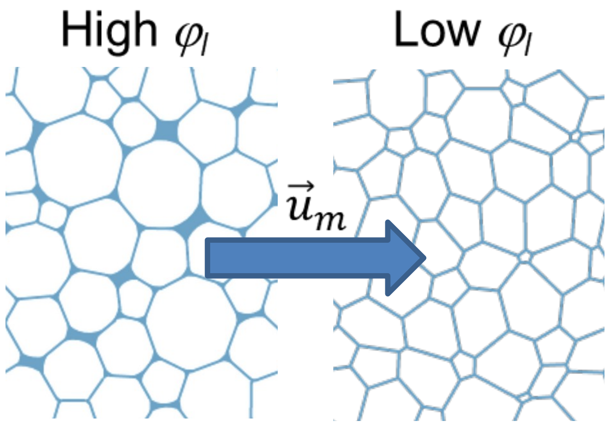
Drainage equation

- Capillary term drives horizontal and vertical drainage flows:

$$\frac{\partial \varphi_l}{\partial t} + \frac{\rho}{\mu}\vec{\nabla}\cdot(\alpha\vec{g}) - \underline{\frac{\gamma}{2\,\delta_b R_e\,\mu}\vec{\nabla}\cdot\left(\frac{\alpha}{{\varphi_l}^{3/2}}\vec{\nabla}\varphi_l\right)} = 0$$

- For mobile interfaces and homogeneous bubble size:

$$\frac{\partial \varphi_l}{\partial t} + \frac{C_v\,\rho\,g}{\mu}\frac{\partial}{\partial z}\left({R_e}^2{\varphi_l}^{3/2}\right) - \frac{C_v {R_e}^2\gamma}{2\,\delta_b R_e \mu}\vec{\nabla}\cdot\left(\vec{\nabla}\varphi_l\right) = 0$$



- Similar to diffusion equation:

$$\frac{\partial \varphi_l}{\partial t} - D\,\vec{\nabla}^2\varphi_l = 0$$

- Capillary term tends to homogenize liquid distribution

Forced drainage

- Gravity term drives a vertical drainage flow

$$\frac{\partial \varphi_l}{\partial t} + \underline{\frac{\rho}{\mu}\vec{\nabla}\cdot(\alpha\vec{g})} - \frac{\gamma}{2\,\delta_b R_e\,\mu}\vec{\nabla}\cdot\left(\frac{\alpha}{{\varphi_l}^{3/2}}\vec{\nabla}\varphi_l\right) = 0$$

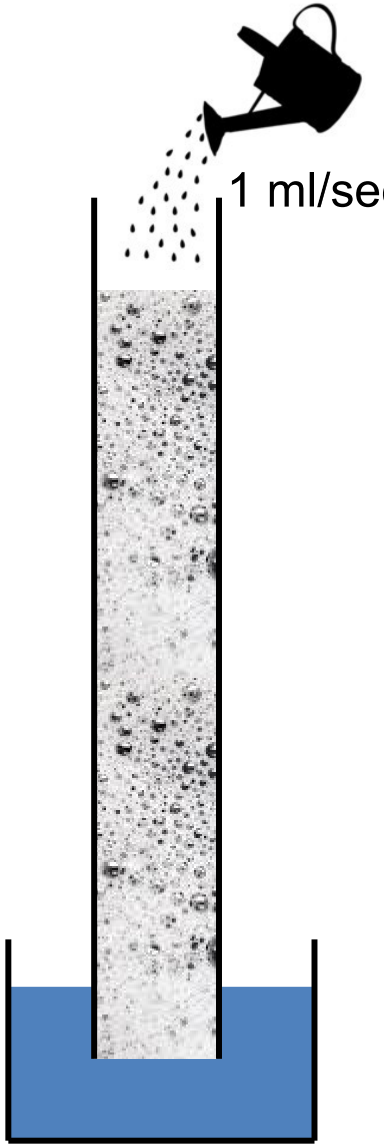
- Steady state, neglecting diffusion, homogeneous bubble size:

