

Dr.-Ing. Sascha Heitkam,

Institute for Process Engineering and Environmental Technology

Lecture 10: Foam Measurement

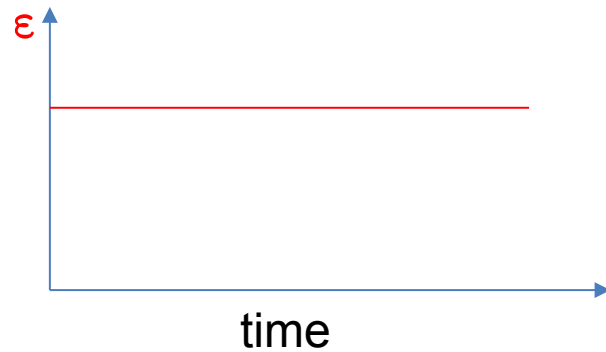
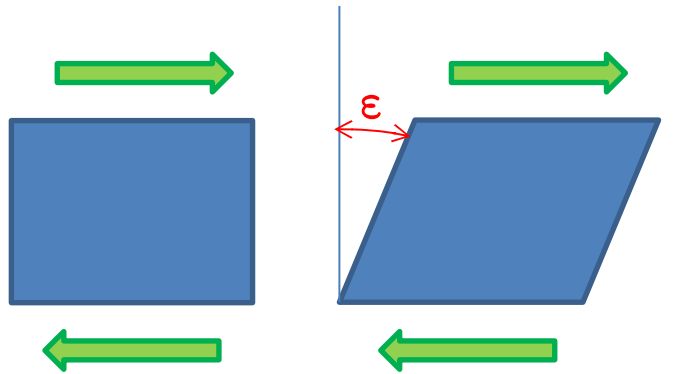
Structure

- Review lecture 9
- Optical measurement
- Resistivity measurement
- Radiography/Tomography
- Velocity measurement

Response to stress:

- Elastic body

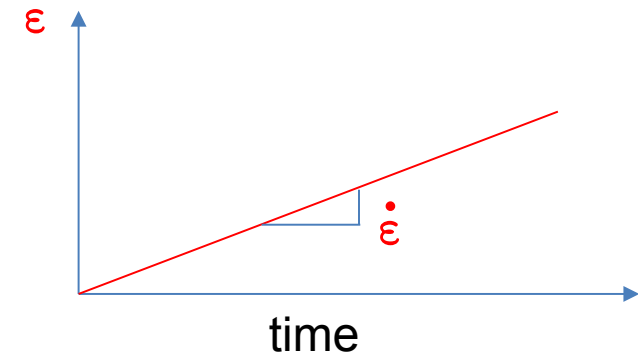
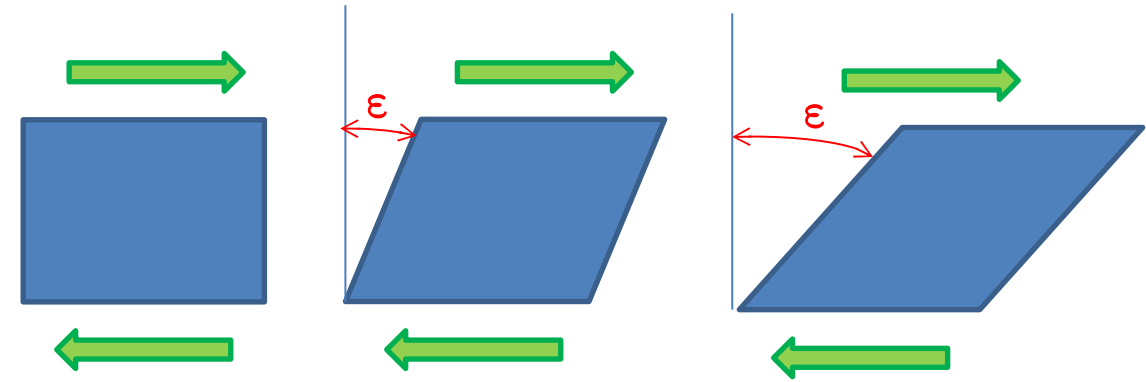
Constant stress $\rightarrow$ constant strain **angle** ϵ



A geometric diagram of a parallelogram with height d and horizontal displacement u . The shear strain angle ϵ is shown. The formula $\epsilon = \frac{u}{d}$ is written inside the parallelogram.

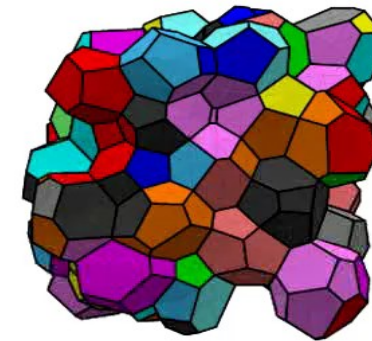
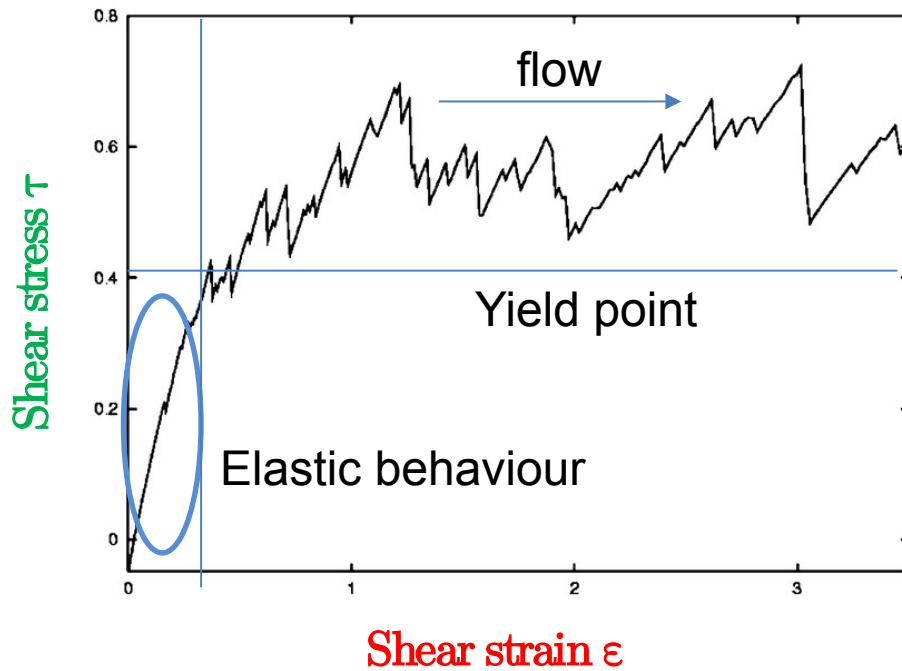
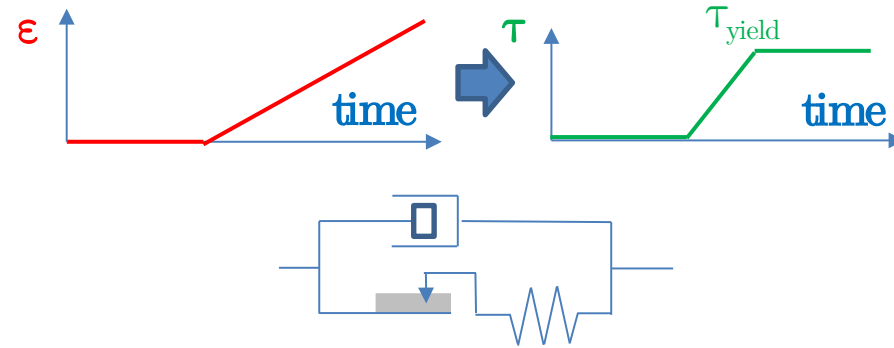
- Newtonian fluid:

