TECHNISCHE UNIVERSITÄT DRESDEN

DRESDEN concept

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Insitute for Process Engineering and Environmental Technology

Lecture 8: Foam Rheology

Structure

- Review lecture 8
- Rheology
- Elastic deformation
- Yield Stress
- Influence of liquid fraction
- Dissipation in foam flow
- Wall effect
- Dilatancy and shear banding
- Measurement of foam rheology

Continuity equation of liquid mass in foam

- Liquid density is constant, foam density results from liquid fraction

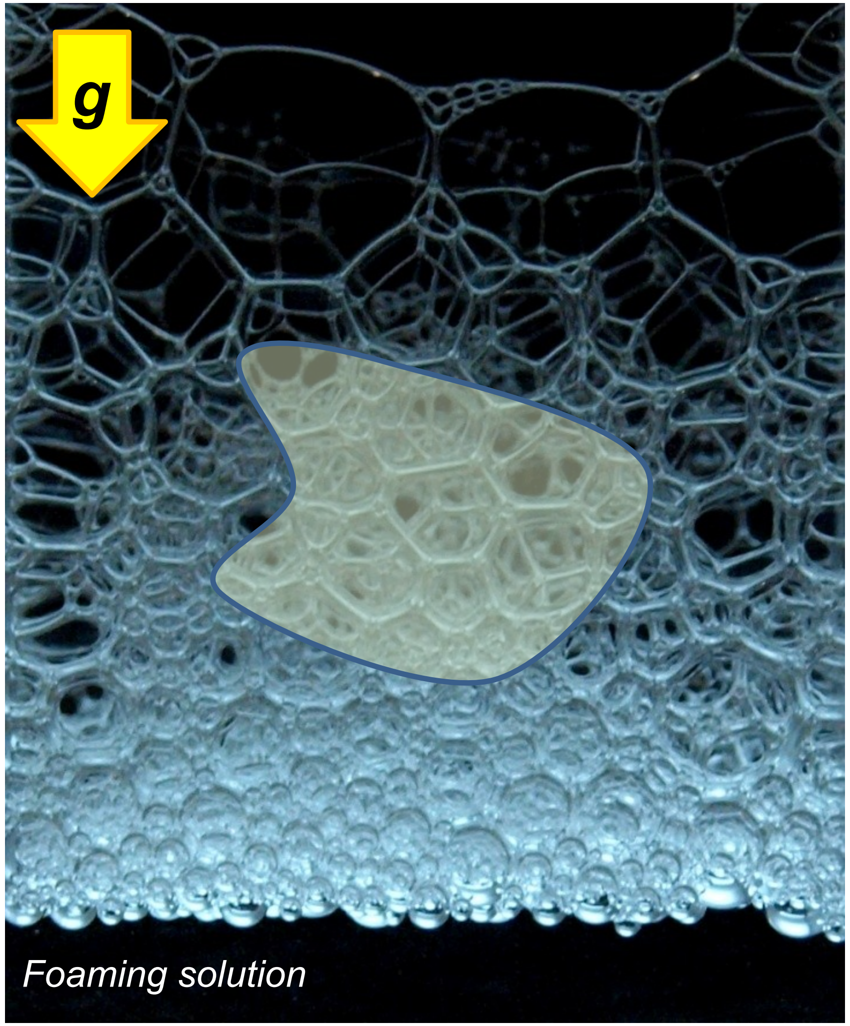
  $\rho_{foam} = \rho_{liquid}\, \varphi_l$

- → Conservation of liquid fraction:

$$\int_V \frac{\partial \varphi_l}{\partial t} dV + \oint_{\partial V} (\varphi_l\, \vec{v})\, \overrightarrow{dS} = 0$$

$$\frac{\partial \varphi_l}{\partial t} + \vec{\nabla} \cdot (\varphi_l\, \vec{v}) = 0$$

$$\frac{\partial \varphi_l}{\partial t} + \frac{\rho}{\mu} \vec{\nabla} \cdot (\alpha \vec{g}) - \frac{\gamma}{2\, \delta_b R_e\, \mu} \vec{\nabla} \cdot \left(\frac{\alpha}{\varphi_l{}^{3/2}} \vec{\nabla} \varphi_l \right) = 0$$



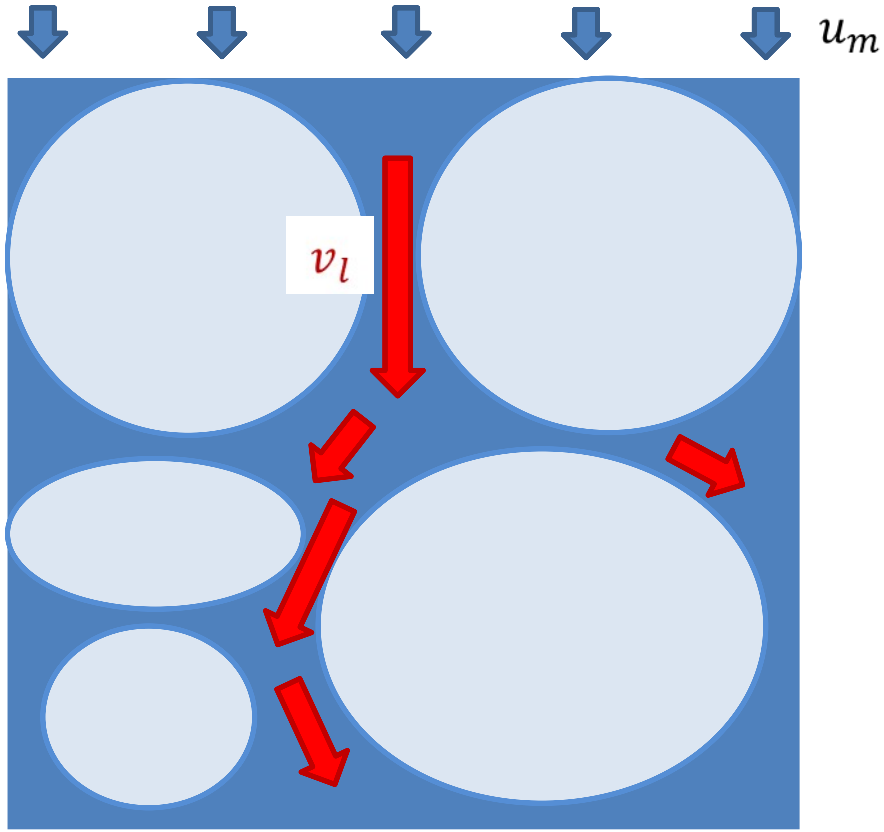
Superficial velocity

- Superficial velocity u_m results from volumetric flow

$$\frac{Q}{A} = u_m$$

- Superficial velocity does not equal local velocity v_l!
- Superficial velocity and average local velocity are linked by the liquid fraction:

$$u_m = \langle v_l \rangle \, \varphi_l$$



Drainage equation

- Based on balance of liquid fraction

$$\frac{\partial \varphi_l}{\partial t} + \vec{\nabla} \cdot (\varphi_l \, \vec{v}) = 0 \qquad \frac{\partial \varphi_l}{\partial t} + \vec{\nabla} \cdot (\vec{u}_m) = 0$$

Origin of Darcy's Law

- Geology, drainage of water in soil

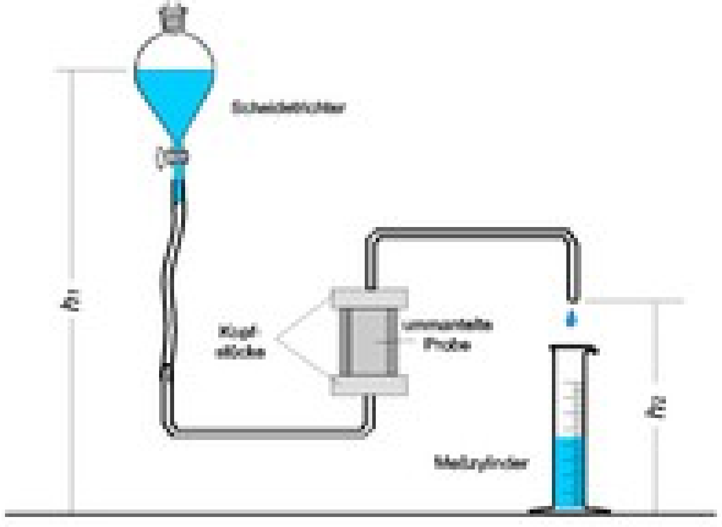
$$\frac{Q}{A} = u_m = \frac{\alpha}{\mu}\frac{dp}{dx} \qquad u_m = \langle v_l \rangle \varphi_l$$

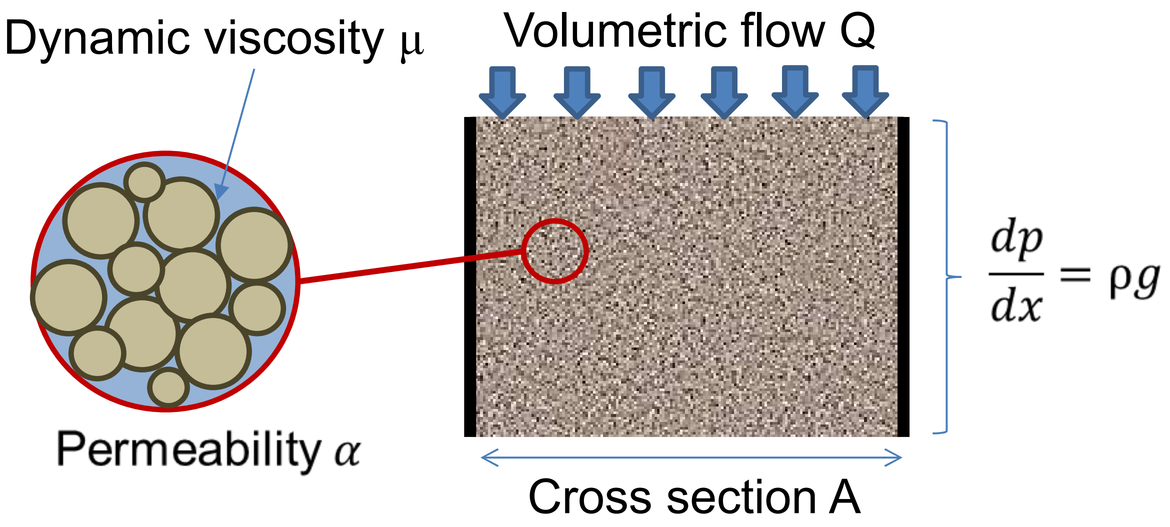
- Units:

$$\frac{m^3/s}{m^2} = \frac{m^2}{Pa\,s}\frac{Pa}{m}$$

- Permeability k is a feature of the packing
- For soil (polydisperse grains) estimated from effective grain diameter:

$$\alpha \approx 0.00116\ {d_{eff}}^2$$





Drainage equation

- Based on balance of liquid fraction

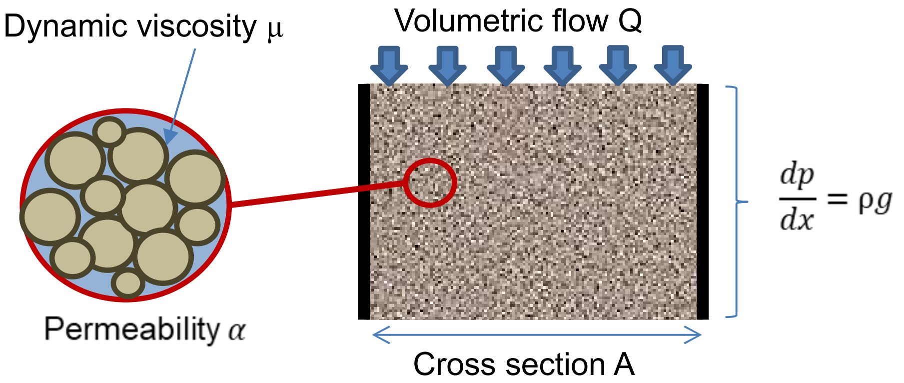
$$\frac{\partial \varphi_l}{\partial t} + \vec{\nabla} \cdot (\varphi_l \, \vec{v}) = 0 \qquad \frac{\partial \varphi_l}{\partial t} + \vec{\nabla} \cdot (\vec{u}_m) = 0$$

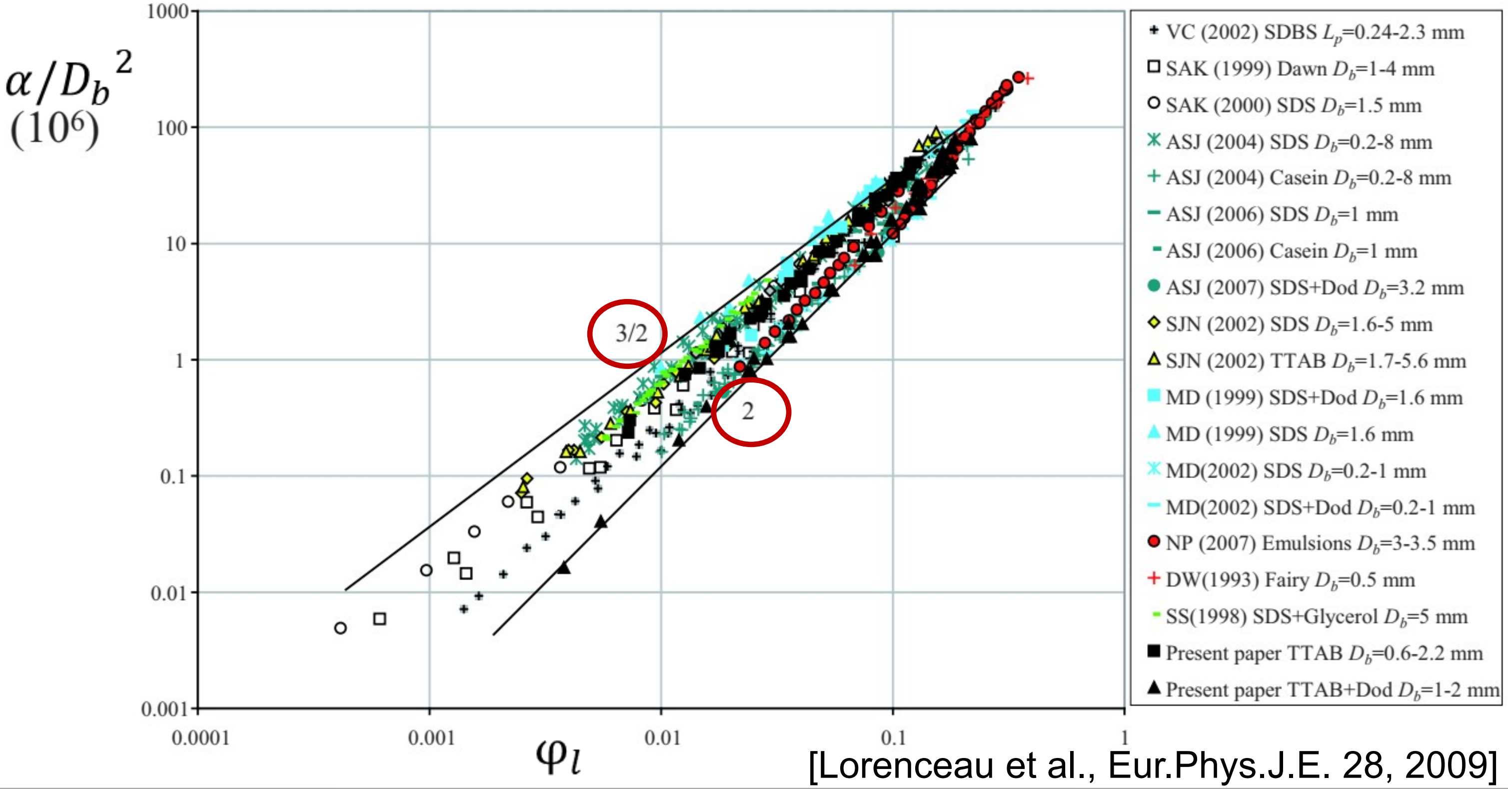
- Superficial velocity $\vec{u}_m$

$$\vec{u}_m = \frac{\alpha}{\mu} \vec{\nabla} p$$

Permeability of foam

- Darcy's law is valid for foam $$\frac{Q}{A} = \frac{\alpha}{\mu}\frac{dp}{dx}$$
- BUT (!) Permeability depends on liquid fraction
- Exponent depends on surfactant
- → SDS: $$\alpha \sim \varphi_l^{3/2} D_b^2$$
- → Casein: $$\alpha \sim \varphi_l^{2} D_b^2$$





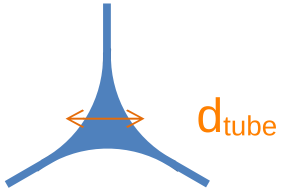
[Lorenceau et al., Eur.Phys.J.E. 28, 2009]

Flow inside the Plateau borders

- Liquid is concentrated in Plateau borders
- Plateau borders form a network (of pipes)
- Plateau border dominated:

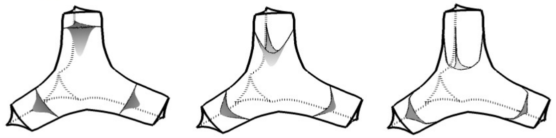
$$Q \sim \frac{{d_{tube}}^4}{\mu}\frac{dp}{dx} \quad \Leftrightarrow \quad \frac{Q}{A} = \frac{\alpha}{\mu}\frac{dp}{dx}$$

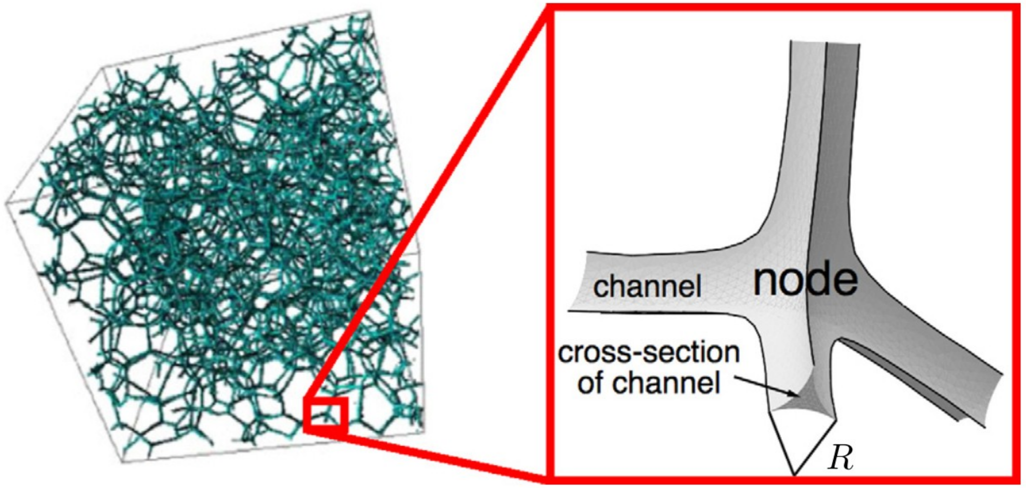
$$\alpha \sim {\varphi_l}^2 {R_e}^2$$



- Vertex dominated:

$$\alpha \sim {\varphi_l}^{3/2} {D_b}^2$$





[Dollet & Raufaste 2015]

Drainage equation

- Based on balance of liquid fraction

$$\frac{\partial \varphi_l}{\partial t} + \vec{\nabla} \cdot (\varphi_l \, \vec{v}) = 0 \qquad \frac{\partial \varphi_l}{\partial t} + \vec{\nabla} \cdot (\vec{u}_m) = 0$$