$$\frac{C_v\,\rho\,g\,{R_e}^2}{\mu}\frac{\partial}{\partial z}\left({\varphi_l}^{3/2}\right) = 0$$

$$\frac{C_p\,\rho\,g\,{R_e}^2}{\mu}\frac{\partial}{\partial z}\left({\varphi_l}^{2}\right) = 0$$

$$\rightarrow \frac{\partial}{\partial z}(\varphi_l) = 0$$

- Yields homogeneous liquid fraction distribution → forced drainage setup



Free drainage

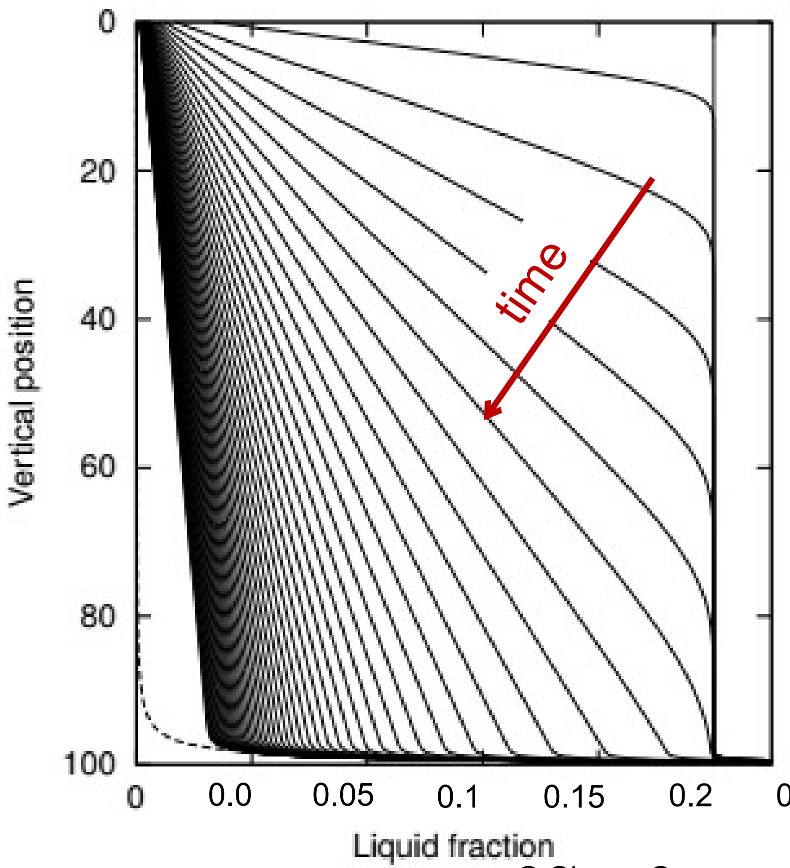
- Foam column with homogeneous bubble size and initial homogeneous liquid fraction
- Numerical simulation of 1D- Drainage equation:

$$\frac{\partial \varphi_l}{\partial t} + \frac{\rho}{\mu}\vec{\nabla}\cdot(\alpha\vec{g}) - \frac{\gamma}{2\,\delta_b R_e\,\mu}\vec{\nabla}\cdot\left(\frac{\alpha}{\varphi_l^{3/2}}\vec{\nabla}\varphi_l\right) = 0$$

$$\frac{\partial \varphi_l}{\partial t} - \frac{C_p}{\mu}\frac{\partial}{\partial z}\left(\rho\, g\, R_e^{2}\varphi_l^{2} + \frac{\gamma}{2\,\delta_b}R_e\varphi_l^{1/2}\frac{\partial \varphi_l}{\partial z}\right) = 0$$

$$\frac{\partial \varphi_l}{\partial t} - \frac{C_v}{\mu}\frac{\partial}{\partial z}\left(\rho\, g\, R_e^{2}\varphi_l^{3/2} + \frac{\gamma}{2\,\delta_b}R_e\frac{\partial \varphi_l}{\partial z}\right) = 0$$