Constant stress $\rightarrow$ constant strain **rate** $\dot{\epsilon}$



Rheology of foam

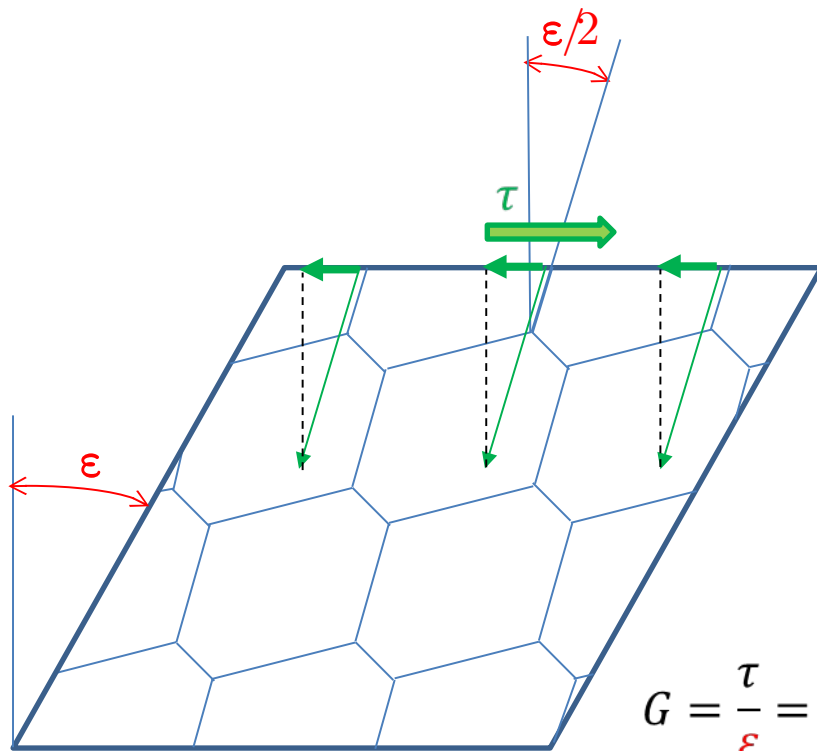
- Low strain: elastic behaviour
- Critical yield stress causes bubble rearrangement
- Viscous behaviour at high stress, strain



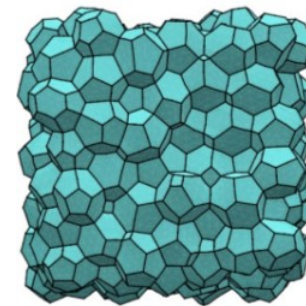
foams@aber.ac.uk

Simple aproach: 2D foam

- Dry, monodisperse 2D foam forms hexagonal pattern
- Shear stress results from surface tension of cutted lamellas



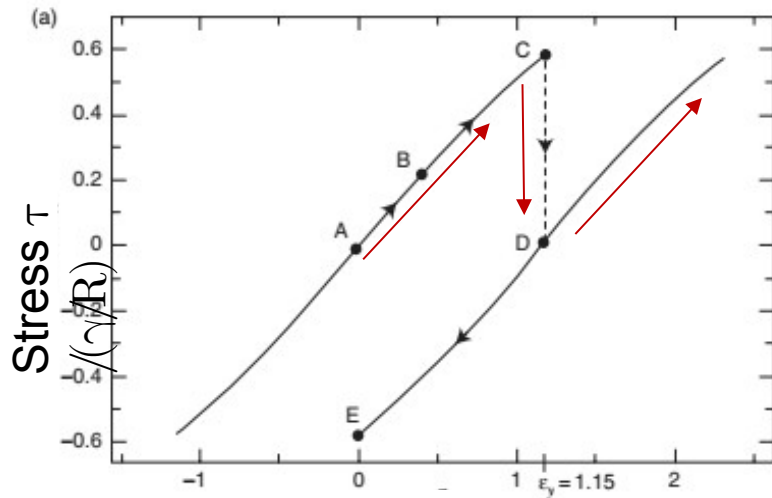
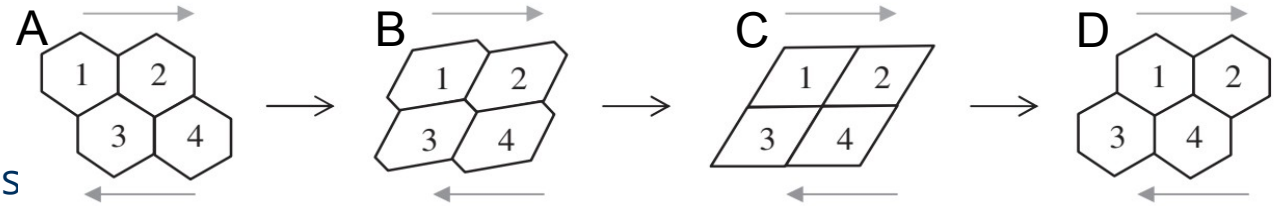
$$G = \frac{\tau}{\epsilon} = 0.525 \frac{\gamma}{R_e}$$



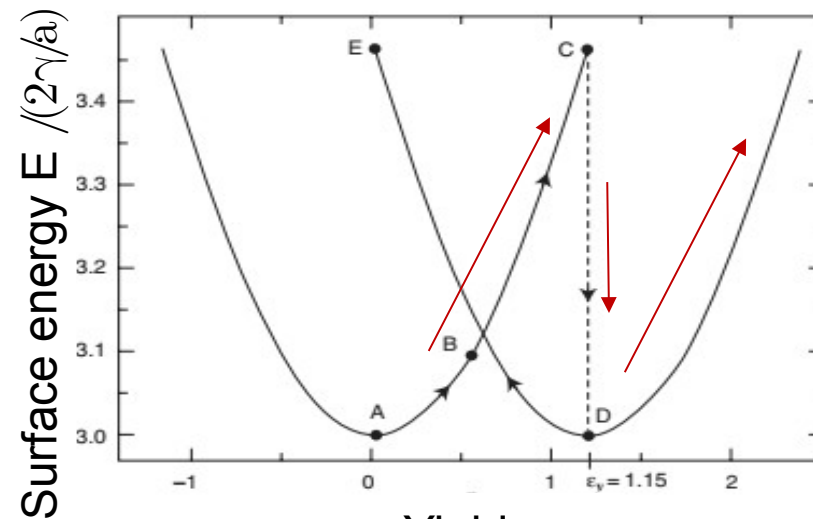
$$G = \frac{\tau}{\epsilon} = 0.51 \frac{\gamma}{R_e}$$

Yield point for 2D foam

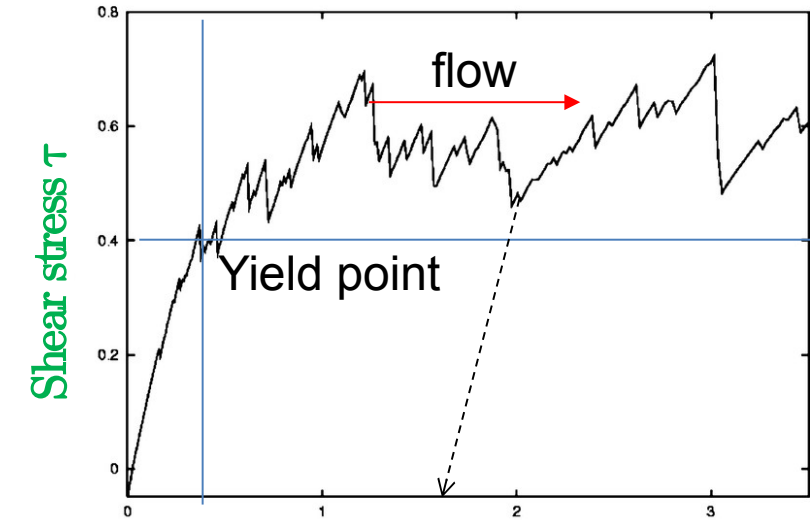
- Flow of foam is a sequence of T1 rearrangements



Yield
strain ϵ



Yield
strain ϵ



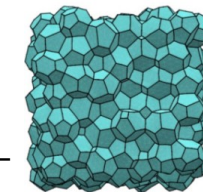
Shear strain ϵ

$$\epsilon_{\text{yield}} = 1.15$$

$$\tau_{\text{yield}} = 0.603 \frac{\gamma}{R_e}$$

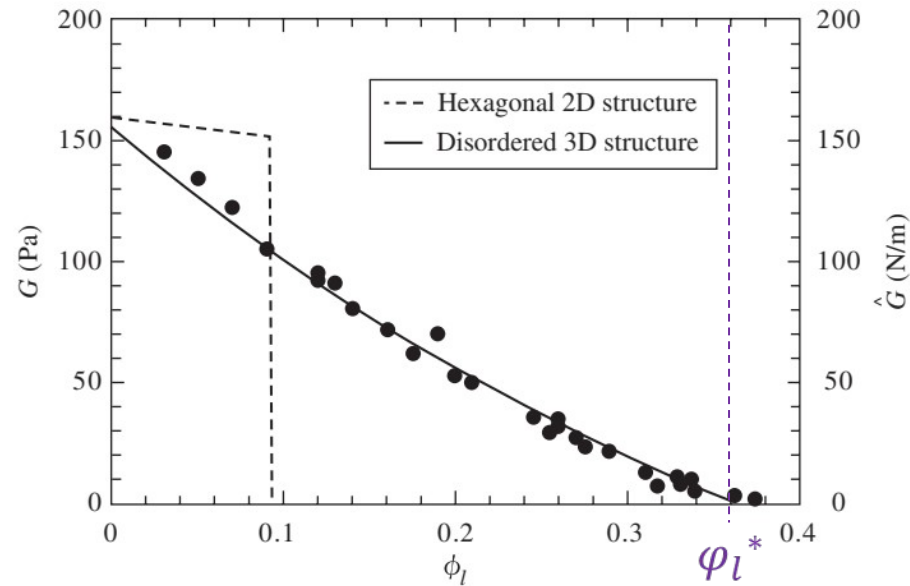
$$\epsilon_{\text{yield}} \approx 0.15 \dots 0.4$$