- Superficial velocity $\vec{u}_m$ includes gravity and osmotic pressure

$$\vec{u}_m = \frac{\alpha}{\mu} \vec{\nabla} p = \frac{\alpha}{\mu} \left(\rho \, \vec{g} + \vec{\nabla} \Pi \right)$$

Osmotic pressure in foam

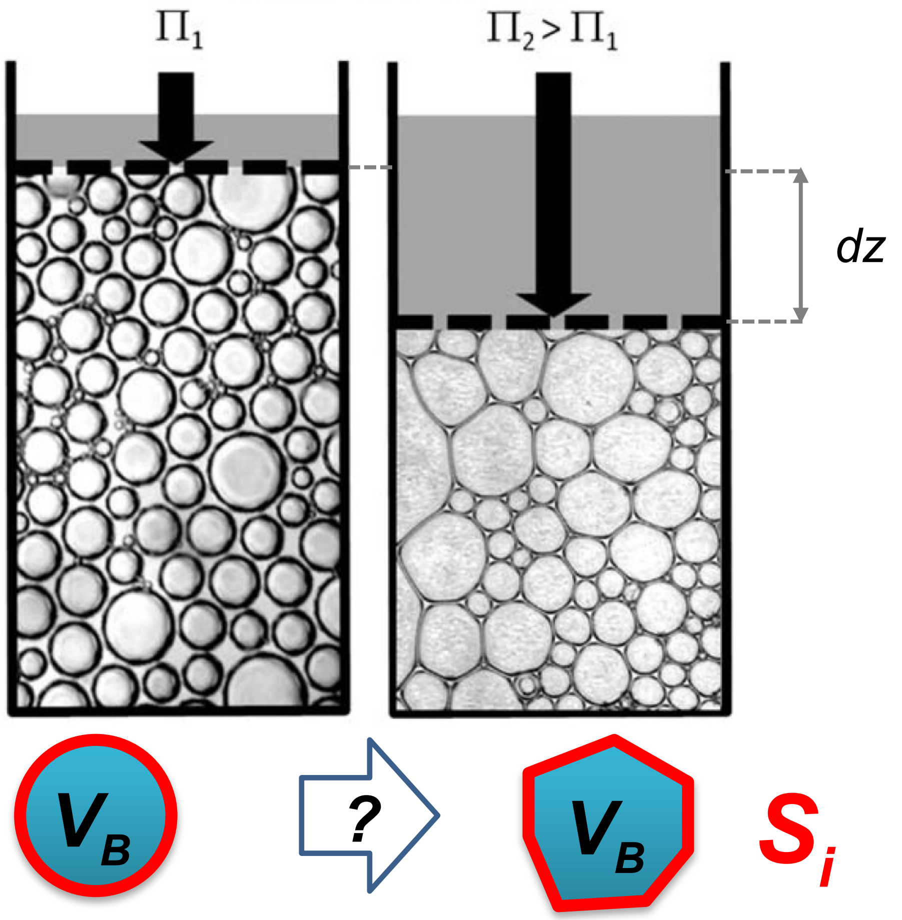
- Bubbles prefer to maintain spherical shape
- Dry foam "sucks in" water
- Mechanical work W required to remove water from foam

$$\mathrm{d}W = -F\,dz = -\Pi\,\underbrace{A\,dz}_{dV_l} = \gamma \sum_i dS_i$$

$$\Pi = -\gamma \sum_i \frac{dS_i}{dV_l}$$

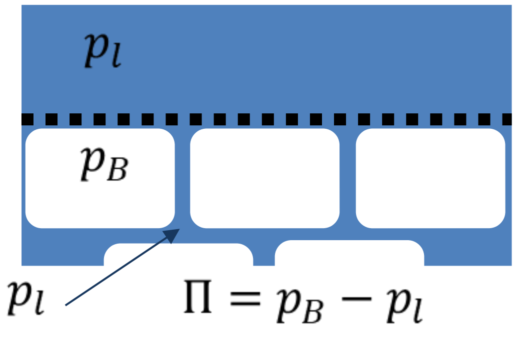
$$\varphi_l = \frac{V_l}{V_l + V_g} \qquad d\varphi_l = d\left(\frac{V_l}{V_l + V_g}\right) = \frac{(1-\varphi_l)^2}{V_g} dV_l$$

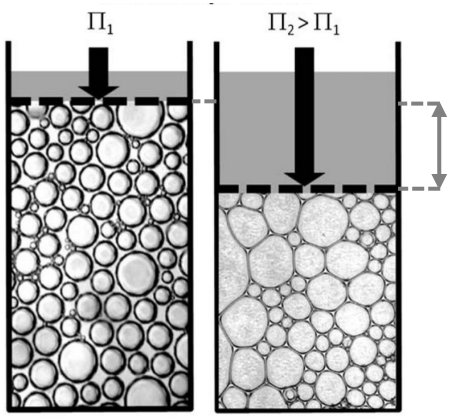
$$\Pi = -\gamma\,(1-\varphi_l)^2 \sum_i \frac{d(S_i/V_g)}{d\varphi_l}$$



Osmotic pressure for dry foam

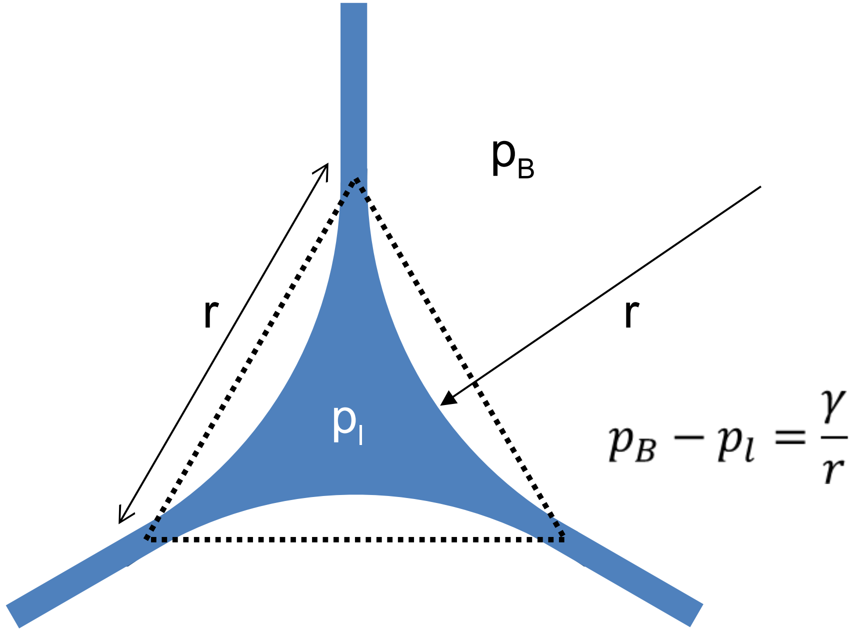
- Osmotic pressure equals capillary pressure p_B-p_l
- Capillary pressure balanced by Plateau border curvature
- Liquid fraction results from decoration theorem





$$\Pi = p_B - p_l = \frac{\gamma}{r} \qquad \varphi_l \approx 0.72 \frac{r^2}{{R_e}^2}$$

$$\Pi = \frac{\gamma}{R_e} \frac{\sqrt{0.72}}{\sqrt{\varphi_l}} \rightarrow \boxed{\Pi \sim \frac{\gamma}{R_e} \frac{1}{\sqrt{\varphi_l}}}$$



Drainage equation

- Based on balance of liquid fraction

$$\frac{\partial \varphi_l}{\partial t} + \vec{\nabla} \cdot (\varphi_l\, \vec{v}) = 0 \qquad \frac{\partial \varphi_l}{\partial t} + \vec{\nabla} \cdot (\vec{u}_m) = 0$$

- Superficial velocity $\vec{u}_m$ includes gravity and osmotic pressure

$$\vec{u}_m = \frac{\alpha}{\mu} \vec{\nabla} p = \frac{\alpha}{\mu} \left(\rho\, \vec{g} + \vec{\nabla}\Pi\right) = \frac{\alpha}{\mu} \left(\rho\, \vec{g} + \vec{\nabla}\frac{\gamma}{r}\right)$$

Substitute r

$$\varphi_l \approx 0.72 \frac{r^2}{{R_e}^2} \rightarrow \frac{1}{r} = \frac{\sqrt{0.72}}{R_e} \frac{1}{\sqrt{\varphi_l}}$$

$$\vec{u}_m = \frac{\alpha}{\mu} \vec{\nabla} p = \frac{\alpha}{\mu} \left(\rho\, \vec{g} + \frac{\gamma}{\delta_b R_e} \vec{\nabla} \frac{1}{\sqrt{\varphi_l}}\right) \qquad \delta_b = 1.73$$

$$\frac{\partial \varphi_l}{\partial t} + \frac{\rho}{\mu} \vec{\nabla} \cdot (\alpha \vec{g}) - \frac{\gamma}{2\, \delta_b R_e\, \mu} \vec{\nabla} \cdot \left(\frac{\alpha}{{\varphi_l}^{3/2}} \vec{\nabla} \varphi_l\right) = 0$$

Free drainage

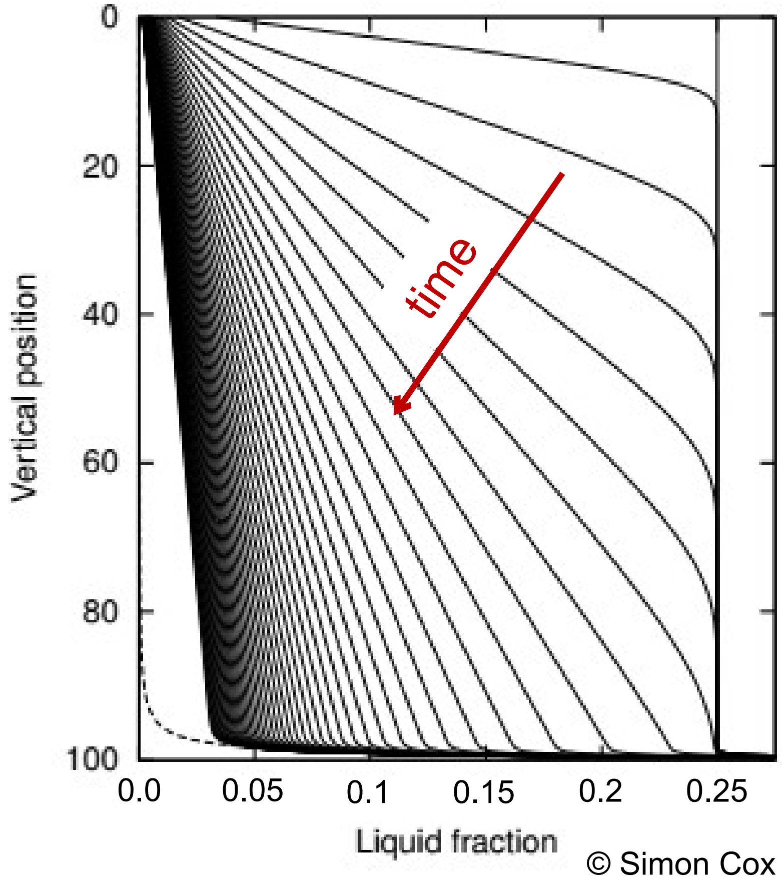
- Foam column with homogeneous bubble size and initial homogeneous liquid fraction
- Numerical simulation of 1D- Drainage equation:

$$\frac{\partial \varphi_l}{\partial t} + \frac{\rho}{\mu}\vec{\nabla}\cdot(\alpha\vec{g}) - \frac{\gamma}{2\,\delta_b R_e\,\mu}\vec{\nabla}\cdot\left(\frac{\alpha}{\varphi_l{}^{3/2}}\vec{\nabla}\varphi_l\right) = 0$$

$$\frac{\partial \varphi_l}{\partial t} - \frac{C_p}{\mu}\frac{\partial}{\partial z}\left(\rho\, g\, R_e{}^2\varphi_l{}^2 + \frac{\gamma}{2\,\delta_b}R_e\varphi_l{}^{1/2}\frac{\partial \varphi_l}{\partial z}\right) = 0$$

$$\frac{\partial \varphi_l}{\partial t} - \frac{C_v}{\mu}\frac{\partial}{\partial z}\left(\rho\, g\, R_e{}^2\varphi_l{}^{3/2} + \frac{\gamma}{2\,\delta_b}R_e\frac{\partial \varphi_l}{\partial z}\right) = 0$$

- Distribution develops towards steady state



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Structure

- Review lecture 8
- Rheology
- Elastic deformation
- Yield Stress
- Influence of liquid fraction
- Dissipation in foam flow
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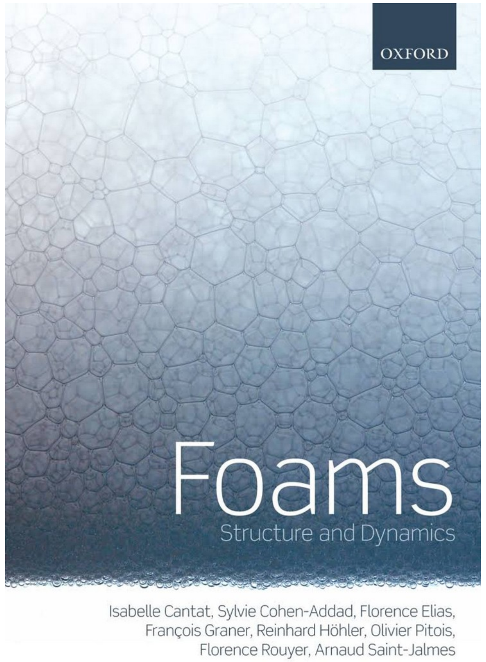
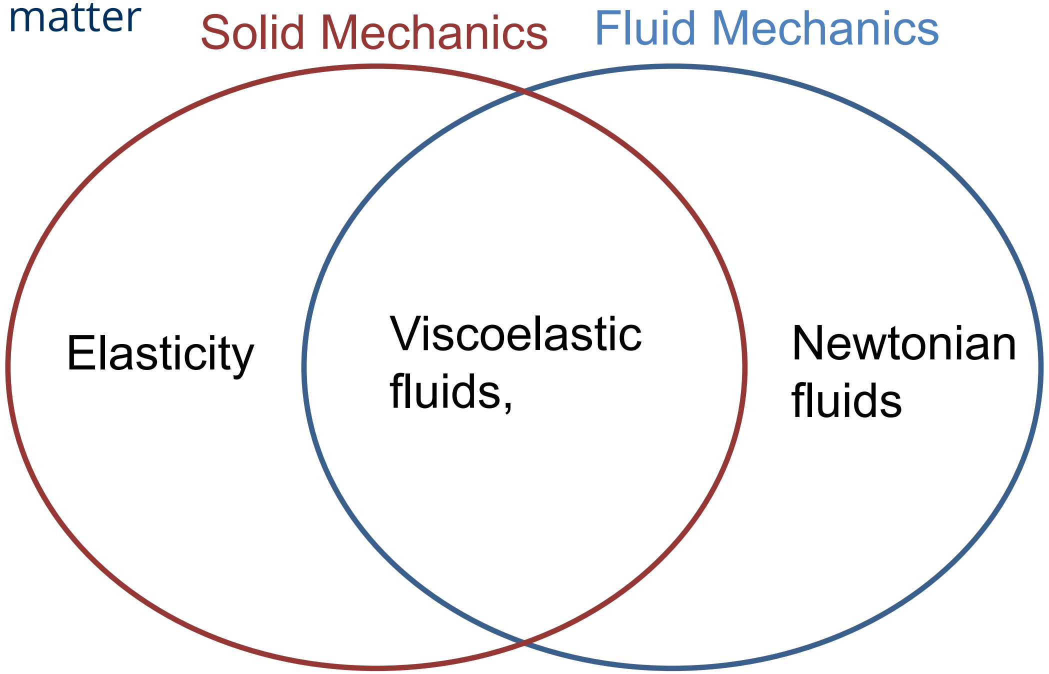
- Measurement of foam rheology



What is rheology

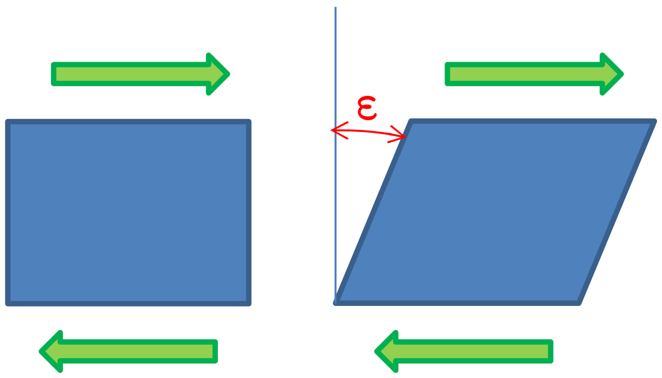
- Describes the flow of matter

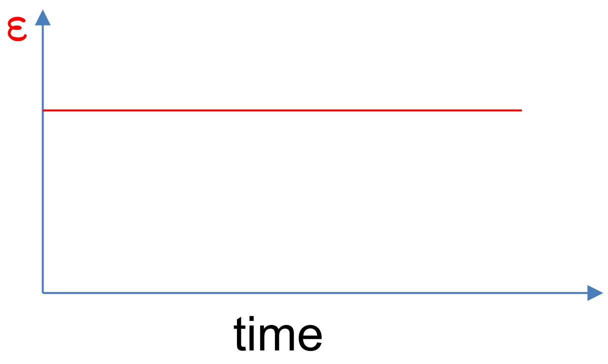


Response to stress:

- Elastic body

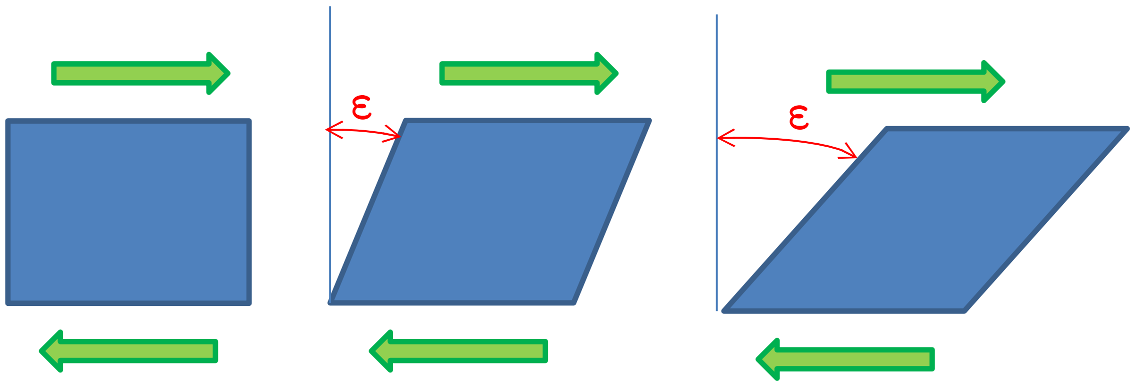
  Constant stress → constant strain **angle** ε

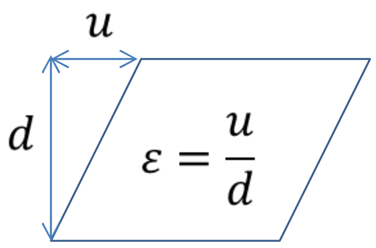


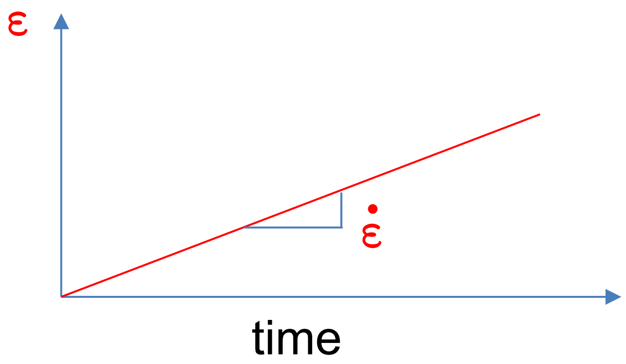


- Newtonian fluid:

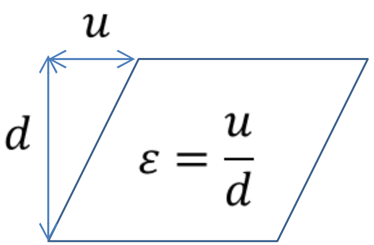
  Constant stress → constant strain **rate** $\dot{\varepsilon}$



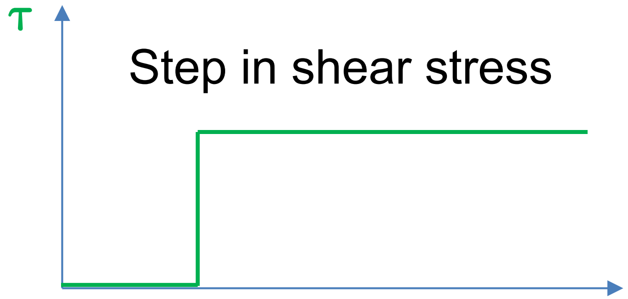




Response on shear stress step

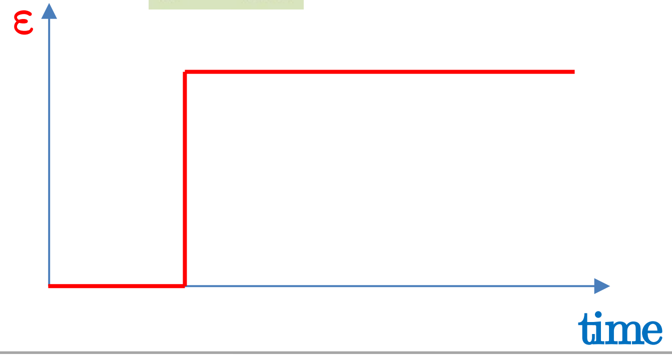


- Sudden stress is provided, corresponding strain is measured
- Relation between stress τ and strain ε



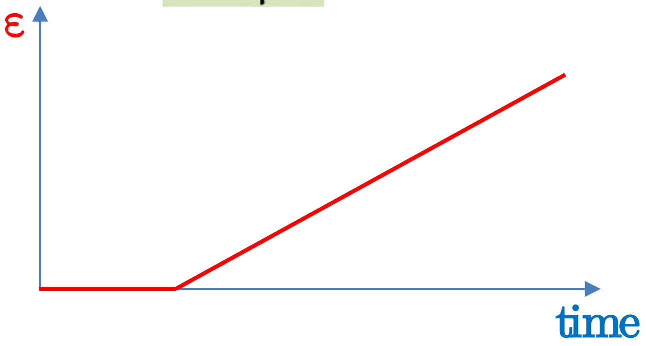
- Elastic body:
  shear modulus G

$$\tau = G\varepsilon$$



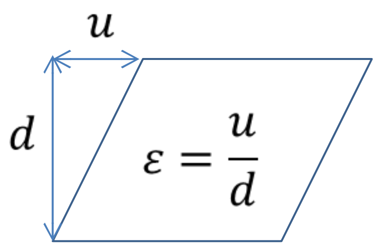
- Newtonian fluid:
  viscosity μ

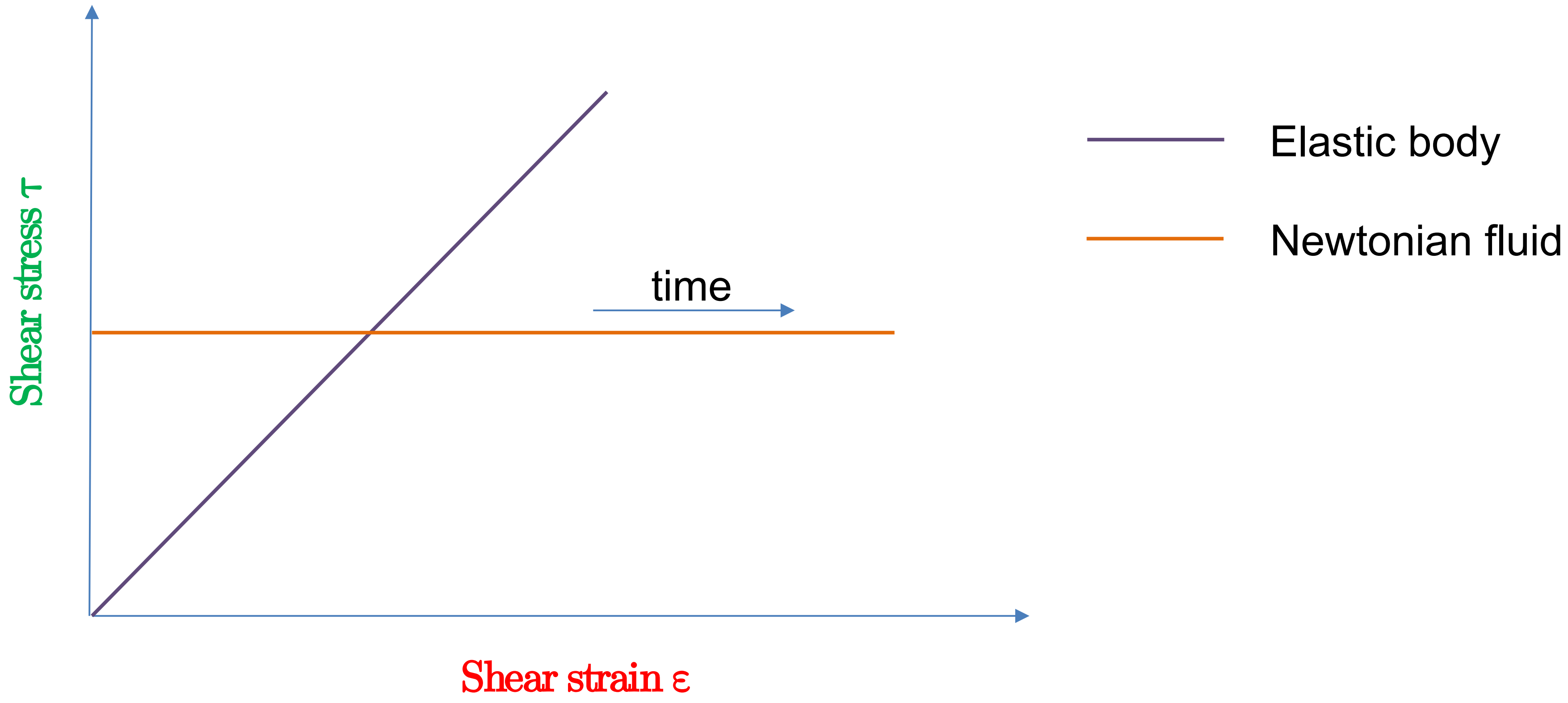
$$\tau = \mu\,\dot{\varepsilon}$$



Constitutive law: stress vs strain

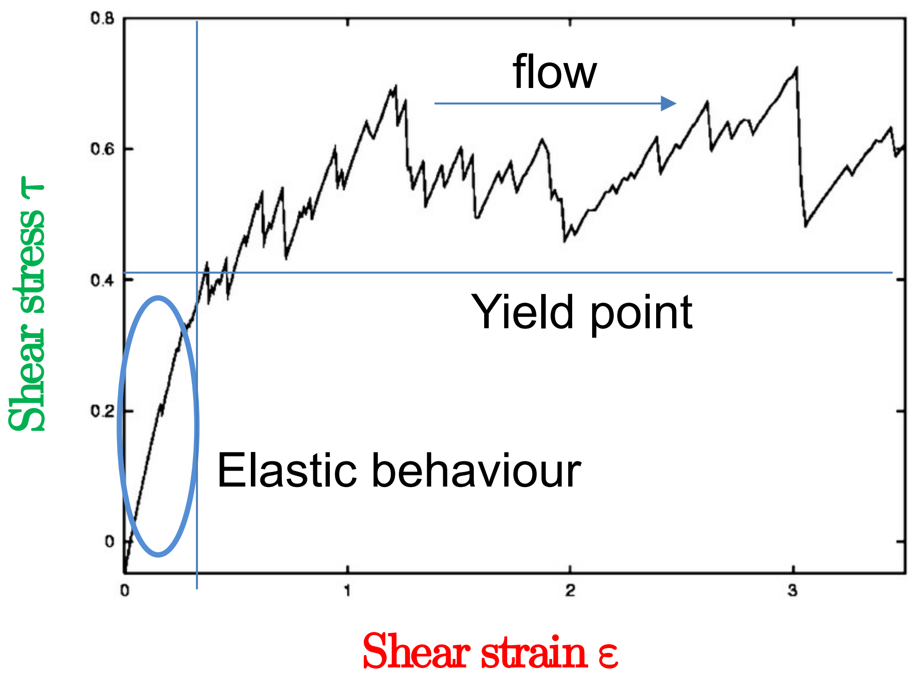
- Sudden stress is provided, corresponding strain is measured
- Relation between stress τ and strain ε

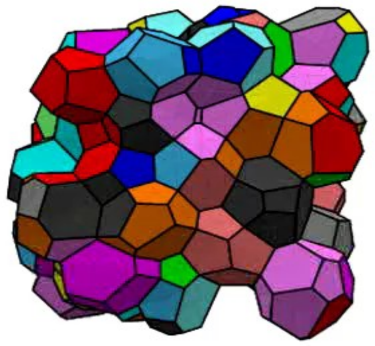




Rheology of foam

- Low strain: elastic behaviour
- Critical yield stress causes bubble rearrangement
- Viscous behaviour at high stress, strain





foams@aber.ac.uk

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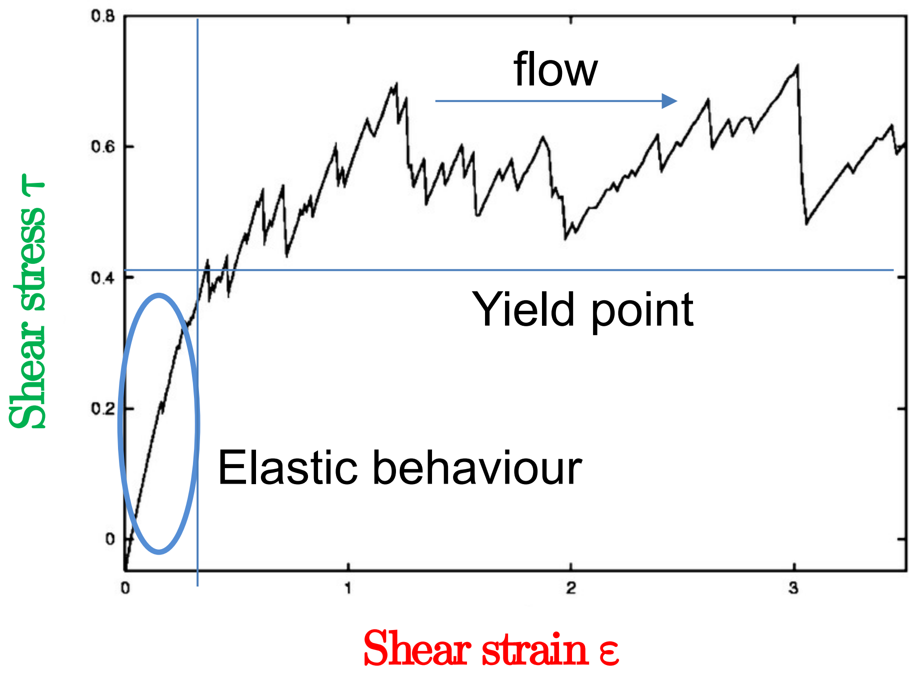
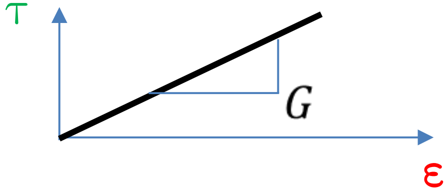
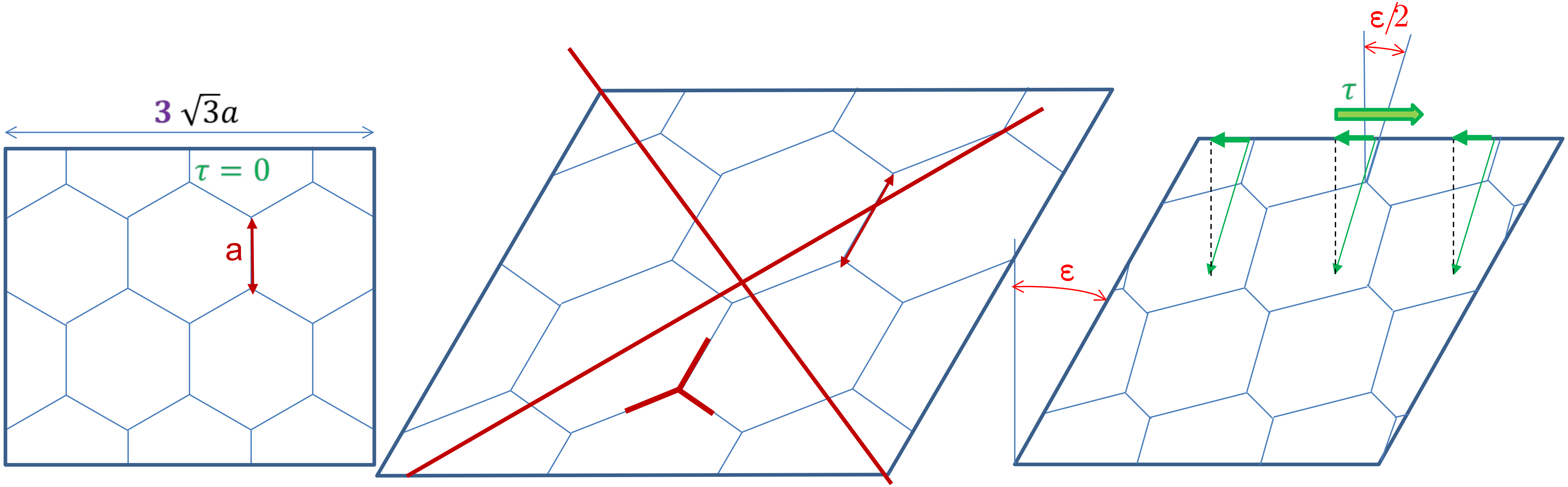


Simple aproach: 2D foam

- Dry, monodisperse 2D foam forms hexagonal pattern
- Shear stress results from surface tension of cutted lamellas



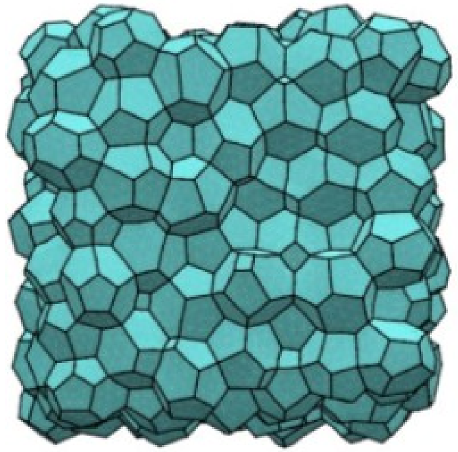
$$\tau = \frac{F}{A} = \frac{3\ 2\gamma\sin(\varepsilon/2)}{3\ \sqrt{3}a} \qquad G = \frac{\tau}{\varepsilon} = \frac{2\gamma\sin(\varepsilon/2)}{\varepsilon\ \sqrt{3}a} \approx 0.58\,\frac{\gamma}{a} = 0.525\,\frac{\gamma}{R_e}$$



Dry 3D foam

- Numerical simulations using surface evolver
- Polydisperse, dry 3D-foam
- Linear relationship between stress and strain, scales with γ/R
- Close to analytical result for 2D foam

$$G = \frac{\tau}{\varepsilon} = 0.51\frac{\gamma}{R_e}$$



Hexagonal 2D foam: $G = 0.525\frac{\gamma}{R_e}$

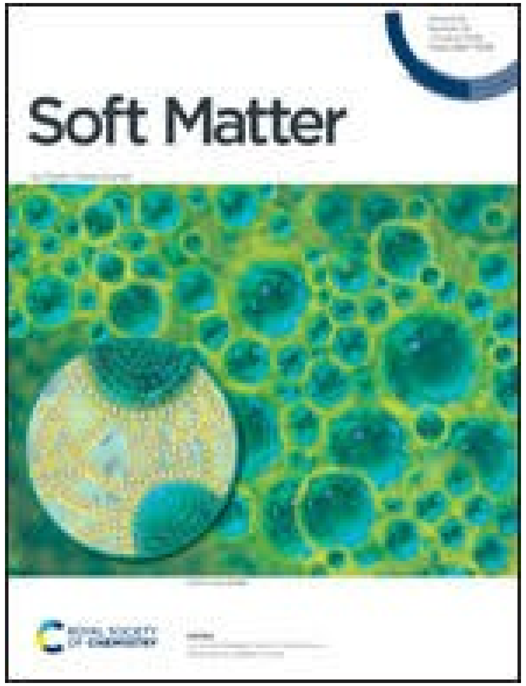
Exercise: Soft matter?

- What is the shear modulus of typical foam?
- Compare with shear modulus of Polycarbonate (700 MPa)

$$G = \frac{\tau}{\varepsilon} = 0.51 \frac{\gamma}{R_e}$$

- What is the compression modulus of foam?
- Compare with compression modulus of Polycarbonate (86 MPa)

$$K = -V \frac{dp}{dV}$$



Soft matter?