- Distribution develops towards steady state



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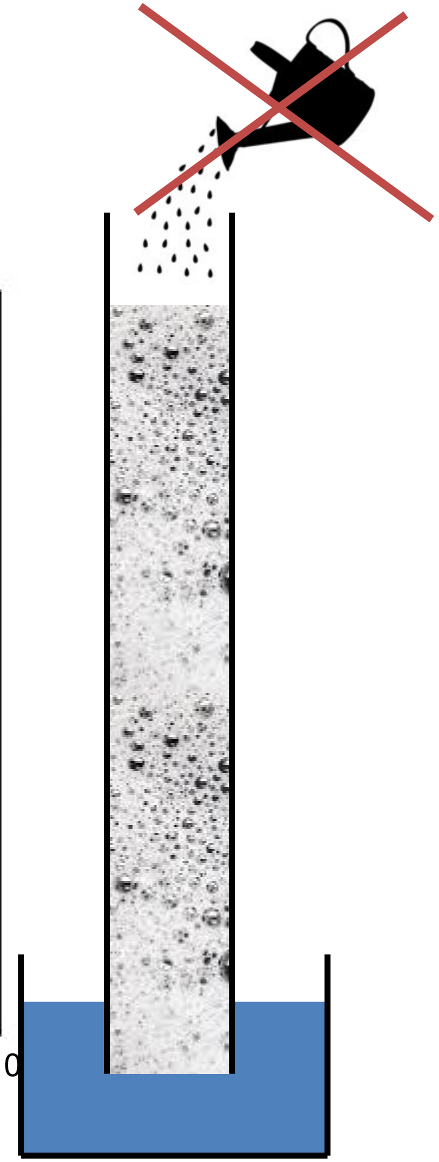


Free drainage – steady state

- Steady state: $\dfrac{\partial \varphi_l}{\partial t} = 0$

$$\lambda_c = \sqrt{\frac{\gamma}{\rho\, g}} = 2mm$$

$$\frac{C_p}{\mu}\frac{\partial}{\partial z}\left(\rho\, g\, R_e{}^2 \varphi_l{}^2 + \frac{\gamma}{2\,\delta_b} R_e \varphi_l{}^{1/2} \frac{\partial \varphi_l}{\partial z}\right) = 0$$

$$\delta_b = \frac{1}{\sqrt{0.33}}$$

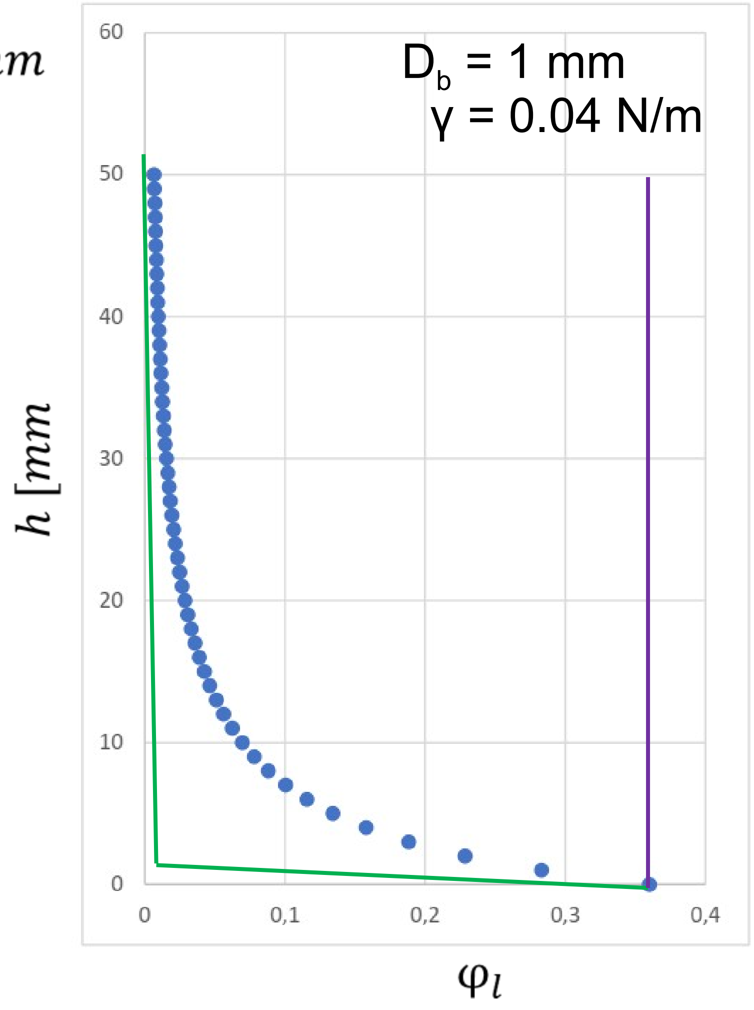
$$\frac{2\,\delta_b\, \rho\, g R_e}{\gamma} \varphi_l{}^{3/2} = -\frac{\partial \varphi_l}{\partial z}$$