$$\tau_{\text{yield}} \approx 0.1 \dots 0.2 \frac{\gamma}{R_e}$$

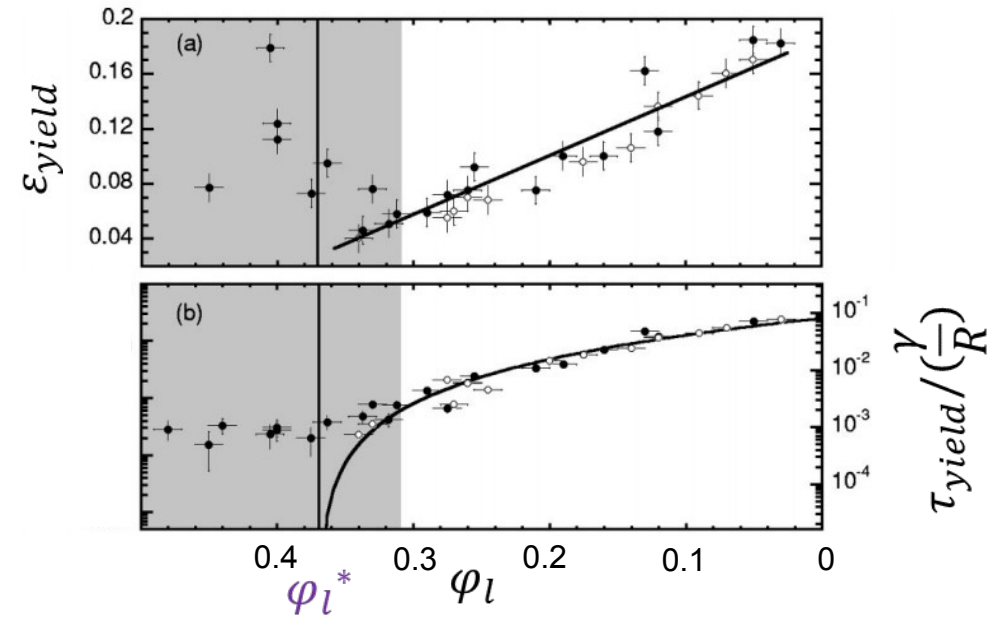


Elastic behaviour of wet foam

- Experimental results: liquid fraction reduces shear modulus
- Loss of rigidity at jamming point (36% liquid fraction)



$$G = 1.4(1 - \phi_l)(\phi_l^* - \phi_l) \frac{\gamma}{R_e}$$

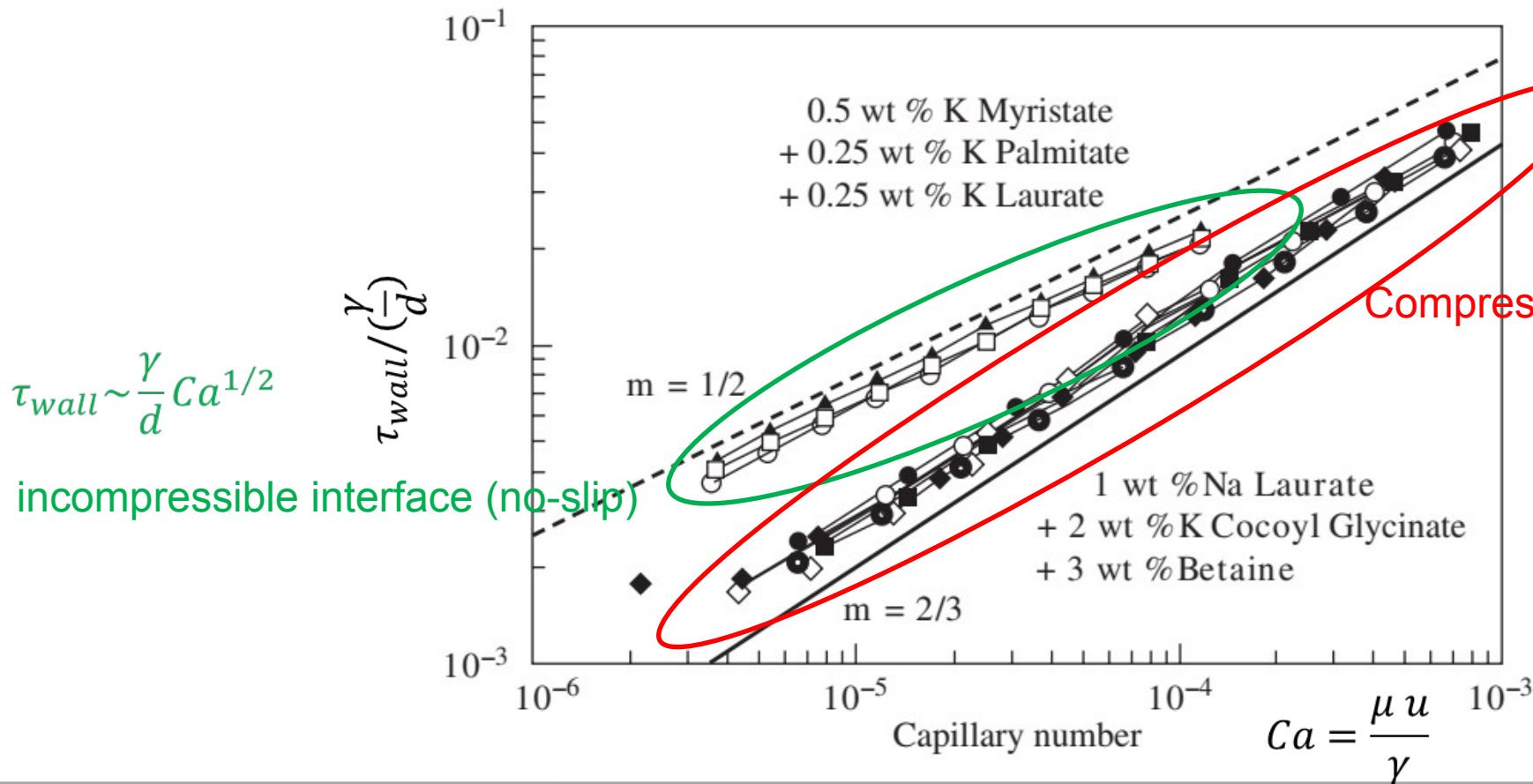
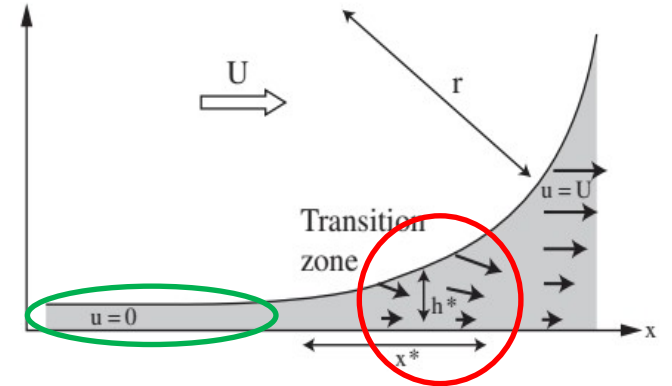


$$\epsilon_{yield} \approx a_1(\phi_l^* - \phi_l), \quad a_1 \approx 0.5 \dots 1.0$$

$$\tau_{yield} \approx a_2 \frac{\gamma}{R_e} (\phi_l^* - \phi_l)^2, \quad a_2 \approx 0.2 \dots 0.5$$

Role of surface compressibility

- Surface compressibility (mobility) depends on surfactant
- Compressible interfaces result in lower wall stress