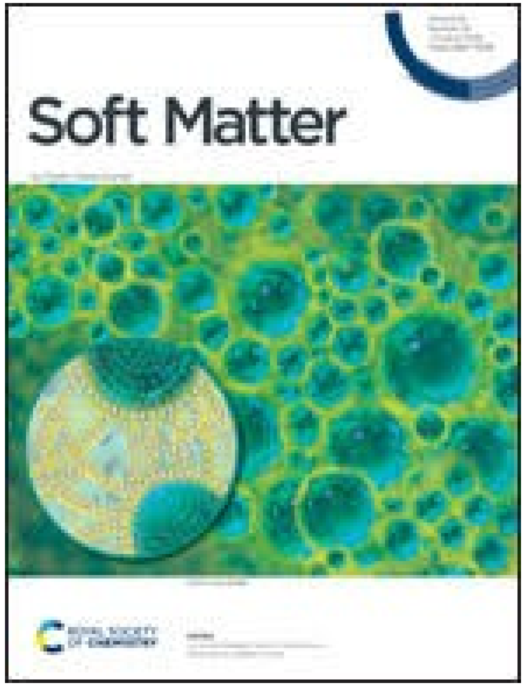
- Coarse foam is more soft
- For γ=30 mN/m and R=0.1 mm → G≈150 Pa
- PC: G=700 MPa = 10^6 x 700 Pa >>> 150 Pa

$$G = \frac{\tau}{\varepsilon} = 0.51 \frac{\gamma}{R_e}$$

- Compression module:
  - Foam ~ ideal gas $pV^k = const$

$$K = -V \frac{dp}{dV} = k\,p = 140\,kPa$$

  - PC: 86 MPa >> 140 kPa



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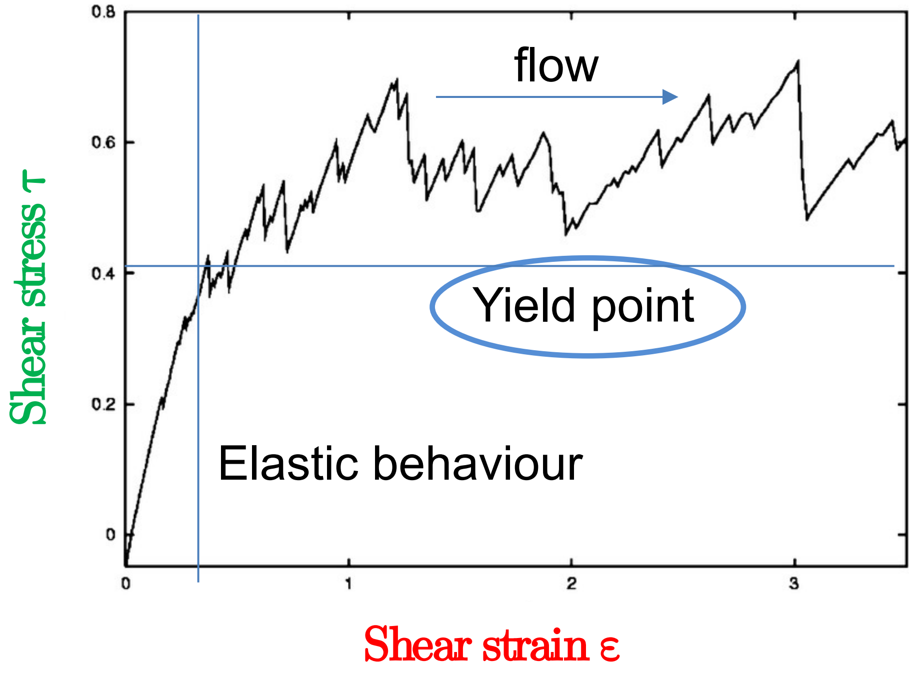
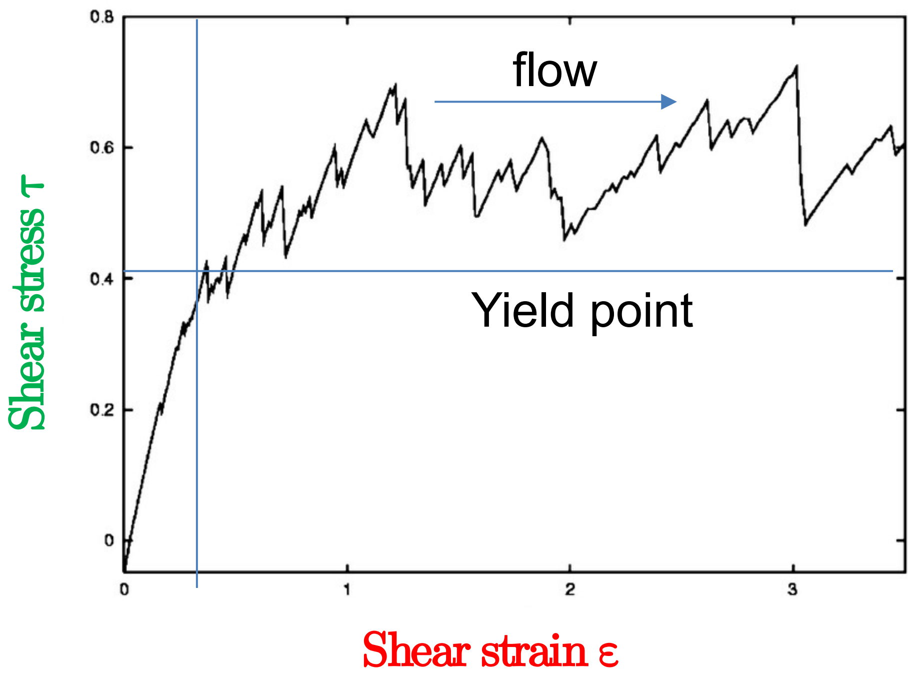
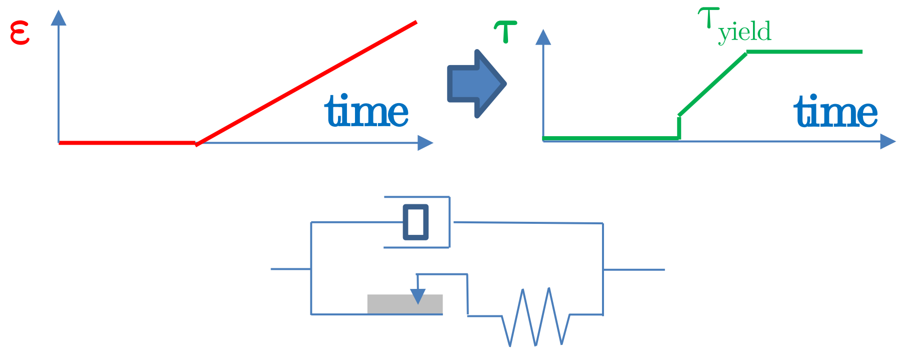


Yielding:

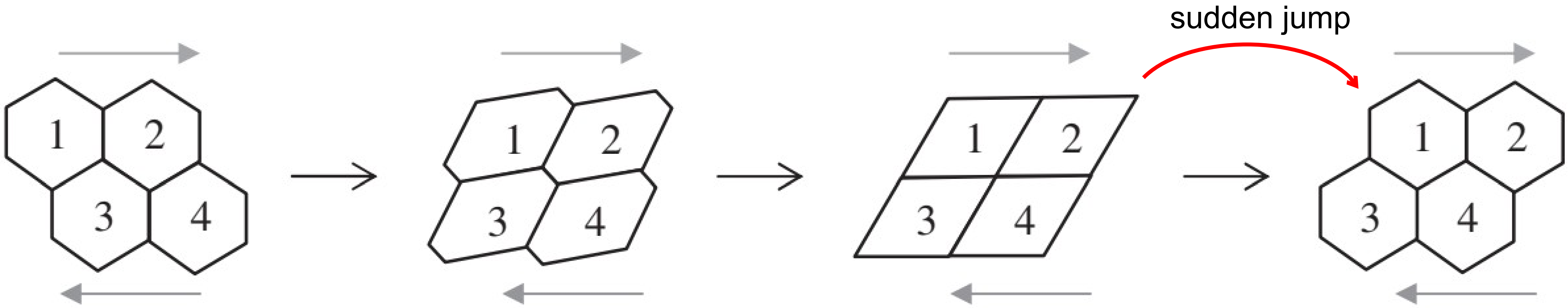
- At a critical yield stress / yield strain foam start to flow
- Above yield point stress remains approximately constant





T1 rearrangements:

- Switch of neighbourhood between two pairs of bubbles:



Yield point for 2D foam

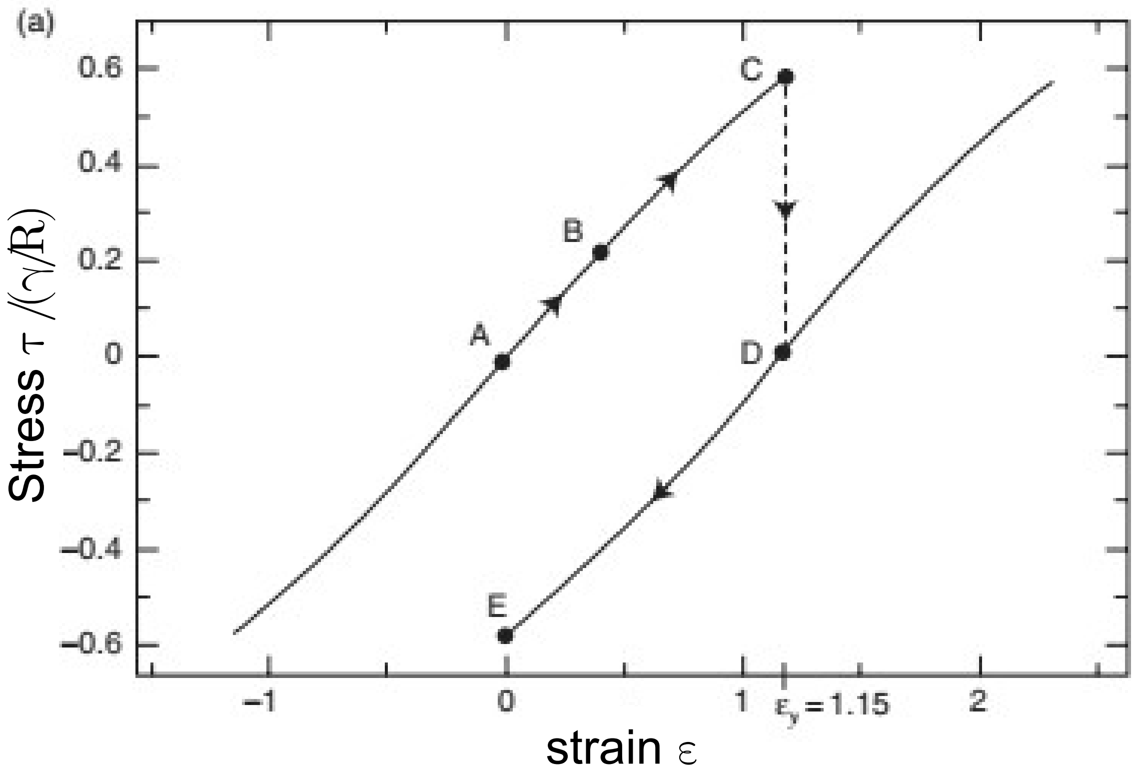
- T1 reduces stress in the structure

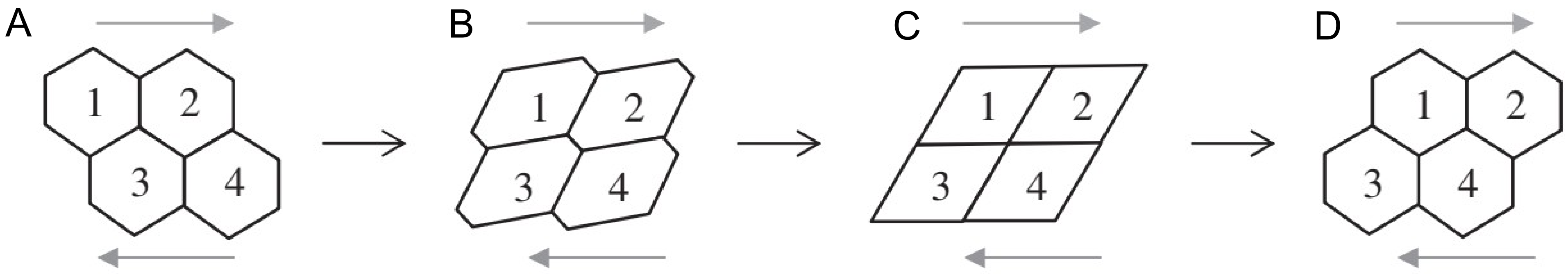
$$\tau = G\varepsilon \qquad G = 0.525\frac{\gamma}{R_e}$$

$$\tau = 0.525\frac{\gamma}{R_e}\ \varepsilon$$

$$\varepsilon_{yield} = 1.15$$

$$\tau_{yield} = 0.603\frac{\gamma}{R_e}$$

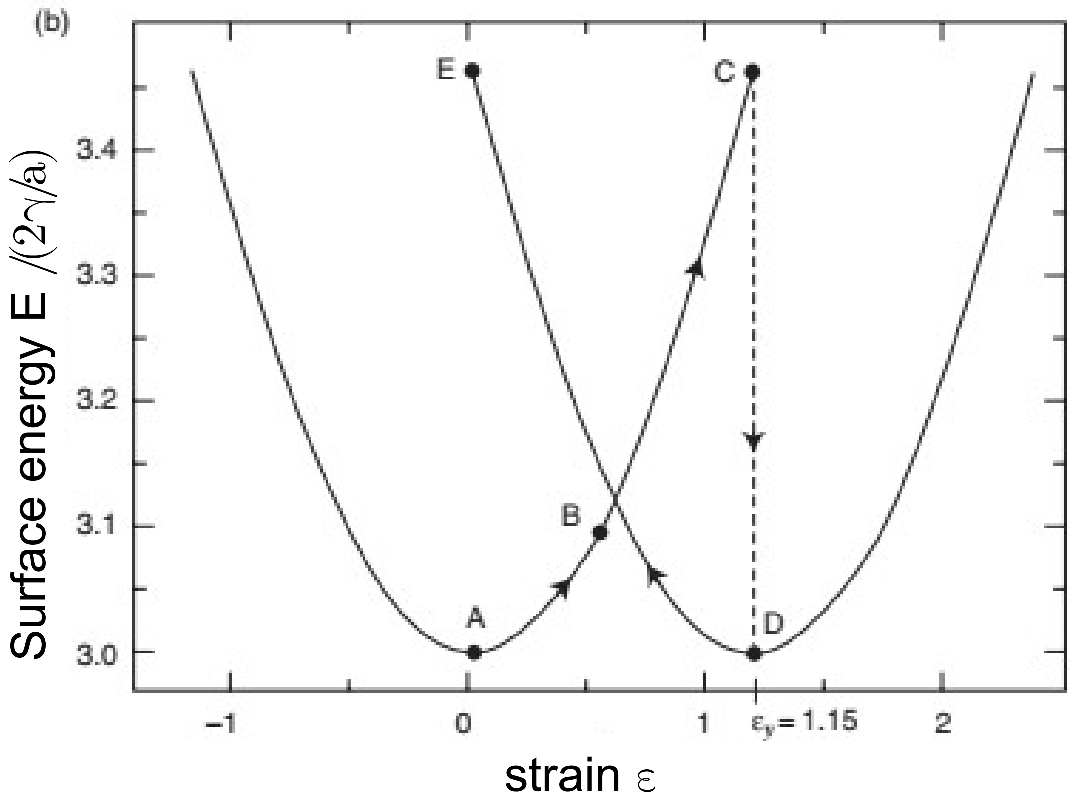


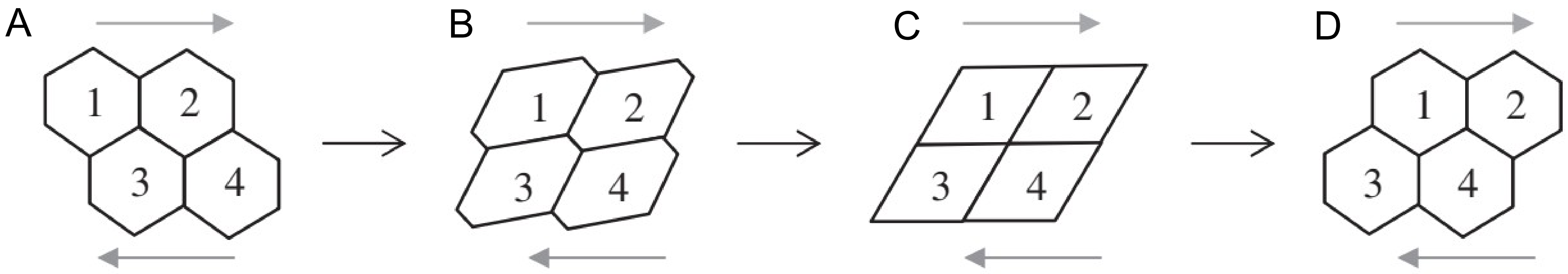


Yield point for 2D foam

- Each foam structure has its own energetical minimum

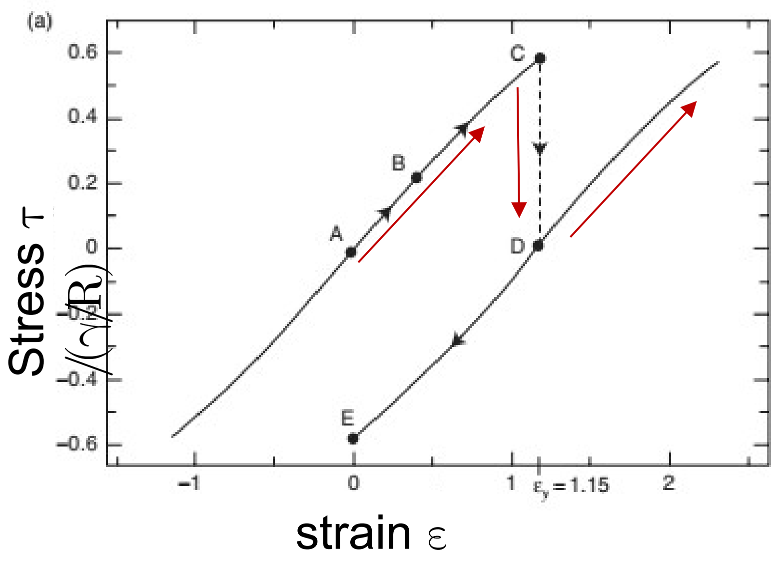
$$\frac{E - E_0}{V} = \frac{1}{2} G\, \varepsilon^2$$

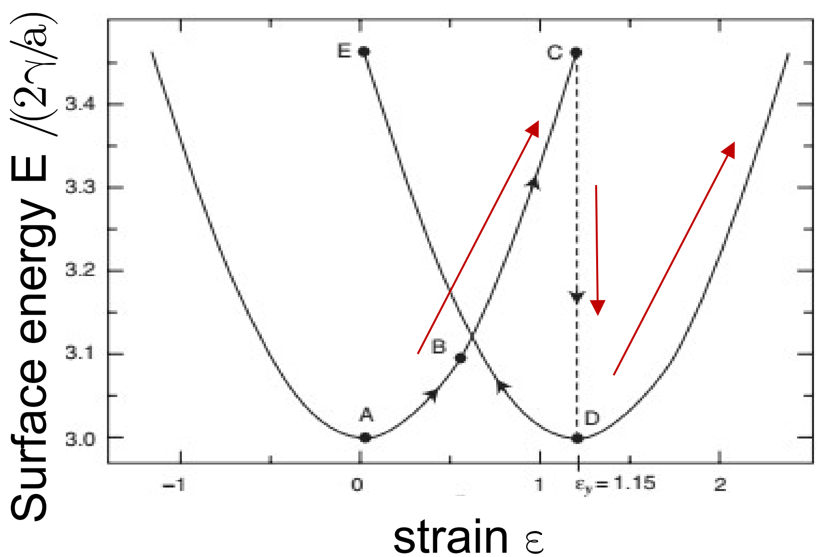


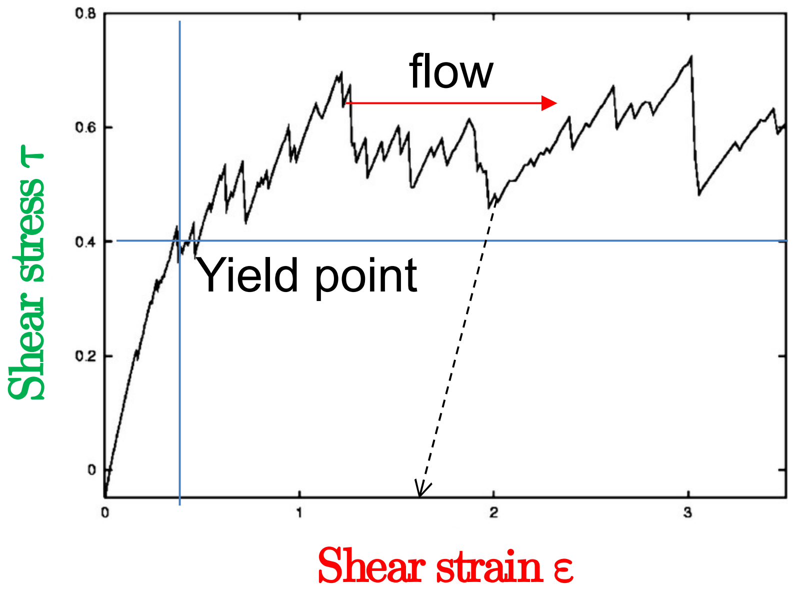


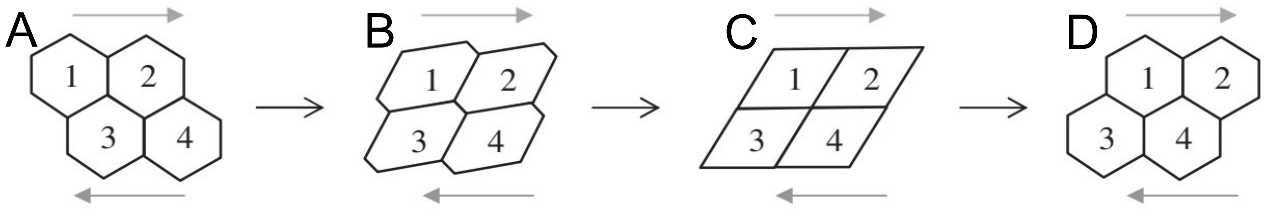
Yield point for 2D foam

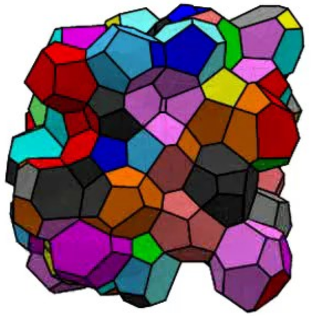
- Flow of foam is a sequence of T1 rearangements