$$\frac{2\,\delta_b\, \rho\, g R_e}{\gamma} \Delta z = 2\, \varphi_l{}^{-1/2} + C$$

$$\frac{\delta_b\, \rho\, g R_e}{\gamma} h = \varphi_l(h)^{-1/2} - 0.36^{-1/2}$$

$1/\lambda_c{}^2$

$$\frac{\delta_b\, \rho\, g R_e}{\gamma} h = \frac{1}{\sqrt{\varphi_l(h)}} - \frac{1}{\sqrt{0.36}} \qquad \left(\frac{R_e\, h}{\lambda_c{}^2 \sqrt{0.33}} + \frac{1}{\sqrt{0.36}}\right)^{-2} = \varphi_l(h)$$



Liquid content of foam

- Most of the liquid concentrated near the bottom
- Infinitely high foam column has finite liquid content

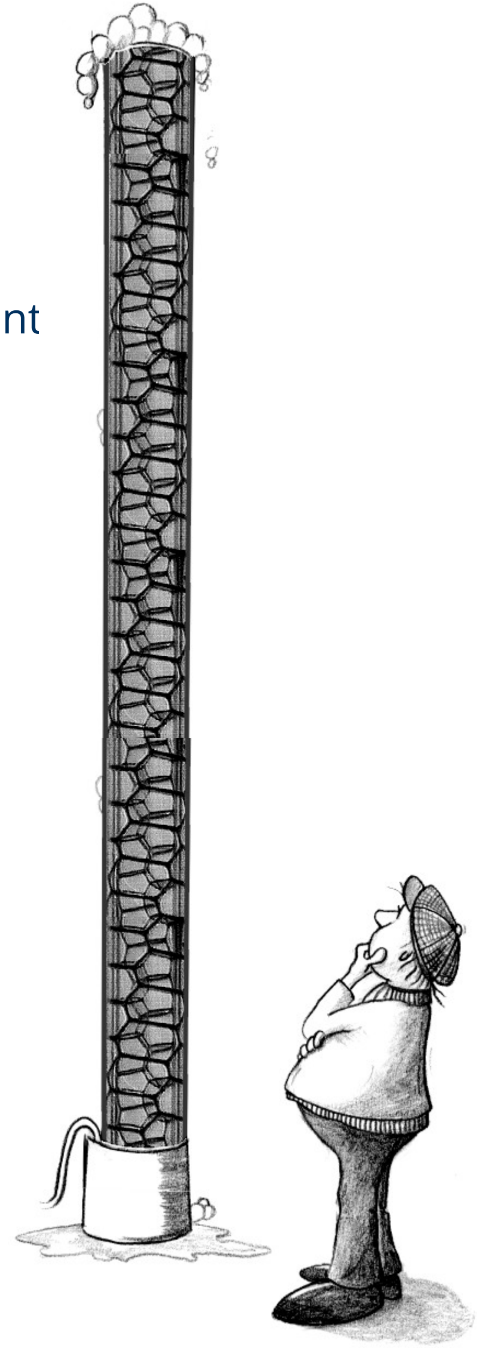
$$V_l = \int_0^{\infty} \varphi_l(h)\, A\, dh = C\, A\, \frac{{\lambda_c}^2}{R_{32}} < \infty\,!!$$