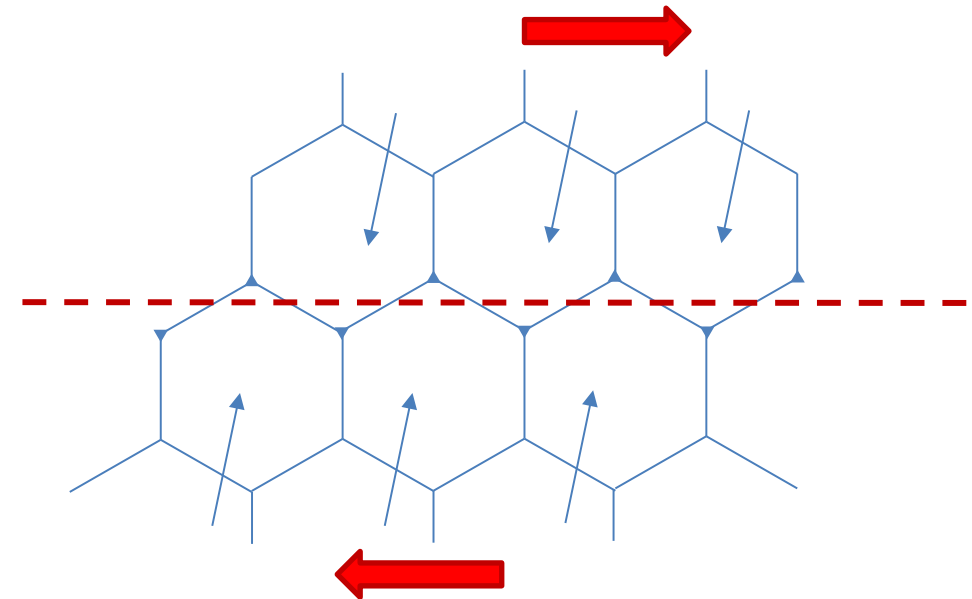
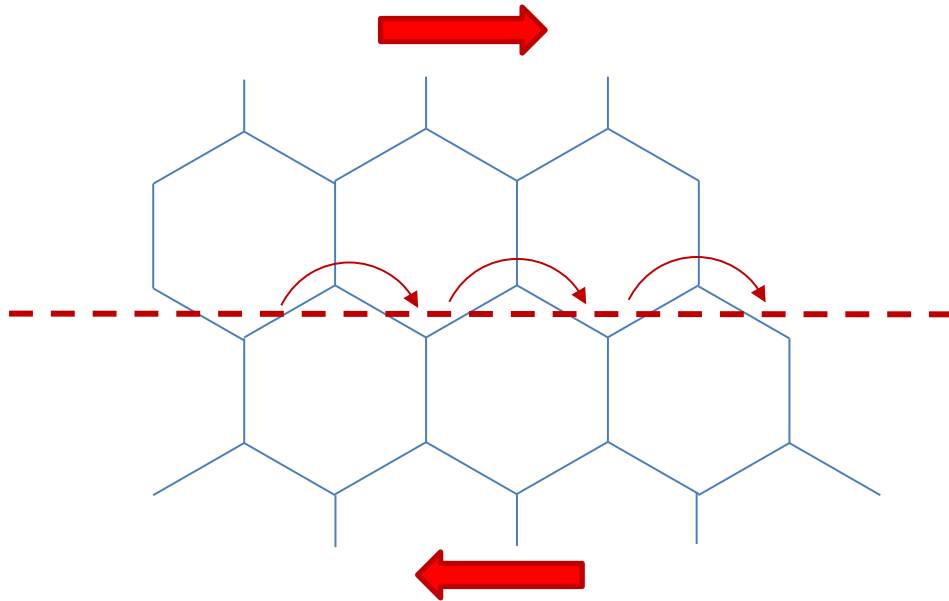
Compressible interface (free-slip)

$$\tau_{wall} \sim \frac{\gamma}{d} Ca^{2/3}$$



Shear banding

- T1 rearrangements are localized in a single sliding layer (band)?
- Local shear might attract more liquid (dilatancy)
- Locally higher liquid fraction reduces shear modulus, stabilizing shear layer



Rheometer

- Review on foam rheology

INSTITUTE OF PHYSICS PUBLISHING

JOURNAL OF PHYSICS: CONDENSED MATTER

J. Phys.: Condens. Matter **17** (2005) R1041–R1069

[doi:10.1088/0953-8984/17/41/R01](https://doi.org/10.1088/0953-8984/17/41/R01)

TOPICAL REVIEW

Rheology of liquid foam

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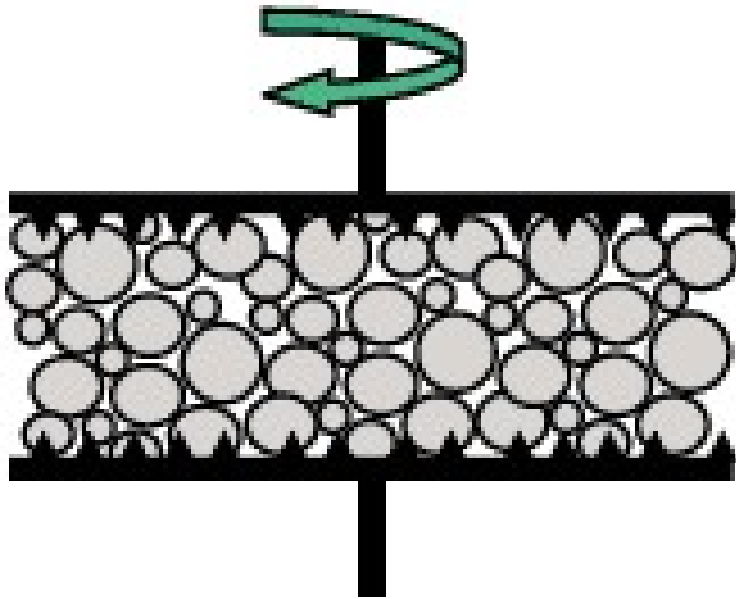
Received 14 June 2005, in final form 4 August 2005

Published 30 September 2005

Online at stacks.iop.org/JPhysCM/17/R1041

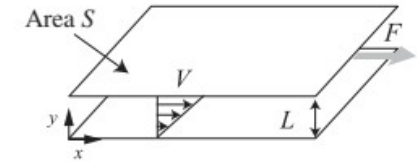
Rheometer

- Measure relation between strain (rate) and stress
- Well defined geometry
- → analytical solution of stress / strain distribution
- Avoid wall slip! → etching grooves, sandpaper



Sliding plate

$$\varepsilon = \frac{Vt}{L} \quad \dot{\varepsilon} = \frac{V}{L} \quad \sigma = \sigma_{xy} = \frac{F}{S}$$



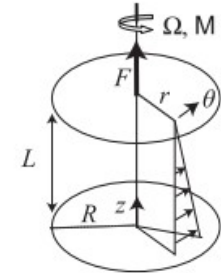
Parallel plate

$$v_{\theta}(r, z) = \frac{r \Omega z}{L}$$

$$\varepsilon(r) = \frac{r\theta}{L} \quad \dot{\varepsilon} = \frac{r\Omega}{L}$$

$$\sigma(r) = \sigma_{\theta z}(r) = \frac{2Mr}{\pi R^4}$$

$$\text{Normal stresses } F = \pi \int_0^R (N_2 - N_1) r \, dr$$

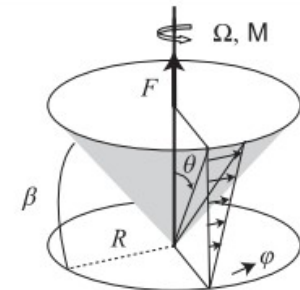


Cone-plate

$$\varepsilon \approx \frac{\varphi}{\beta} \quad \dot{\varepsilon} \approx \frac{\Omega}{\beta} \quad (\text{for small } \beta)$$

$$\sigma = \sigma_{\varphi\theta} = \frac{3M}{2\pi R^3}$$

$$\text{Normal stresses } N_1 = \frac{2F}{\pi R^2}$$



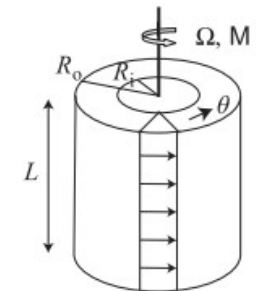
Cylindrical Couette

$$\dot{\varepsilon} = r \frac{\partial}{\partial r} \left(\frac{v_{\theta}}{r} \right)$$

$$\varepsilon \approx \frac{R_o + R_i}{2(R_o - R_i)} \Omega t \quad \dot{\varepsilon} \approx \frac{R_o + R_i}{2(R_o - R_i)} \Omega$$