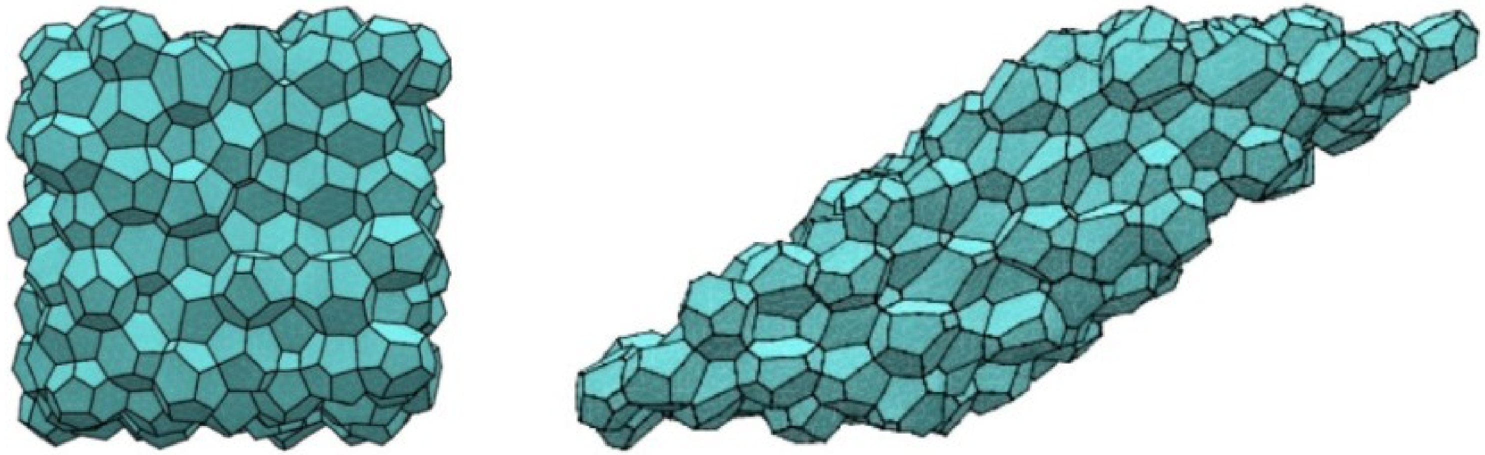
foams@aber.ac.uk

Yield point for 3D foam

- Numerical simulations using surface evolver
- Polydisperse, dry 3D-foam
- Yields point shows high scatter, depending on structure, polydispersity, sample generation

$$\varepsilon_{yield} \approx 0.15 \; \dots 0.4$$

$$\tau_{yield} \approx 0.1 \; \dots 0.2 \frac{\gamma}{R_e}$$



Hexagonal 2D foam: $\varepsilon_{yield} = 1.15$

$$\tau_{yield} = 0.603 \frac{\gamma}{R_e}$$

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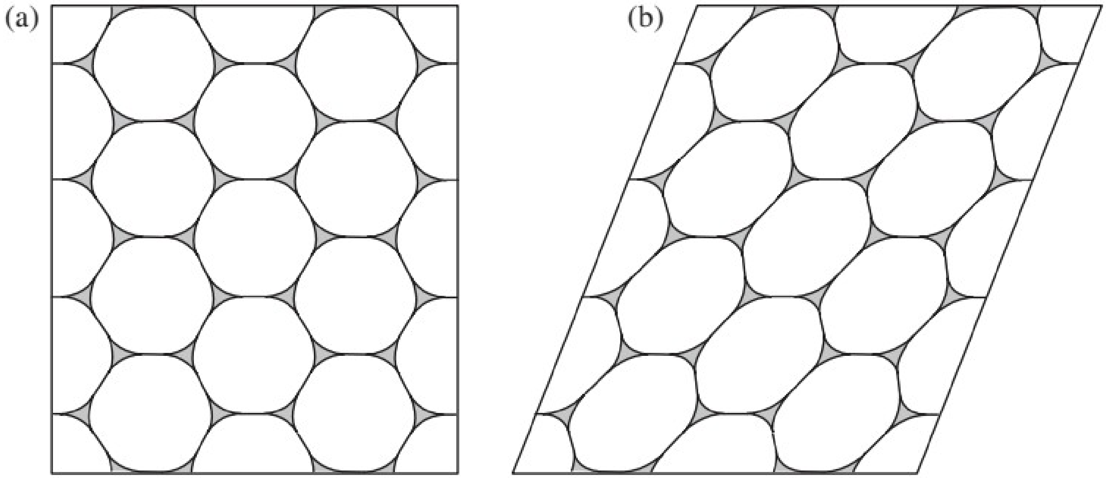
Elastic behaviour of wet foam

- Shear modulus results from change of surface energy under shear
- Decoration theorem: Plateau borders are unaffected by shear
- → Liquid fraction reduces change of surface energy under shear

$$\frac{dE}{V} = \tau \, d\varepsilon \qquad \tau = G\varepsilon$$

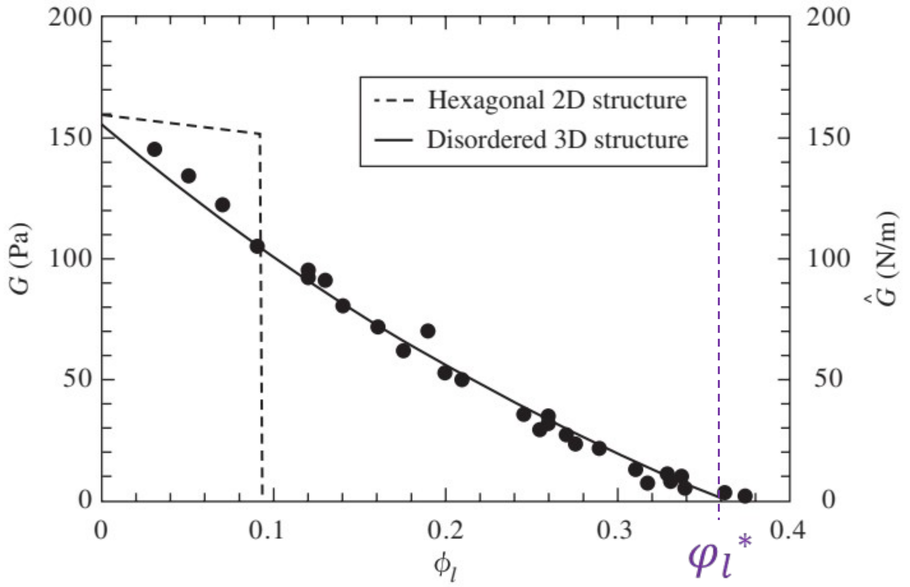
$$G = \frac{1}{V\varepsilon}\frac{dE}{d\varepsilon}$$



2D foam: $G = 0.525\frac{\gamma}{R_e}(1-\varphi_l)^{1/2}$

Elastic behaviour of wet foam

- Experimental results: liquid fraction reduces shear modulus
- Loss of rigidity at jamming point (36% liquid fraction)



3D polydisperse foam: $\boxed{G = 1.4(1-\varphi_l)(\varphi_l{}^* - \varphi_l)\frac{\gamma}{R_e}}$

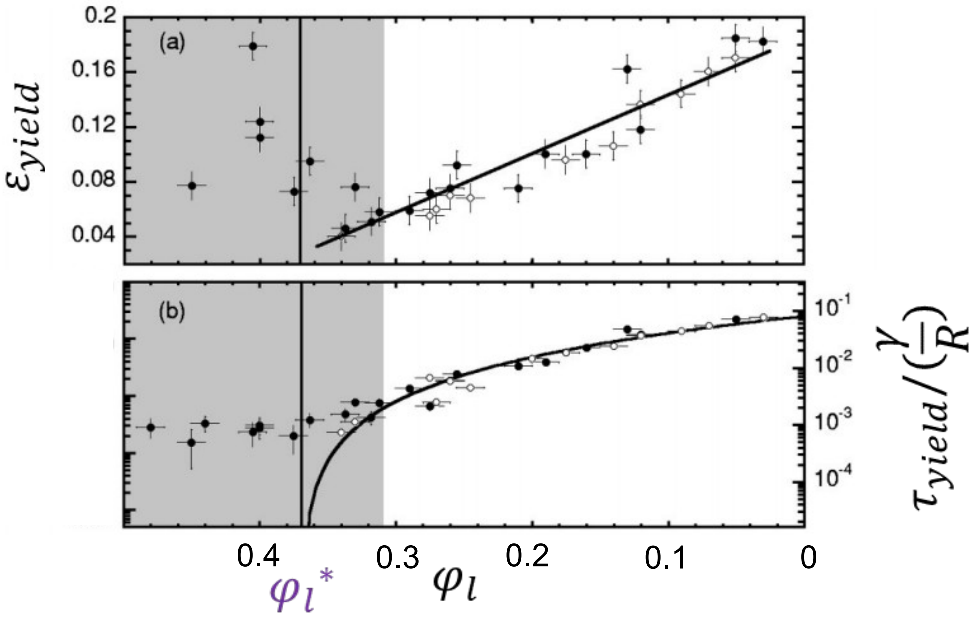
Dry foam:

$$\xrightarrow[\varphi_l = 0]{\varphi_l{}^* = 0.36} \quad G = 0.504\frac{\gamma}{R_e}$$

(surface evolver: $G = 0.51\frac{\gamma}{R_e}$)

Yielding of wet foam

- 3D experiments are very challenging, because gravity varies liquid fraction
- Experiments in an oscillating Couette rheometer



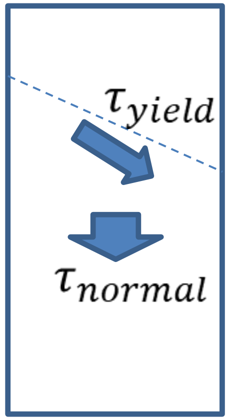
[Saint-Jalmes & Durian, 1999]

$$\varepsilon_{yield} \approx a_1({\varphi_l}^* - \varphi_l), \qquad a_1 \approx 0.5\,..\,1.0$$

$$\tau_{yield} \approx a_2 \frac{\gamma}{R_e}({\varphi_l}^* - \varphi_l)^2, \qquad a_2 \approx 0.2\,..\,0.5$$

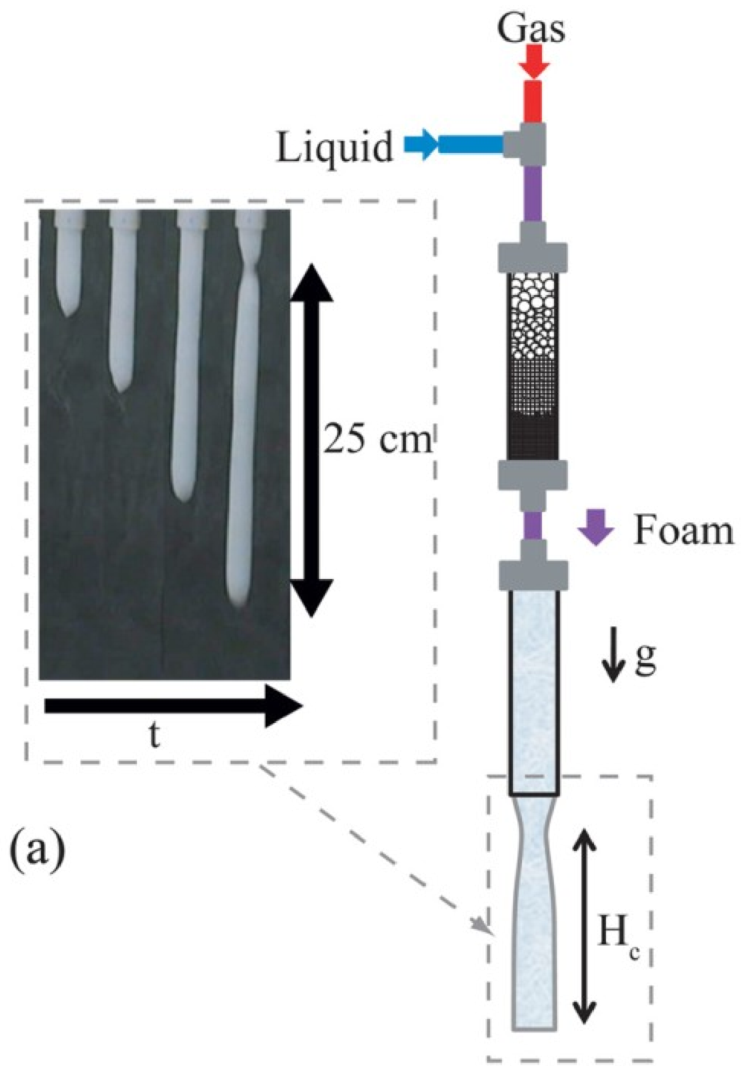
Exercise: Yielding of a foam column

- Column of foam (100 mm diameter), hanging from a cylinder
- What is the maximum length H_c when the column starts to rupture
- (a) 10% liquid fraction, 0.1 mm bubble radius
- (b) 0.1% liquid fraction, 5 mm bubble radius



$$\tau_{yield} \approx 0.3 \frac{\gamma}{R_e} (\varphi_l{}^* - \varphi_l)^2$$

$$\tau_{yield} = \frac{\tau_{normal}}{\sqrt{3}}$$



[Guillermic et al. 2013]

Exercise: Yielding of a foam column

- Column of foam (100 mm diameter), hanging from a cylinder
- What is the maximum length H_c when the column starts to rupture
- (a) 10% liquid fraction, 0.1 mm bubble radius
- (b) 0.1% liquid fraction, 5 mm bubble radius

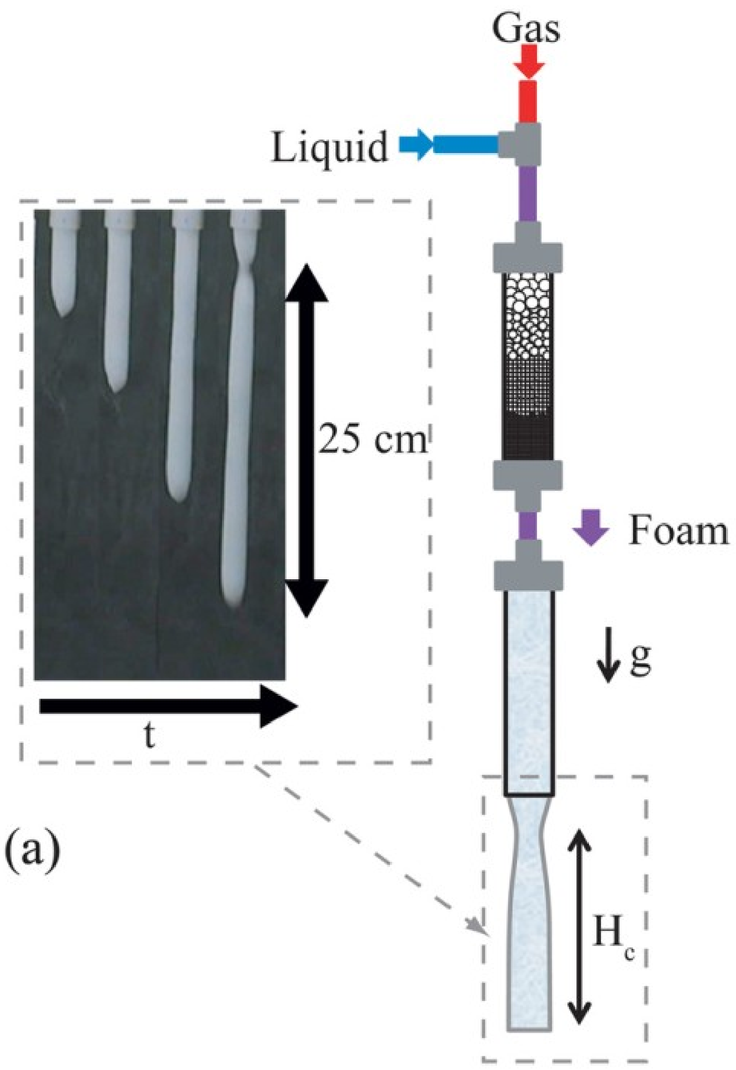
$$\tau_{yield} = \frac{\tau_{normal}}{\sqrt{3}} \qquad \tau_{normal} = H_c \rho g \varphi_l$$

$$\tau_{yield} \approx 0.3 \frac{\gamma}{R_e} (\varphi_l{}^* - \varphi_l)^2$$

$$H_c = \frac{1}{\rho g \varphi_l} \sqrt{3}\, 0.3 \frac{\gamma}{R_e} (\varphi_l{}^* - \varphi_l)^2$$

$a) \rightarrow H_c =$10 cm

b) $\rightarrow H_c =$ 40 cm



[Guillermic et al. 2013]