Princen: C $\approx$ 0.28 [Princen, Langmuir 1986]

$$\lambda_c = \sqrt{\frac{\gamma}{\rho\, g}}$$

D_b=1 mm   γ = 0.04 N/m

D_{Column}=30 cm



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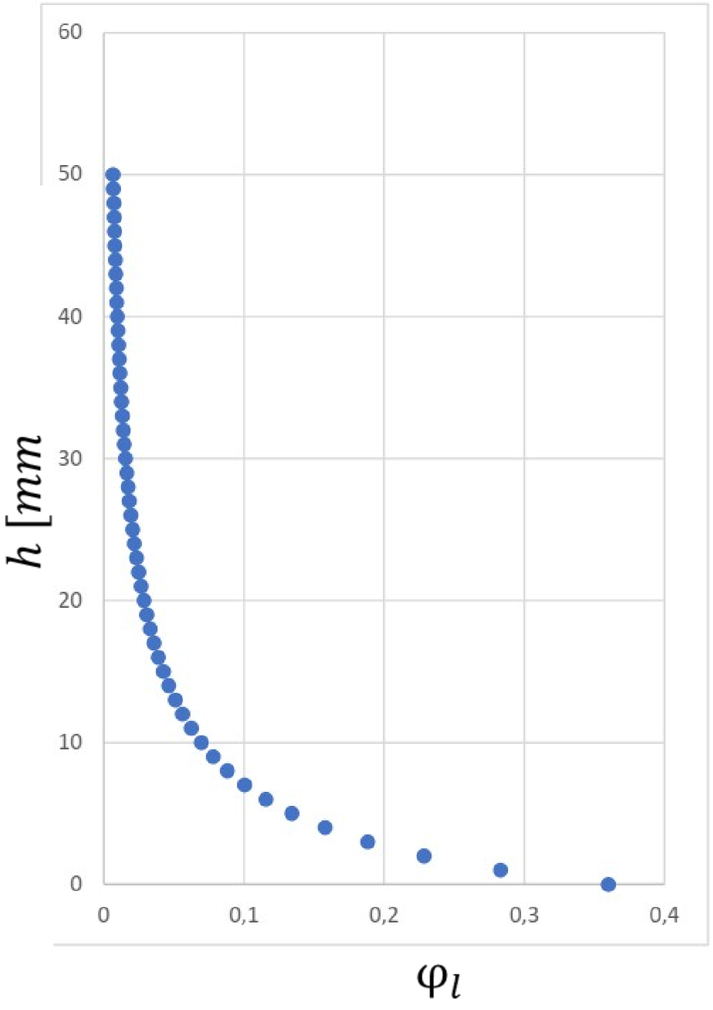

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- Infinitely high foam column has finite liquid conte

$$V_l = \int_0^\infty \varphi_l(h)\, A\, dh = C\, A\, \frac{{\lambda_c}^2}{R_{32}} < \infty\,!!$$

Princen: C $\approx$ 0.28 (Langmuir 1986)

$$\lambda_c = \sqrt{\frac{\gamma}{\rho\, g}} = 2mm$$

D_b=1 mm → R_{32} = 0.5 mm

D_{Column}=30 cm → V = 160 ml → h_{fill} = 2.2 mm