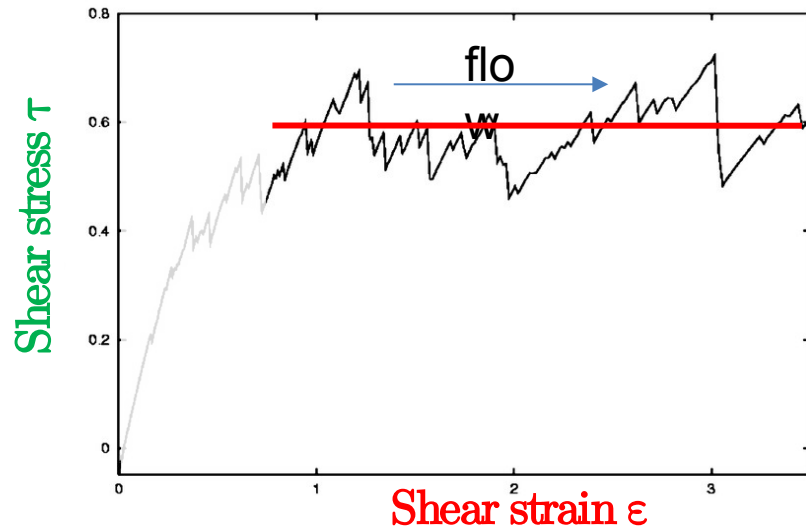
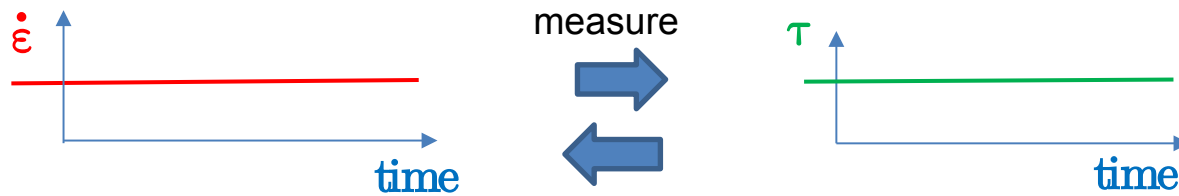
(for a narrow gap)

$$\sigma(r) = \sigma_{\theta r}(r) = \frac{M}{2\pi r^2 L}$$

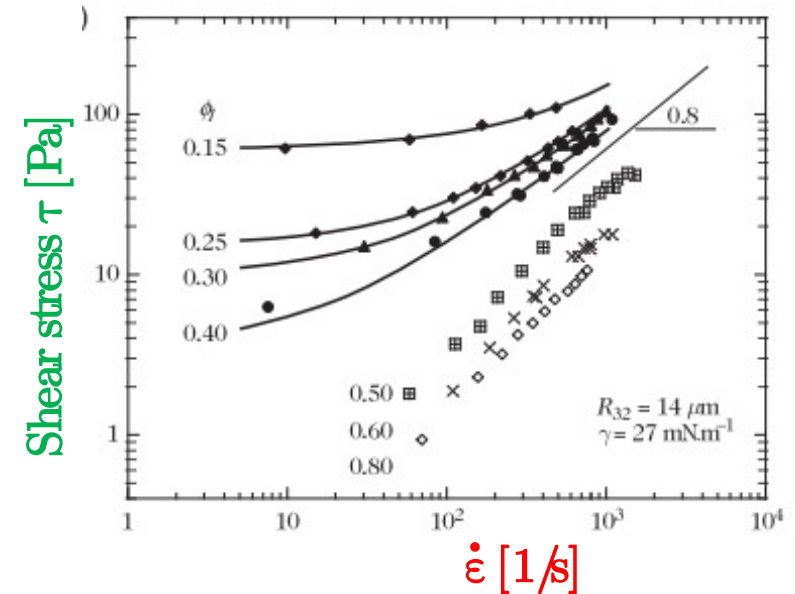


Steady shear

- Set steady shear rate (or steady stress)
- Wait for steady state and measure stress (or shear rate)
- Test different shear rates

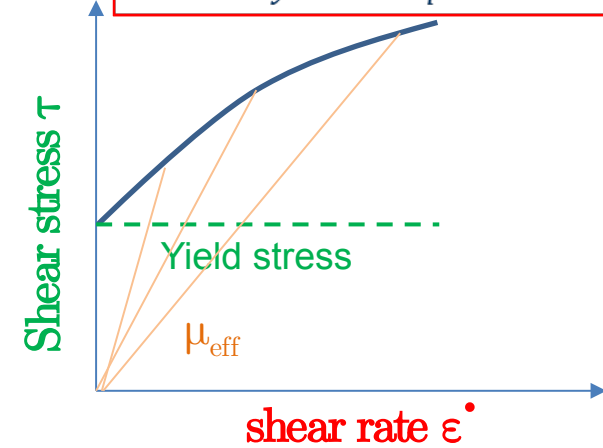


- Effective viscosity $\tau/\dot{\epsilon}$ in flow state



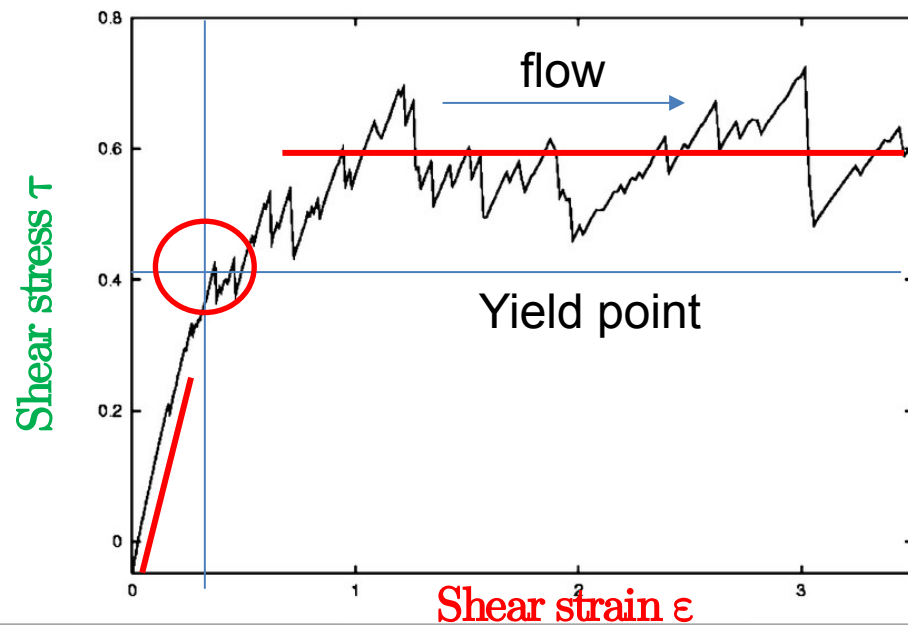
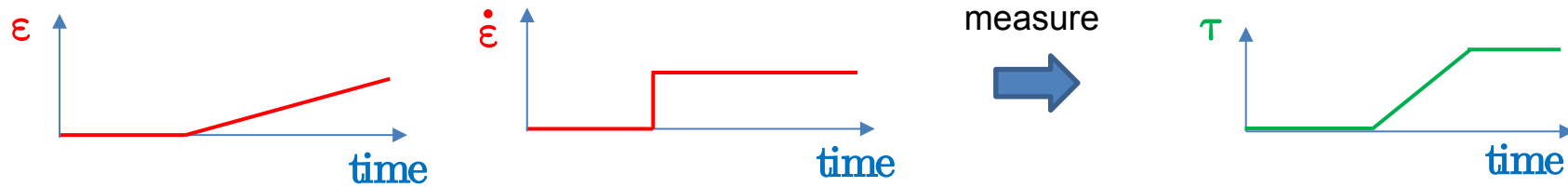
Herschel-Bulkley law

$$\tau = \tau_{\text{yield}} + \eta_p \dot{\epsilon}^\beta, \quad \beta < 1$$



Shear start-up

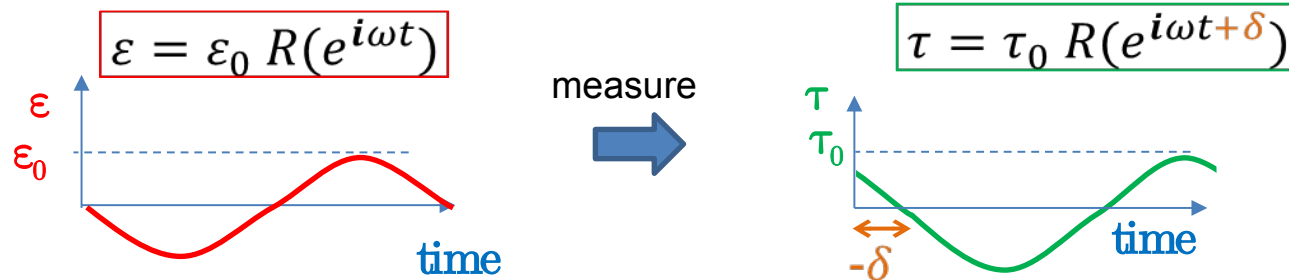
- Shear start-up
- Start to shear with constant shear rate, measure corresponding shear stress