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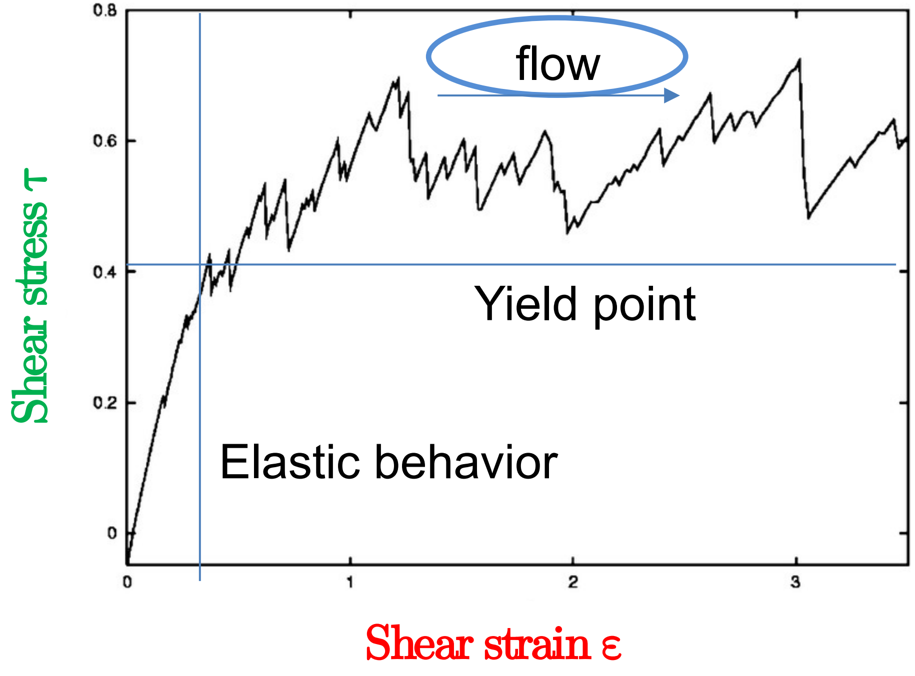
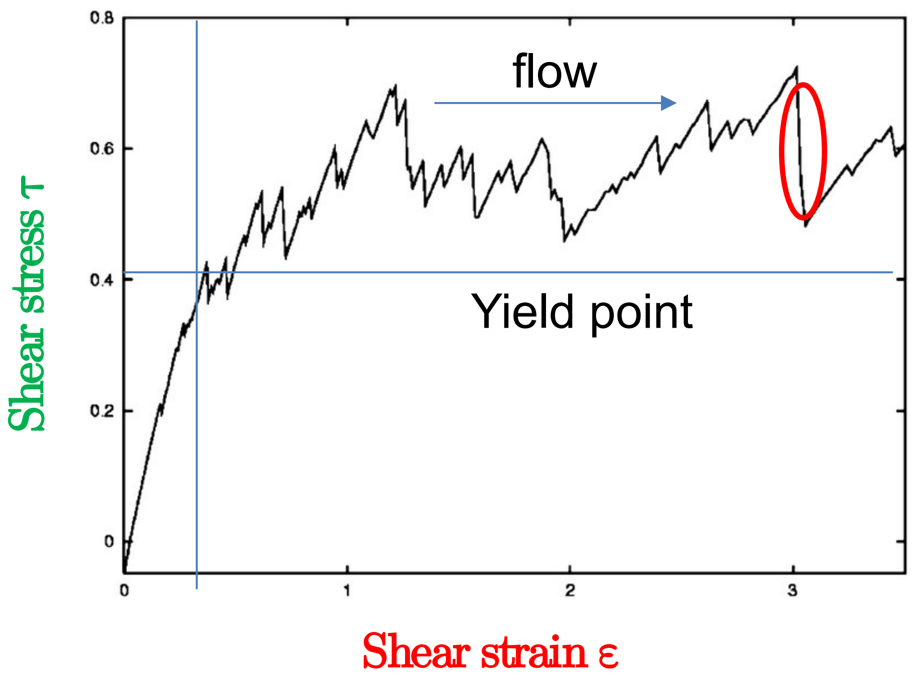
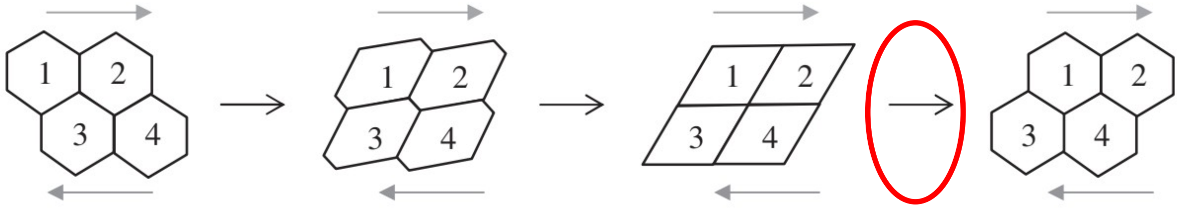


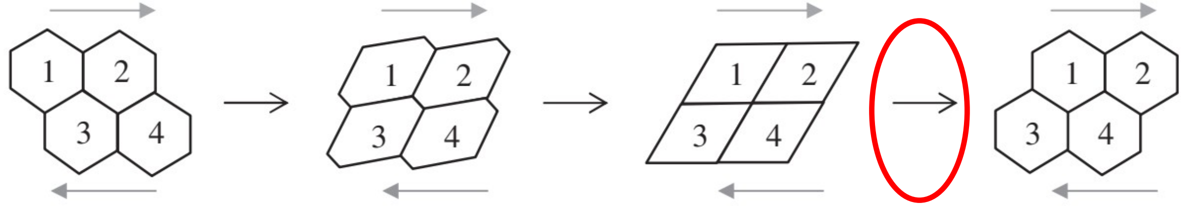
Dissipation in T1s

- Fast movement during rearrangement → viscous effects lead to dissipation
- Strain rate defines rate of T1s
- Dissipation during T1 less dependent on shear rate
- Different dissipation mechanisms discussed, but difficult to tell apart





Dissipation in T1s



- Incompressible interfaces – Mode A
- Interfaces are shifted relative to each other → shear flow inside the film

$$D \sim \mu \frac{d}{h} \dot{\varepsilon}_l^{\,2}$$

- Compressible interfaces – Mode B
- Interfaces are expanded / compressed (maintained thickness)
- → shear flow due to transport of liquid between films

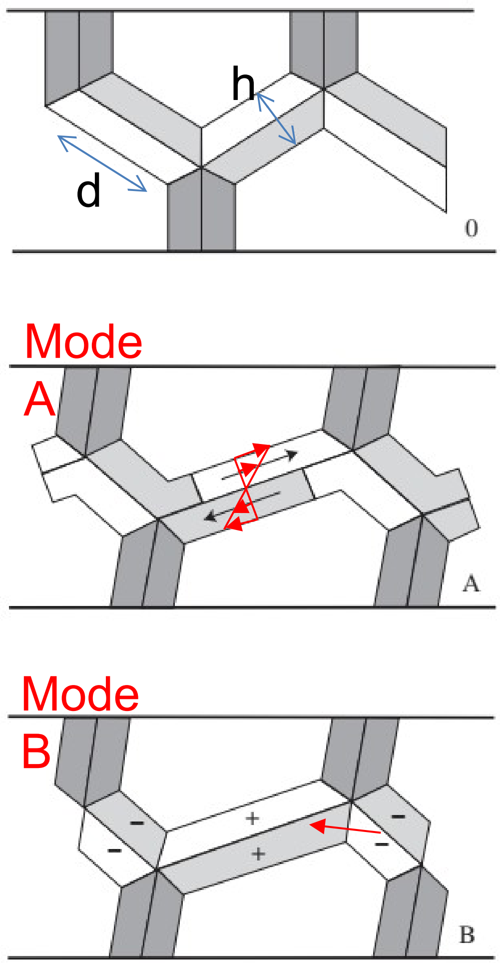
$$D \sim \mu \frac{h}{d} \dot{\varepsilon}_l^{\,2}$$

- Compression/expansion of interfaces leads to ad/desorption of surfactants – Mode B
- → surface dilational viscosity $\mu_{s,d}$ causes additional loss

$$D \sim \frac{\mu_{s,d}}{d} \dot{\varepsilon}_l^{\,2}$$

- Loss due to surface shear viscosity $\mu_{s,s}$ (Mode A and B)

$$D \sim \frac{\mu_{s,s}}{d} \dot{\varepsilon}_l^{\,2}$$



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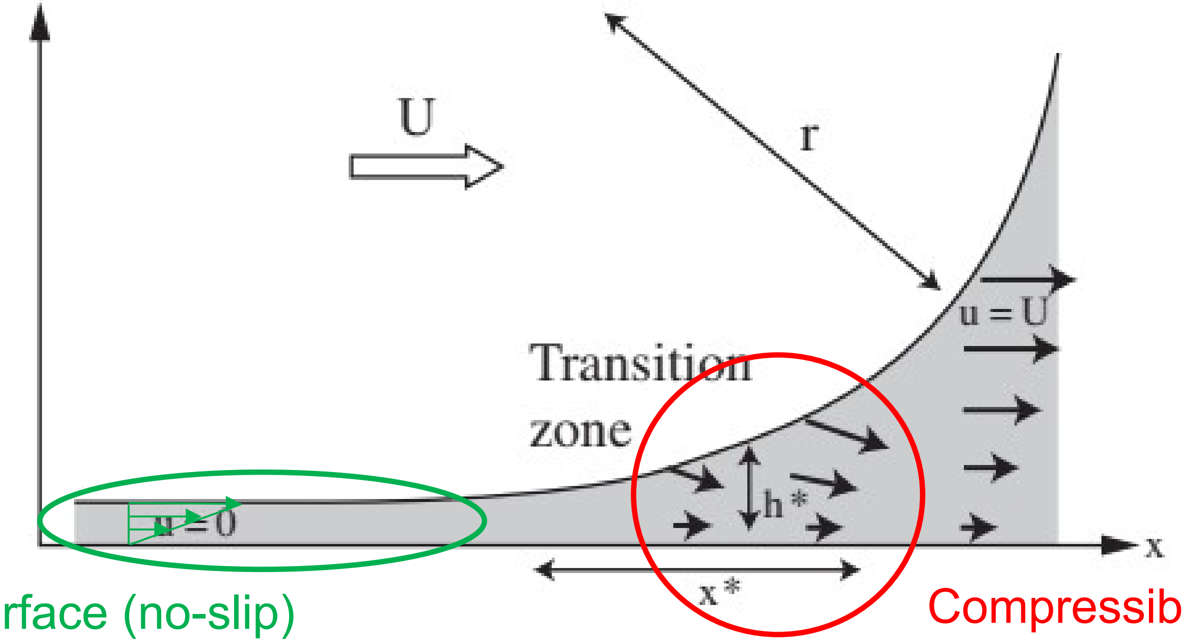
Role of surface compressibility

- Surface compressibility defines mode of dissipation
- Depends on bubble diameter d, surface tension γ and Capillary number Ca $Ca = \frac{\mu u}{\gamma}$



$$U \frac{\partial h}{\partial x} \sim -\frac{\gamma}{\mu} \frac{\partial}{\partial x}\left(h^3 \frac{\partial^3 h}{\partial x^3}\right)$$

experiments

$$\tau_{wall} \sim \frac{\gamma}{d} Ca^{1/2}$$

incompressible interface (no-slip)

Scale analysis

$$\tau_{wall} \sim \frac{\gamma}{d} Ca^{2/3}$$

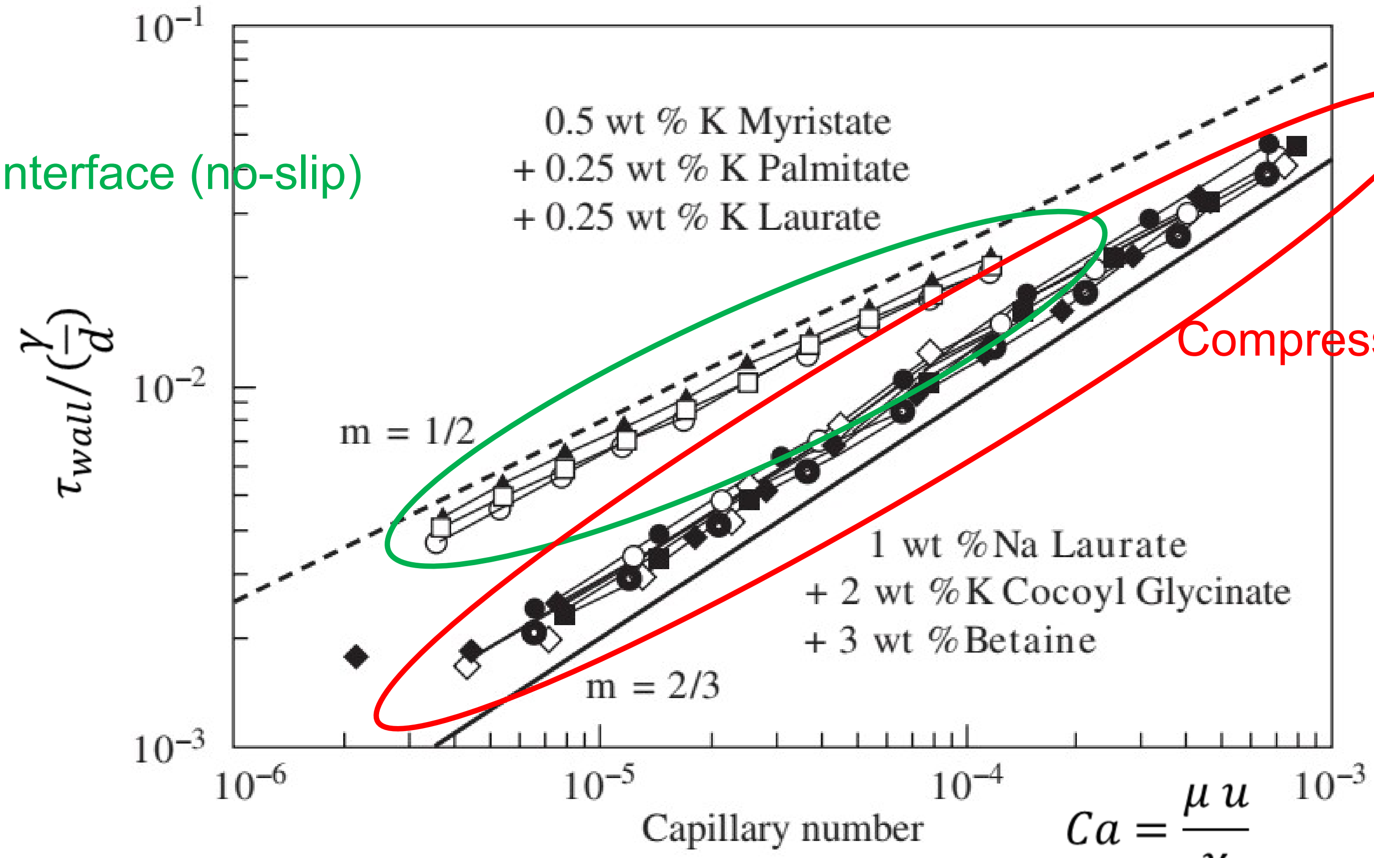
Compressible interface (free-slip)

Role of surface compressibility

- Surface compressibility (mobility) depends on surfactant
- Compressible interfaces result in lower wall stress

$\tau_{wall} \sim \frac{\gamma}{d} Ca^{1/2}$

incompressible interface (no-slip)



$Ca = \frac{\mu u}{\gamma}$

Compressible interface (free-slip)

$\tau_{wall} \sim \frac{\gamma}{d} Ca^{2/3}$

Wall stress vs. yield stress

- Wall slip is often smaller than yield stress
- Often plug-flow (tooth paste)

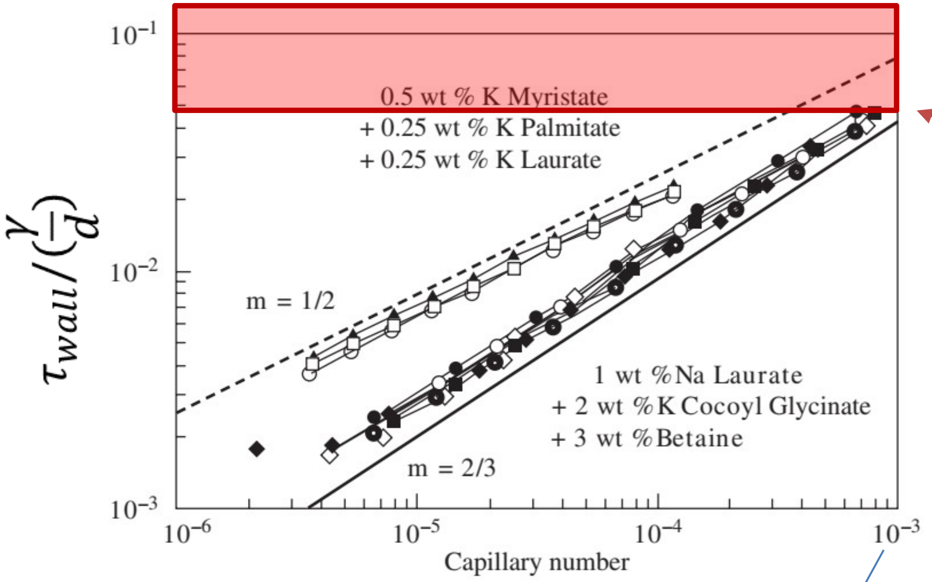
$$\tau_{yield} \approx a_2 \frac{\gamma}{R_e} (\varphi_l{}^* - \varphi_l)^2, \qquad a_2 \approx 0.2\,..\,0.5$$

$$\varphi_l{}^* \approx 0.36$$

$$\rightarrow \tau_{yield} \approx 0.025..0.065 \frac{\gamma}{R_e}$$

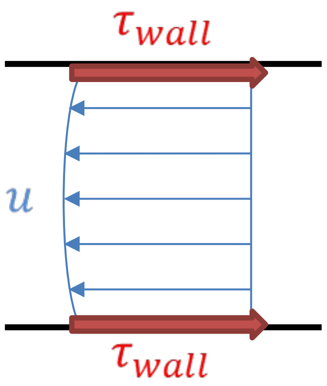
$$\rightarrow \tau_{yield} \approx 0.05..0.13 (\frac{\gamma}{d})$$

u=1 cm/s



$$Ca = \frac{\mu\, u}{\gamma}$$

$$u \approx 3\ cm/s$$



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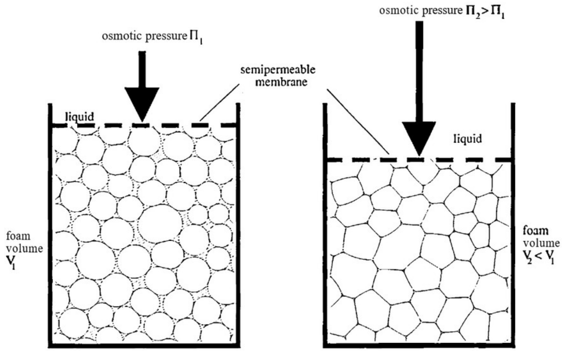
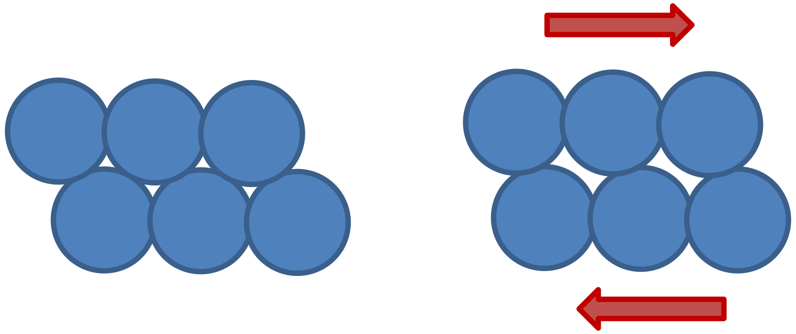
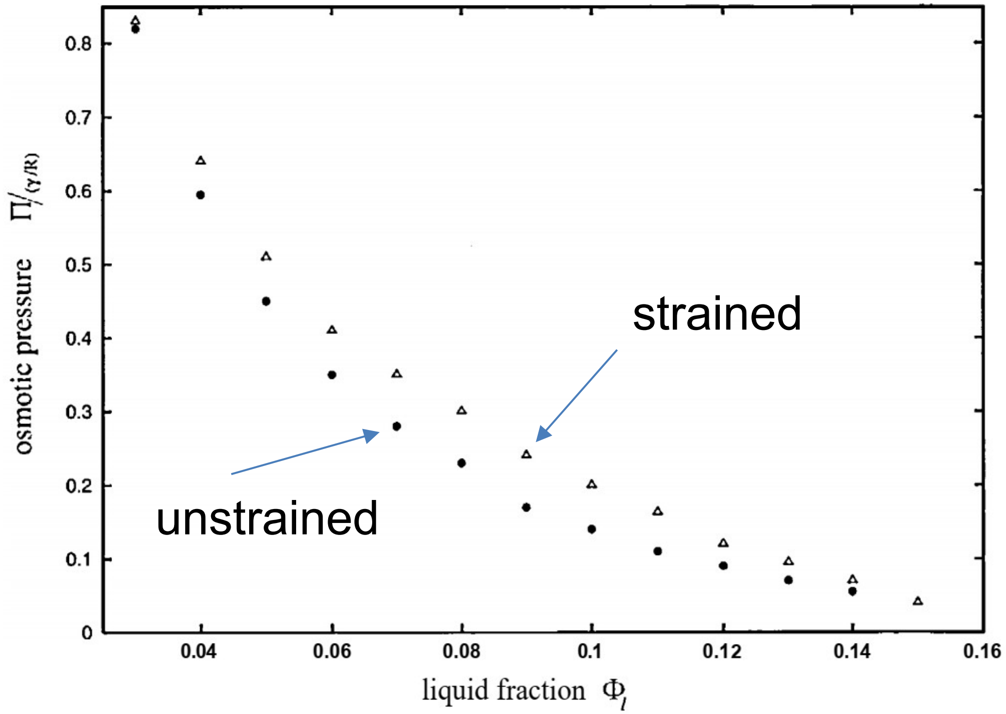
Dilatancy