- Elastic shear modulus
- Yield stress

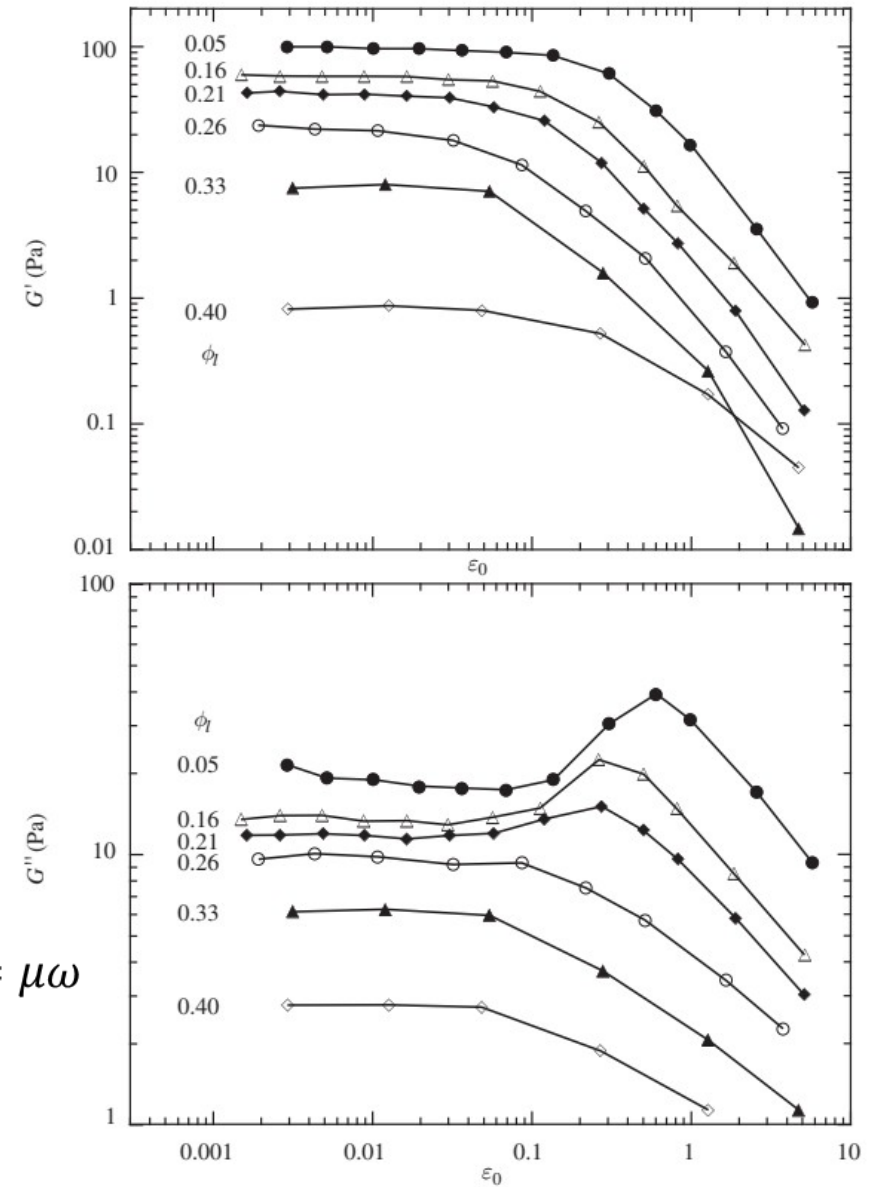
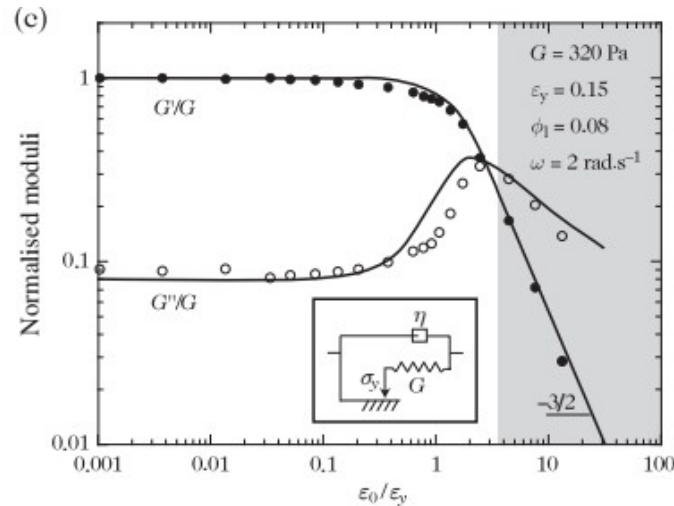
Oscillatory shear

- Sinusoidal strain of amplitude ε_0 and frequency ω
- Wait for steady state and measure resulting stress
- Gives complex shear modulus $G^*(\omega)$



$$\tau = \varepsilon_0 R(G^* e^{i\omega t})$$

$$G^* = G' + iG''$$

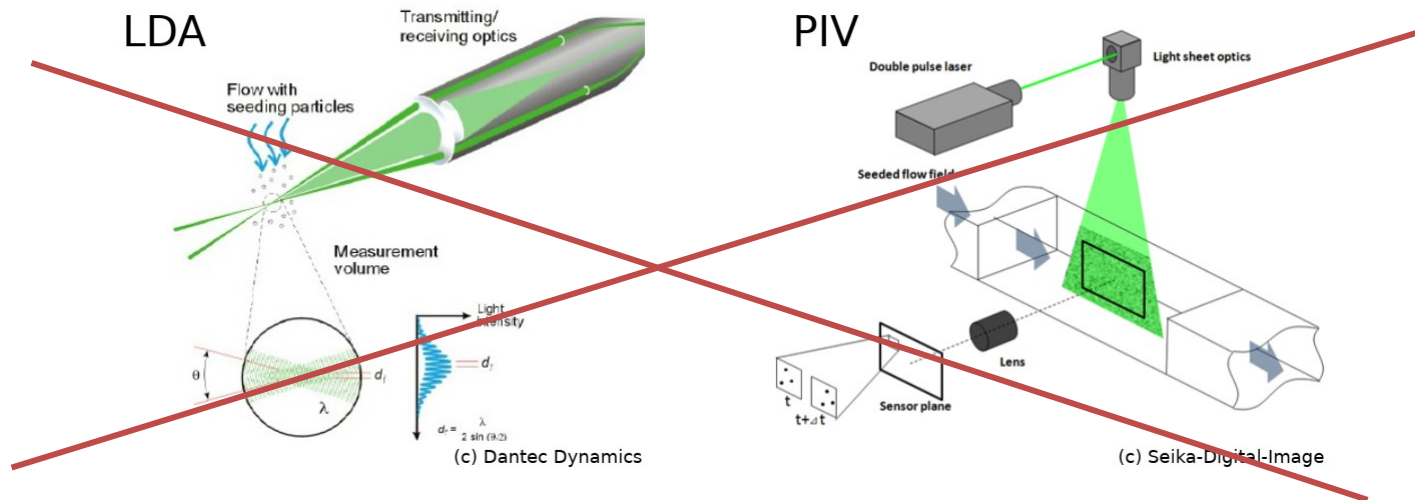


Structure

- Review lecture 9
- Optical measurement
- Resistivity measurement
- Radiography/Tomography
- Velocity measurement

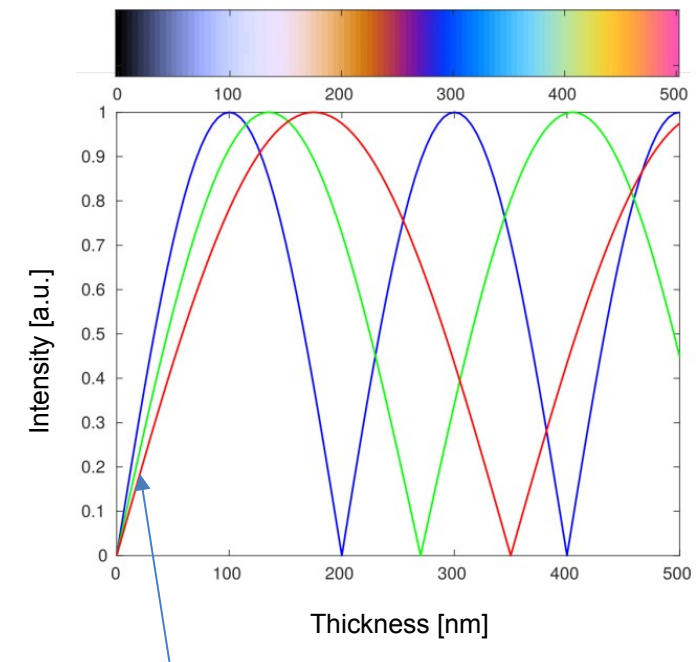
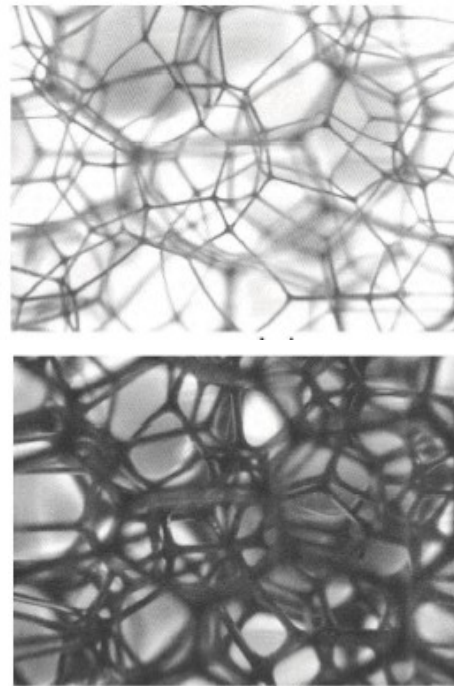
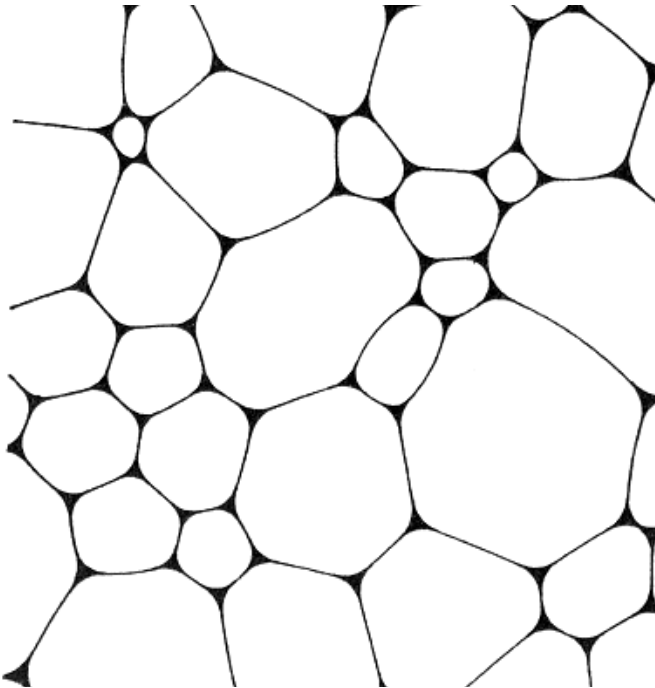
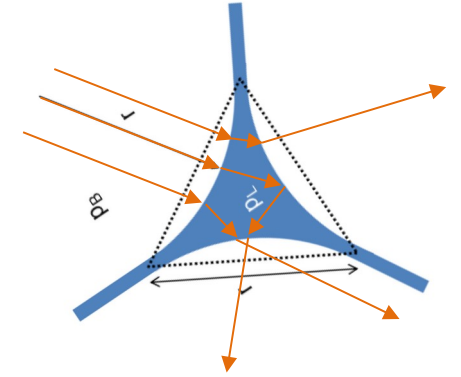
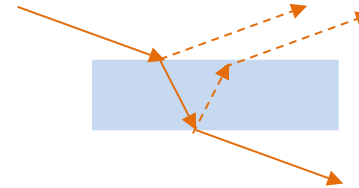
Optical measurement

- Most convenient, but: **Foam is opaque**
- Typically limited to surface of a foam cluster
- Many established measurement techniques are not applicable



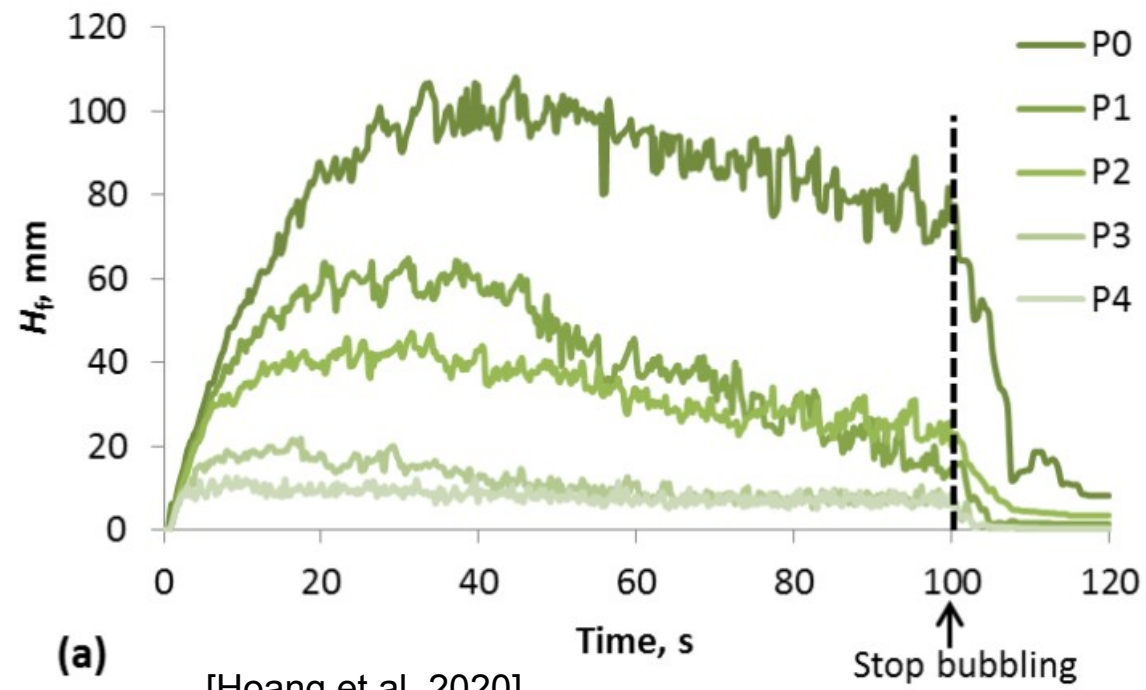
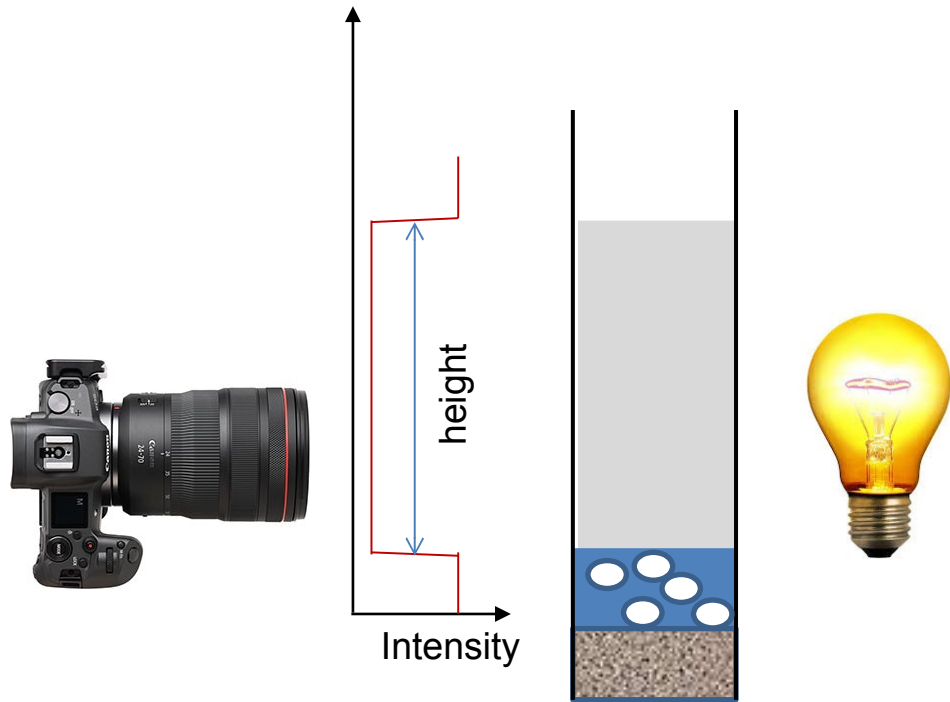
Light refraction

- Thick lamellas – partially reflected, CBF lamellas – no reflection
- Light is refracted/reflected by Plateau borders

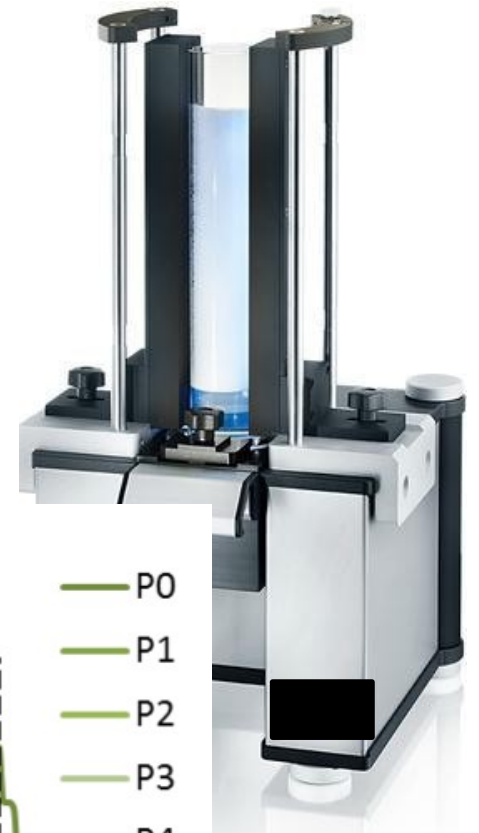


Foam height

- Automated measurement of foam height, using transmittency
- Commercialized in foam test devices