- Sheared granular media (foam) sucks in liquid
- Osmotic pressure describes tendency of foam to attract liquid
- Shearing increases osmotic pressure of foam



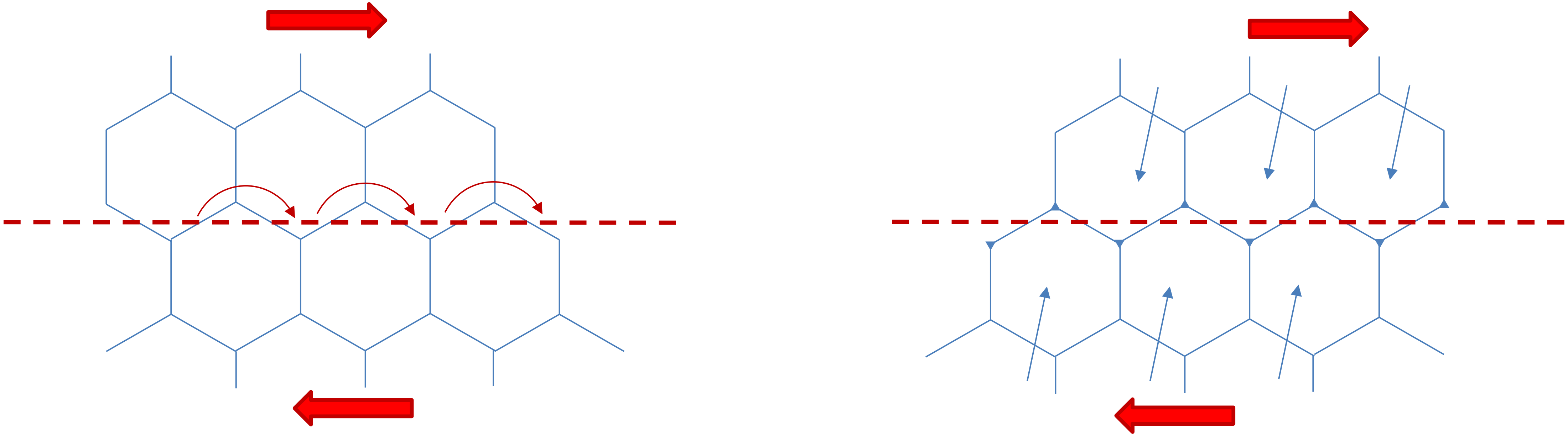




[Weaire & Hutzler 2008]

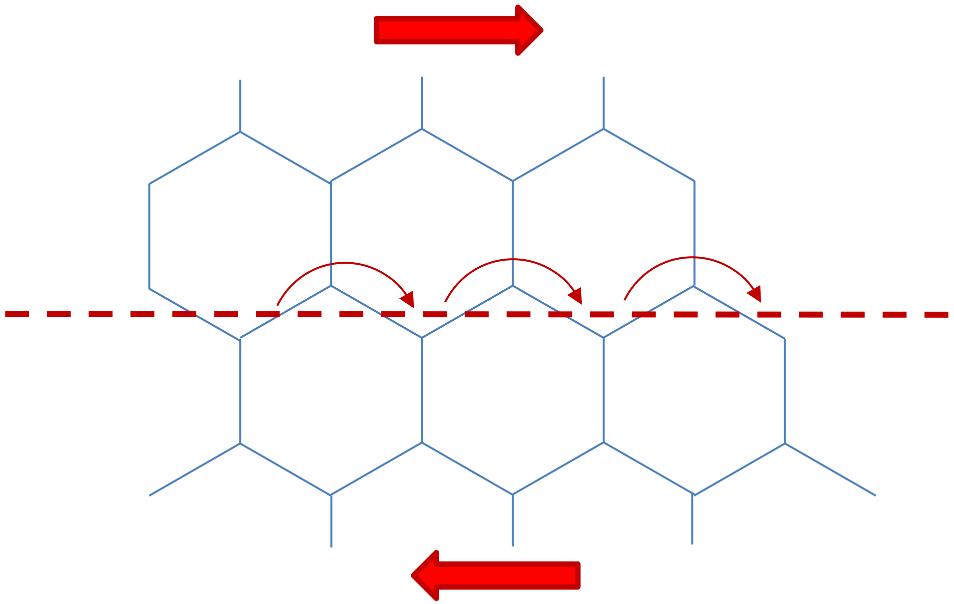
Shear banding

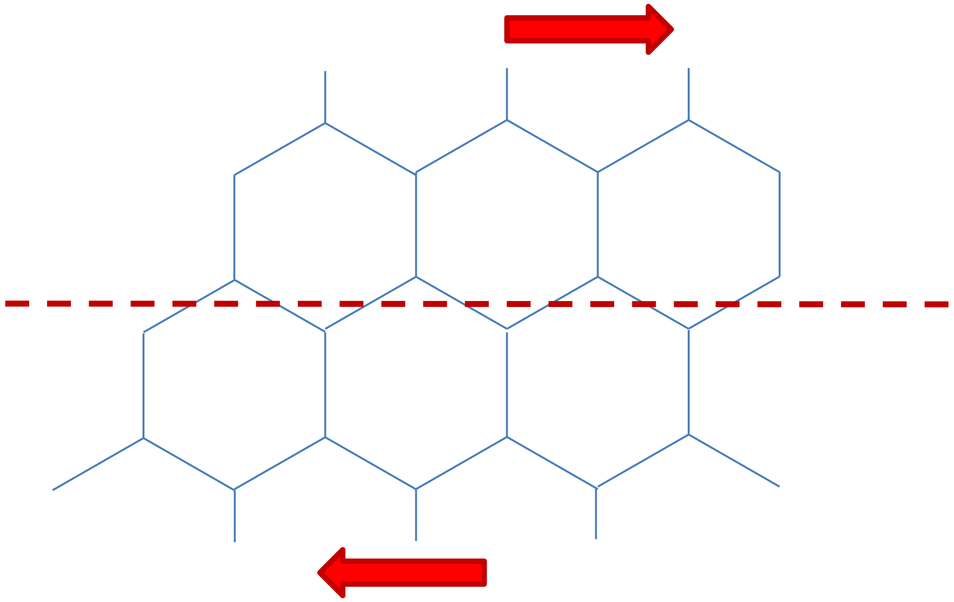
- T1 rearrangements are localized in a single sliding layer (band)?
- Local shear might attract more liquid (dilatancy)
- Locally higher liquid fraction reduces shear modulus, stabilizing shear layer



Shear banding

- Some experiments/simulations have observed shear bandings [Dennin 2008, Wyn et al. 2008, Van Hecke 2010, Weaire et al. 2010]
- Some experiments have not [Ovarlez et al. 2008, 2010]
- Presumably more likely in monodisperse foam, supressed at high polydispersity
- Maybe an effect of inhomogeneous foam, shear start-up





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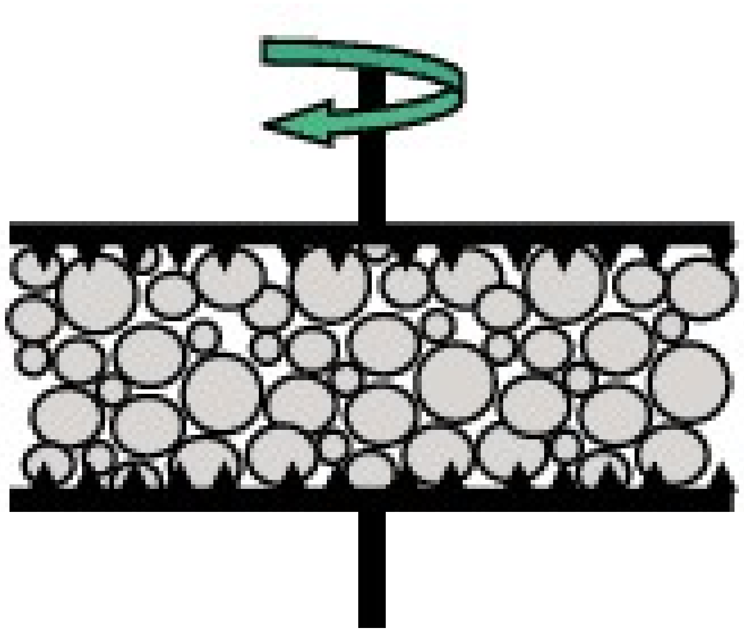
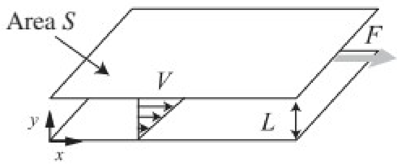
Rheometer

- Measure relation between strain (rate) and stress
- Well defined geometry
- → analytical solution of stress / strain distribution
- Avoid wall slip! → etching grooves, sandpaper



Sliding plate

$$\varepsilon = \frac{Vt}{L} \qquad \dot{\varepsilon} = \frac{V}{L} \qquad \sigma = \sigma_{xy} = \frac{F}{S}$$



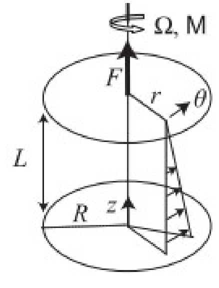
Parallel plate

$$v_\theta(r,z) = \frac{r\,\Omega z}{L}$$

$$\varepsilon(r) = \frac{r\theta}{L} \qquad \dot{\varepsilon} = \frac{r\Omega}{L}$$

$$\sigma(r) = \sigma_{\theta z}(r) = \frac{2Mr}{\pi R^4}$$

Normal stresses $F = \pi \int_0^R (N_2 - N_1)\, r\, \mathrm{d}r$

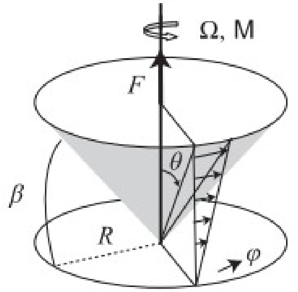


Cone-plate

$$\varepsilon \approx \frac{\varphi}{\beta} \qquad \dot{\varepsilon} \approx \frac{\Omega}{\beta} \quad (\text{for small } \beta)$$

$$\sigma = \sigma_{\varphi\theta} = \frac{3M}{2\pi R^3}$$

Normal stresses $N_1 = \dfrac{2F}{\pi R^2}$



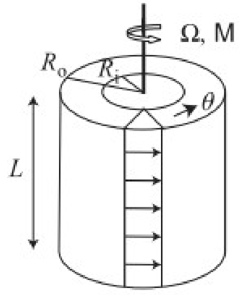
Cylindrical Couette

$$\dot{\varepsilon} = r\frac{\partial}{\partial r}\left(\frac{v_\theta}{r}\right)$$

$$\varepsilon \approx \frac{R_o + R_i}{2(R_o - R_i)}\,\Omega t \qquad \dot{\varepsilon} \approx \frac{R_o + R_i}{2(R_o - R_i)}\,\Omega$$

(for a narrow gap)

$$\sigma(r) = \sigma_{\theta r}(r) = \frac{M}{2\pi r^2 L}$$

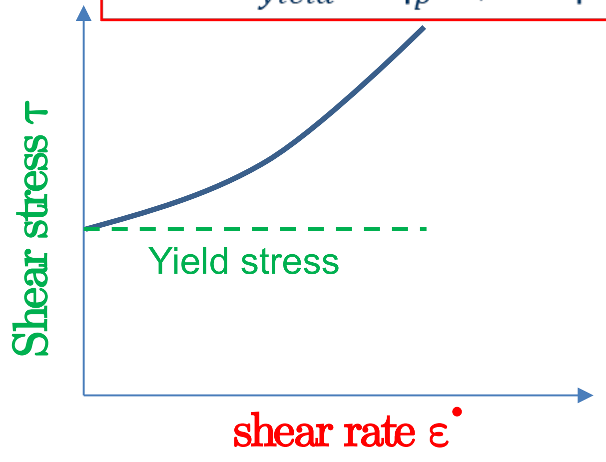


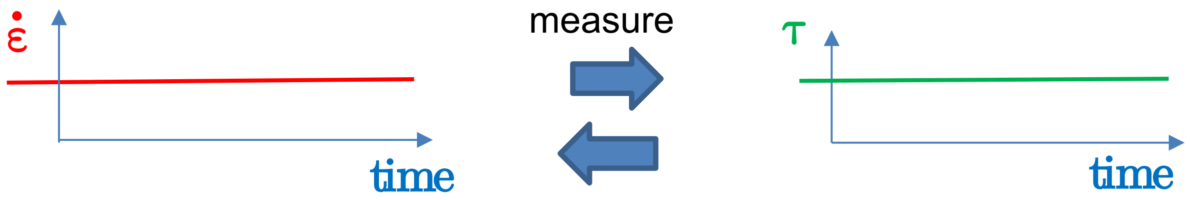
Steady shear

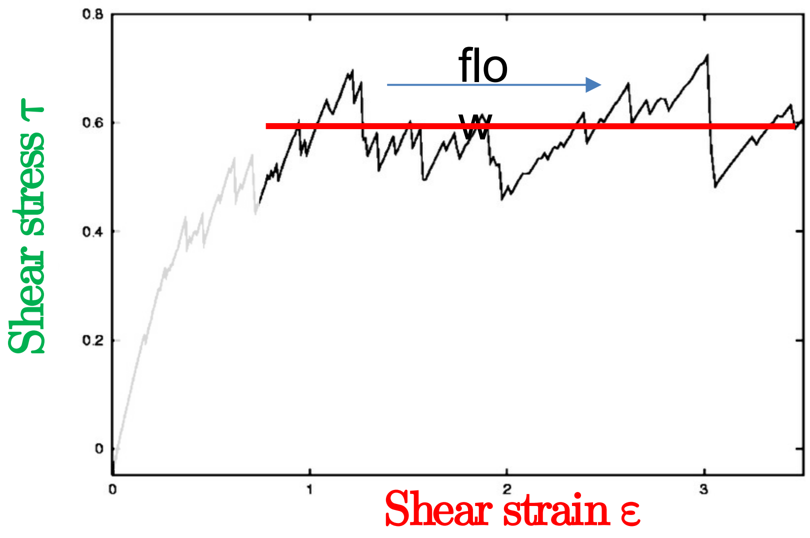
- Set steady shear rate (or steady stress)
- Wait for steady state and measure stress (or shear rate)
- Test different shear rates

Herschel-Bulkley law

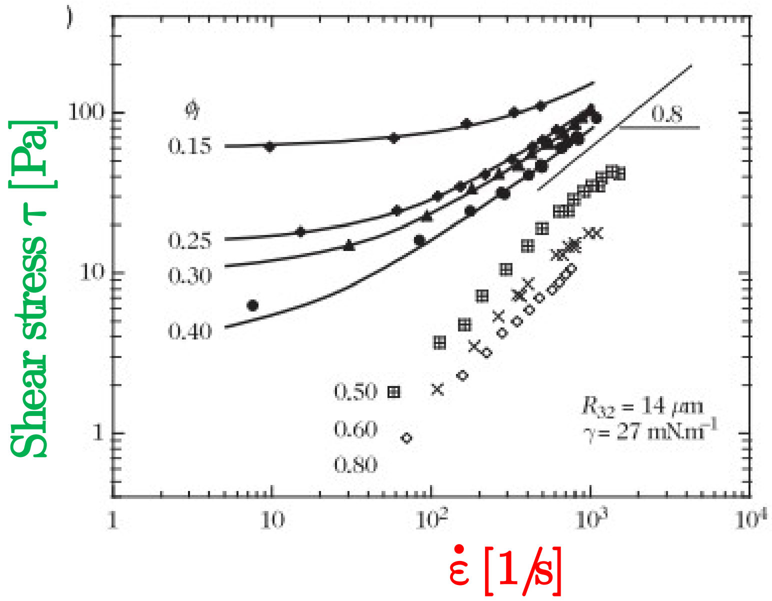
$$\tau = \tau_{yield} + \eta_p \dot{\varepsilon}^{\beta}, \qquad \beta < 1$$





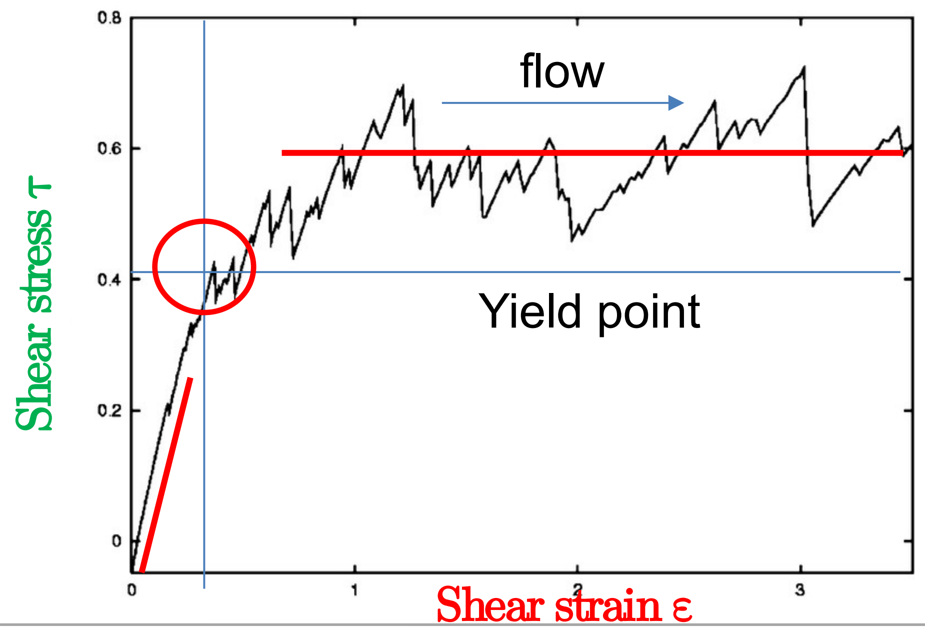


- Effective viscosity $\tau/\dot{\varepsilon}$ in flow state



Shear start-up

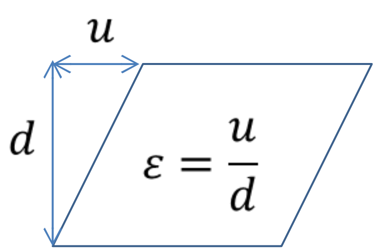
- Shear start-up
- Start to shear with constant shear rate, measure corresponding shear stress

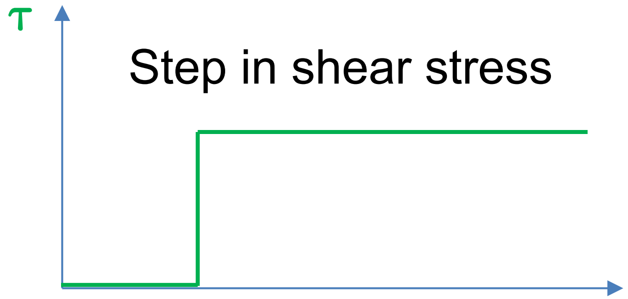


- Elastic shear modulus
- Yield stress
- Effective viscosity $\tau/\dot{\varepsilon}$ in flow state

Response on shear stress step

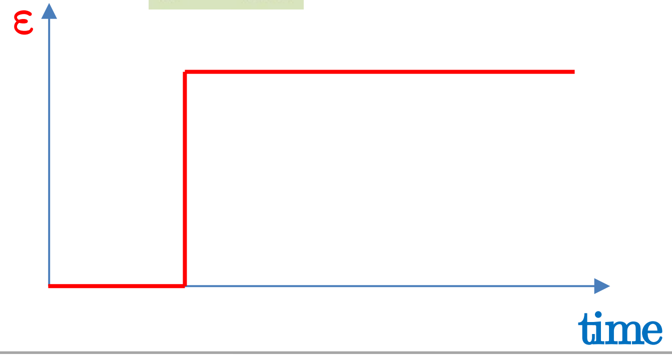
- Sudden stress is provided, corresponding strain is measured
- Relation between stress τ and strain ε





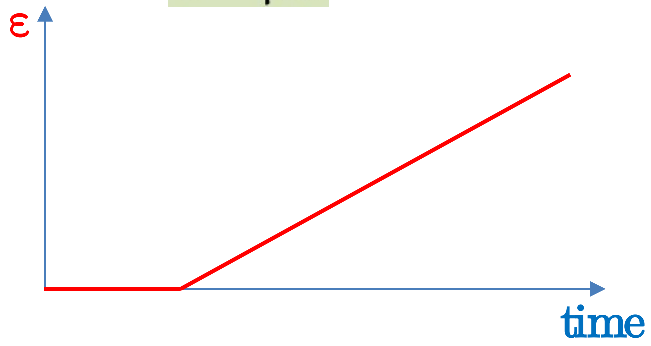
- Elastic body:
  shear modulus G

$$\tau = G\varepsilon$$

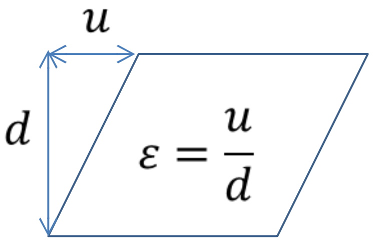


- Newtonian fluid:
  viscosity μ

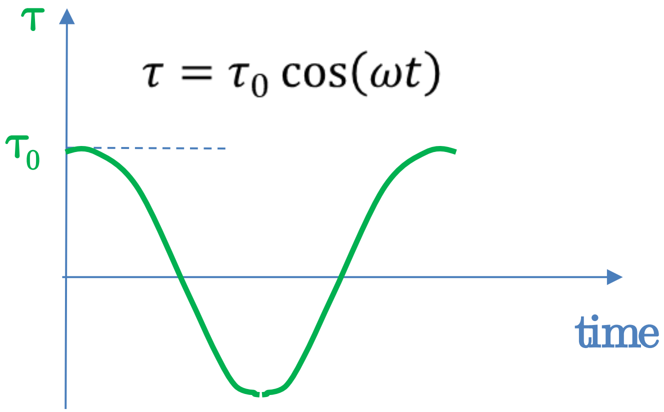
$$\tau = \mu\,\dot{\varepsilon}$$



Response on oscillatory stress



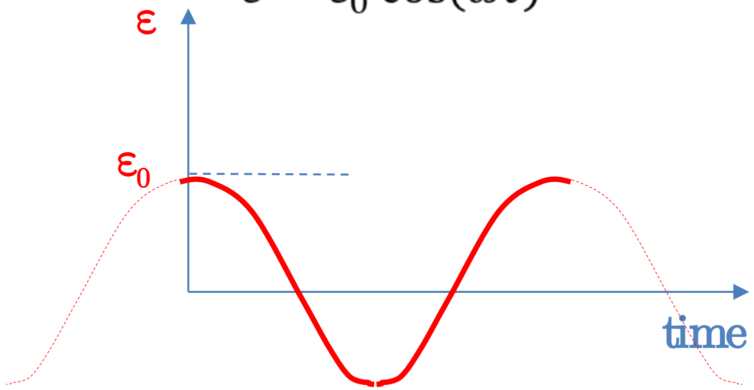
- Sinusoidal stress is provided, strain response is measured



- Elastic body:

$$\tau = G\varepsilon$$

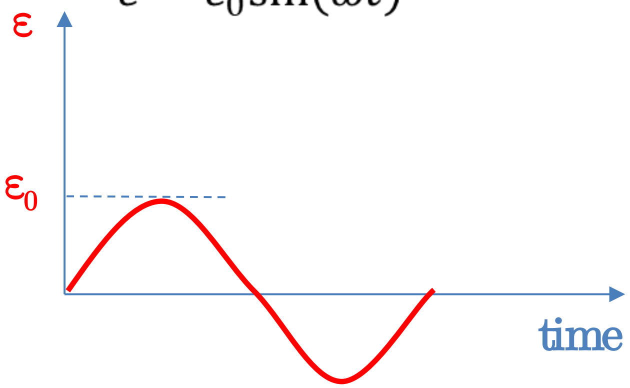
$$\varepsilon = \varepsilon_0 \cos(\omega t)$$



- Newtonian fluid:

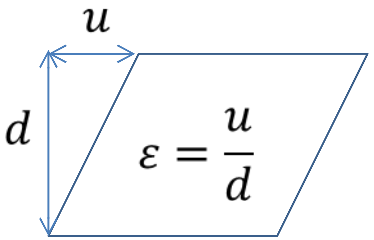
$$\tau = \mu\, \dot{\varepsilon}$$

$$\varepsilon = \varepsilon_0 \sin(\omega t)$$

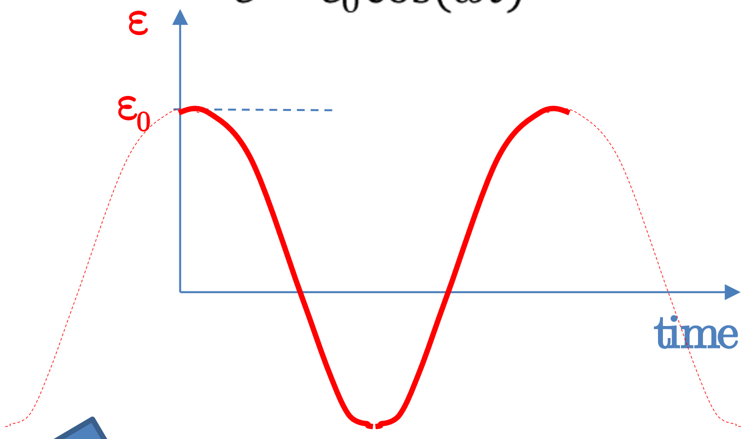


Response on oscillatory strain

- Sinusoidal strain is provided, stress response is measured



$$\varepsilon = \varepsilon_0 \cos(\omega t)$$

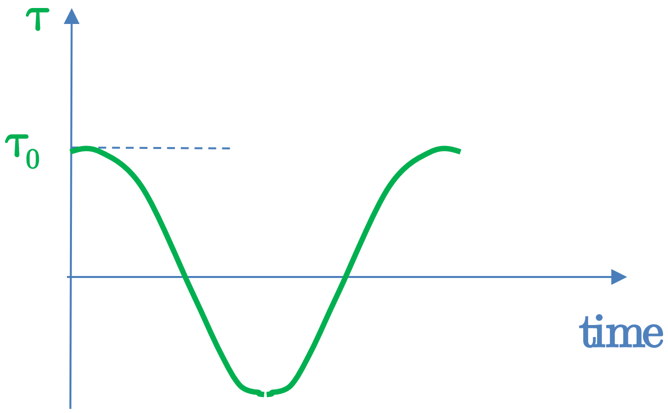


- Elastic body:

$$\tau = G\varepsilon$$

$$\tau = \tau_0 \cos(\omega t) = G\varepsilon_0 \cos(\omega t)$$

$$\tau_0 = G\varepsilon_0$$

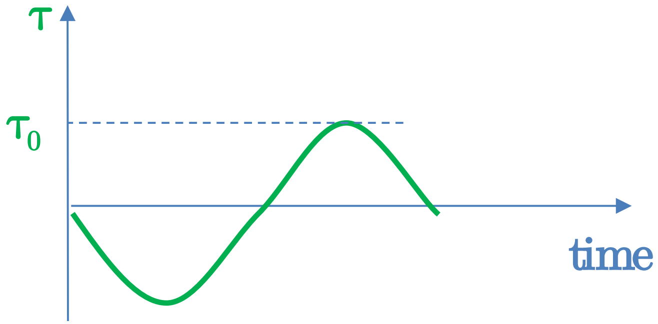


- Newtonian fluid:

$$\tau = \mu\, \dot{\varepsilon}$$

$$\tau = -\,\tau_0 \sin(\omega t) = -\mu\omega\, \varepsilon_0 \sin(\omega t)$$

$$\tau_0 = \mu\omega\, \varepsilon_0$$



Complex shear modulus G*

$$e^{i\omega t} = \cos(\omega t) + \boldsymbol{i}\sin(\omega t)$$

$$R(e^{i\omega t}) = \cos(\omega t)$$

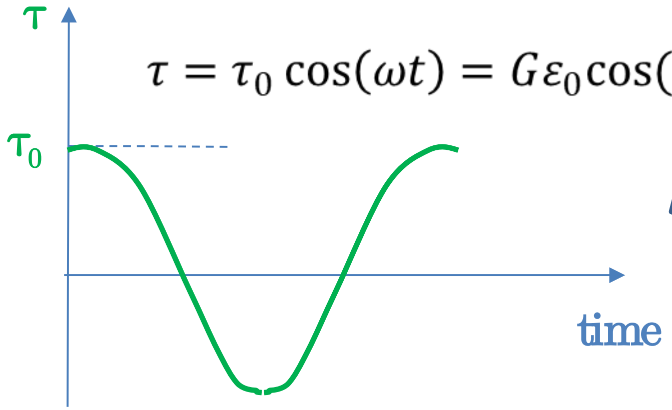
$$R(\boldsymbol{i}e^{i\omega t}) = -\sin(\omega t)$$

- Combined elastic and viscous response in a single (complex) modulus
- Storage modulus G' describes capacity to store elastic energy
- Loss modulus G" describes dissipation of energy

- Elastic body:

$$\tau = G\varepsilon$$

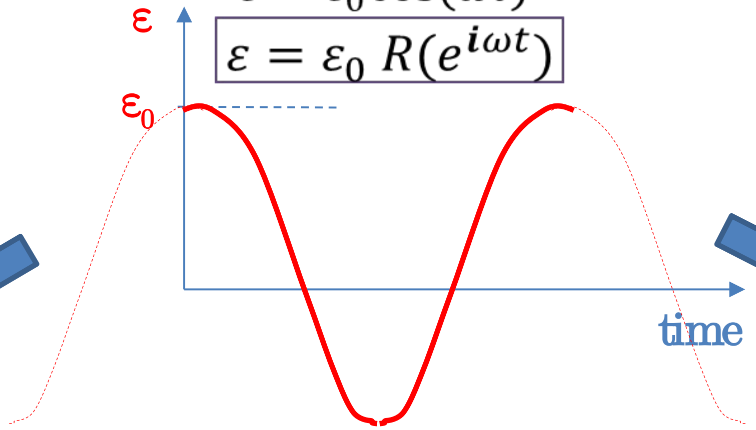
$$\tau = \tau_0 \cos(\omega t) = G\varepsilon_0 \cos(\omega t)$$



$$\boxed{\rightarrow \tau = \varepsilon_0\, G\, R(e^{i\omega t})}$$

$$\varepsilon = \varepsilon_0 \cos(\omega t)$$

$$\boxed{\varepsilon = \varepsilon_0\, R(e^{i\omega t})}$$



$$\boxed{\tau = \varepsilon_0\, R(G^* e^{i\omega t})}$$

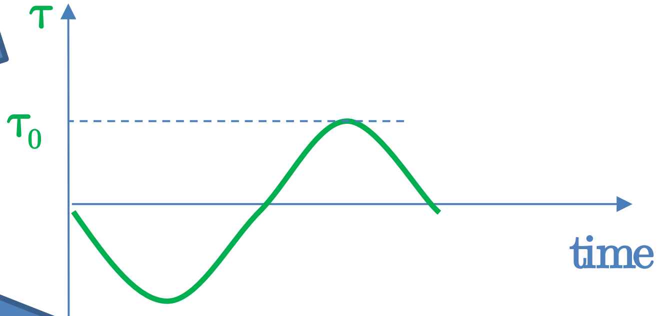
$$\boxed{G^* = G' + \boldsymbol{i}G'' = G + \boldsymbol{i}\mu\omega}$$

$$\boxed{\tau = \varepsilon_0 G' \cos(\omega t) - \varepsilon_0 G'' \sin(\omega t)}$$

- Newtonian fluid:

$$\tau = \mu\, \dot{\varepsilon}$$

$$\tau = -\tau_0 \sin(\omega t) = -\mu\omega\, \varepsilon_0 \sin(\omega t)$$

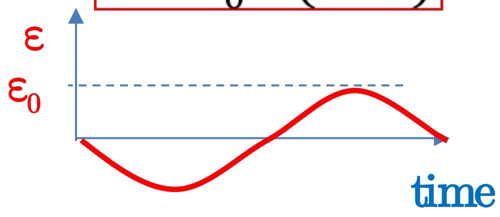


$$\boxed{\rightarrow \tau = \varepsilon_0 \mu\omega\, R(\boldsymbol{i}e^{i\omega t})}$$

Oscillatory shear

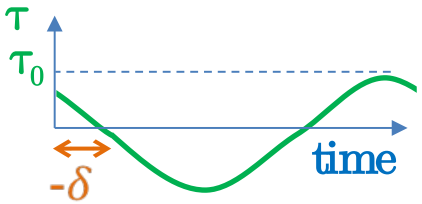
- Sinusoidal strain of amplitude ε_0 and frequency ω
- Wait for steady state and measure resulting stress
- Gives complex shear modulus G*(ω)

$$\boxed{\varepsilon = \varepsilon_0\, R(e^{i\omega t})}$$



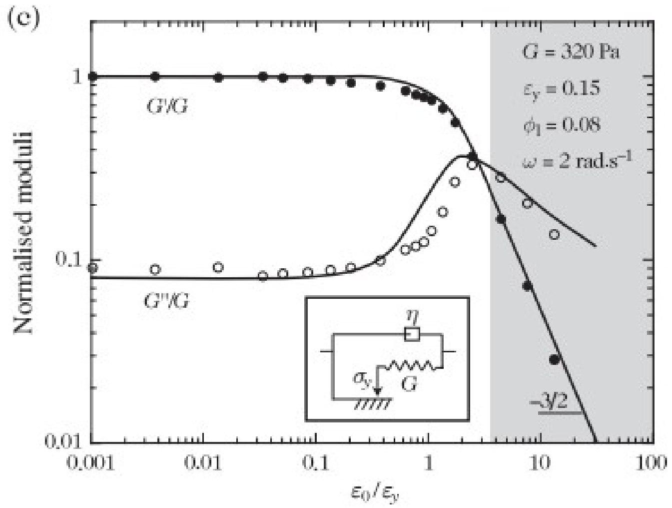
measure

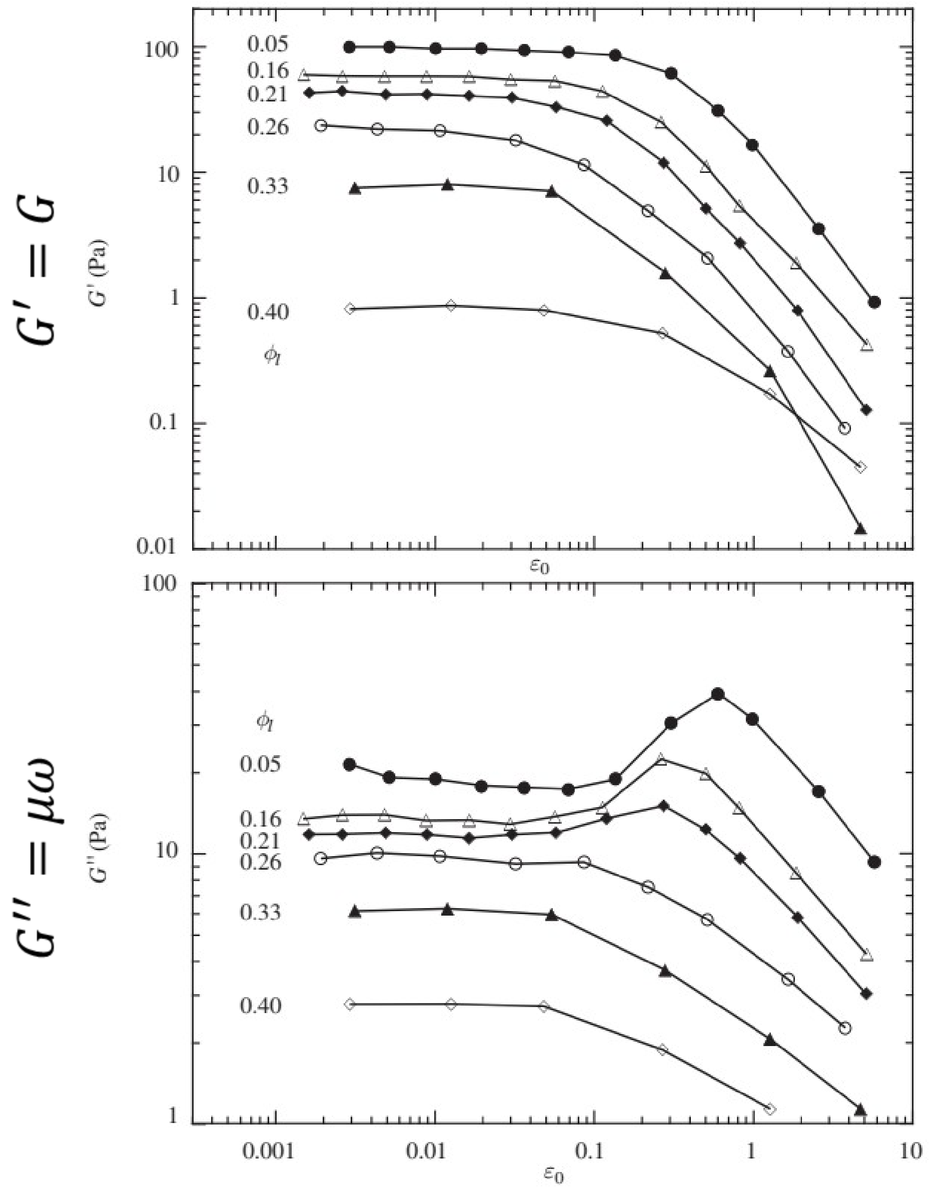
$$\boxed{\tau = \tau_0\, R(e^{i\omega t+\delta})}$$



$$\boxed{\tau = \varepsilon_0\, R(G^* e^{i\omega t})}$$

$$\boxed{G^* = G' + iG''}$$





Remarks

- Difficult to perform proper measurements because of foam aging, drainage, wall-slip
- No optical reference to probe shear field
- Not that much data available, particularly for high liquid fractions
- (Surface Evolver) simulations are difficult, because energy dissipation not resolved
- Shear banding still under discussion
- Emulsions as a model for foam (no drainage)