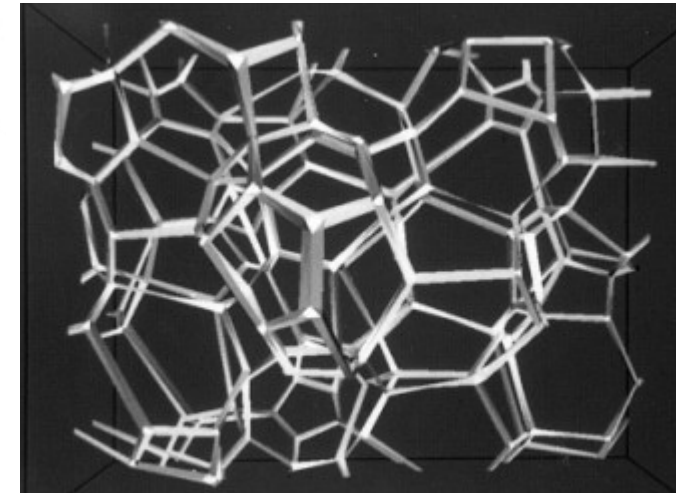
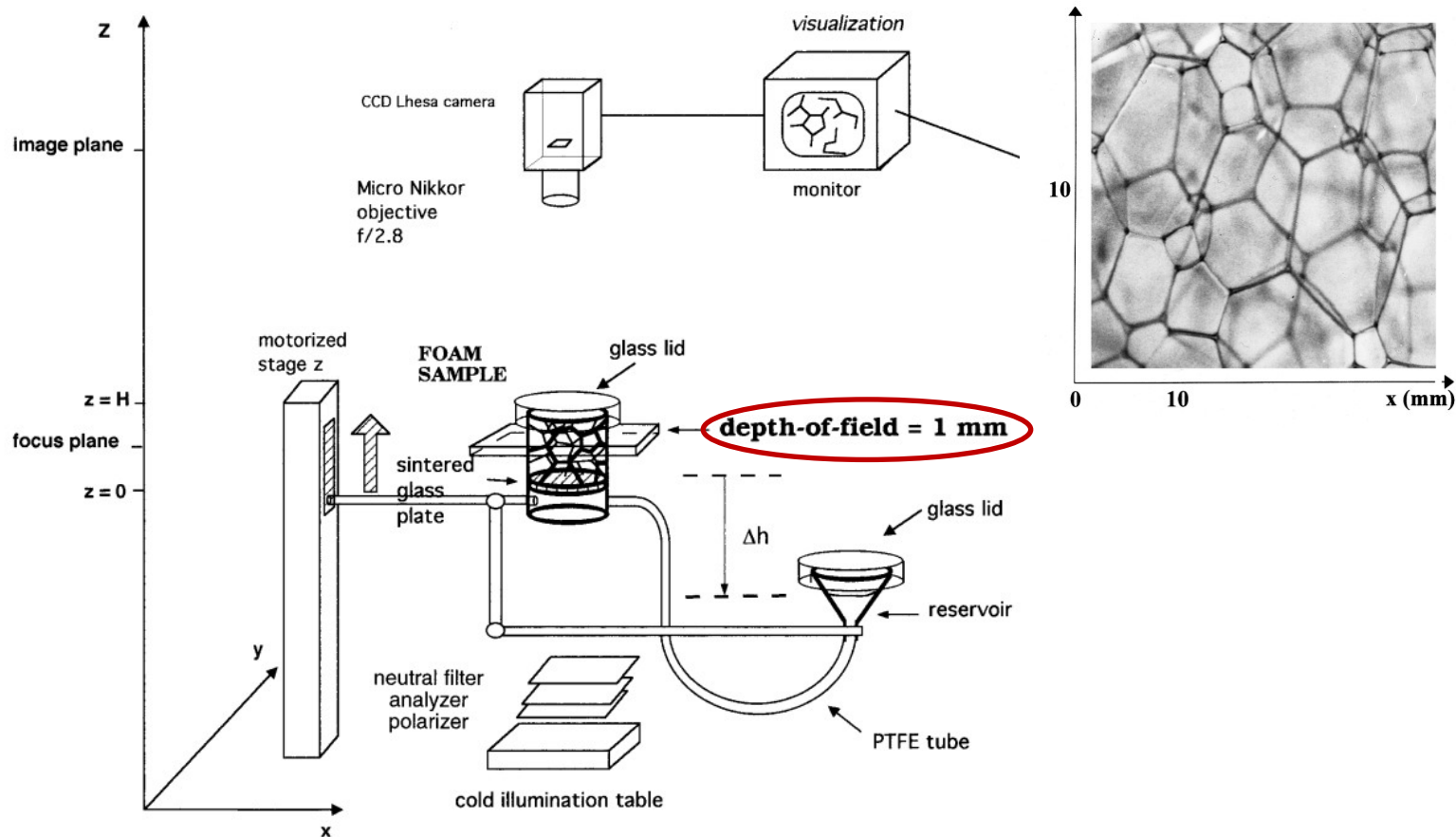


[Hoang et al. 2020]



Optical tomography

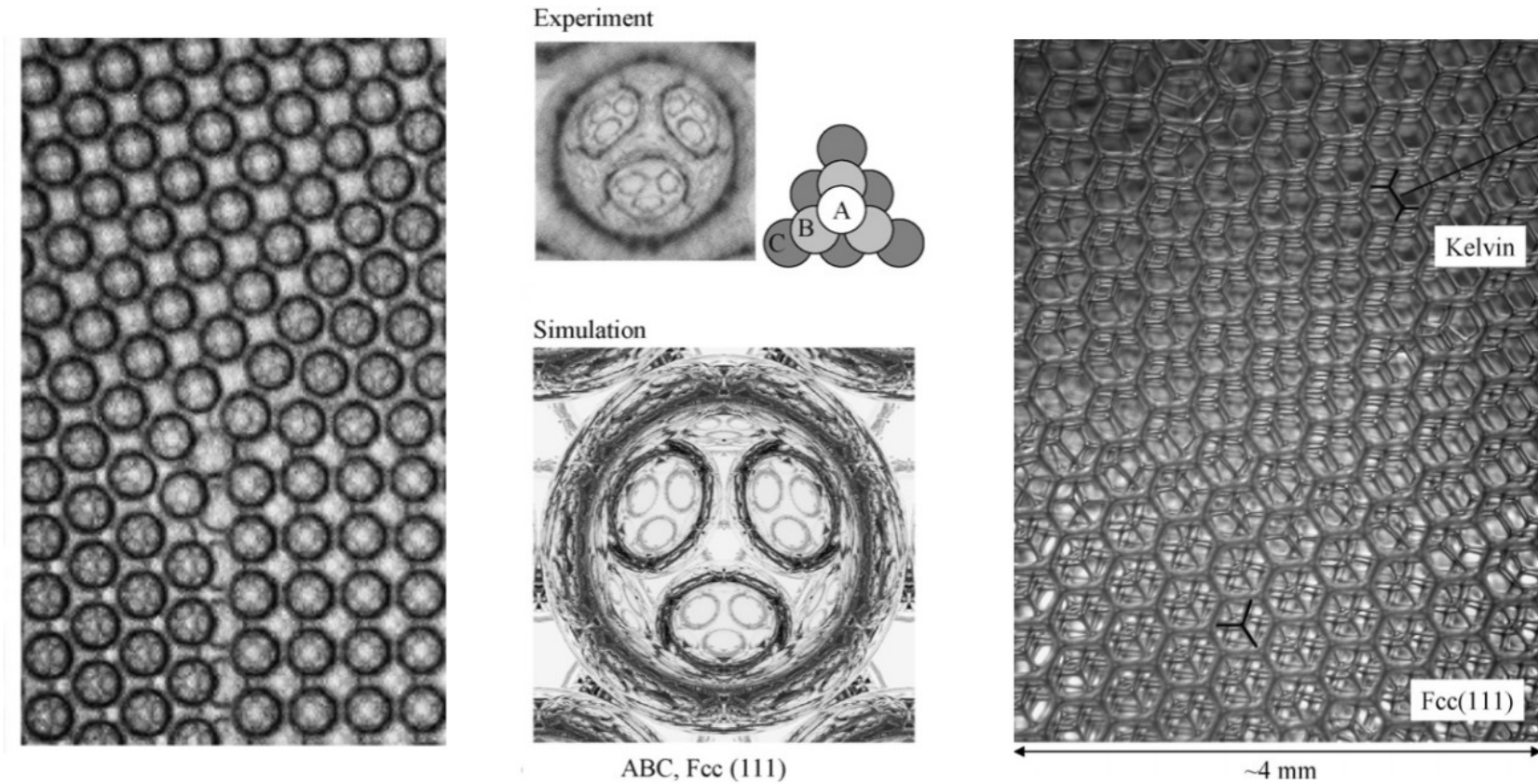
- Reconstruction of Plateau border structure
- Suitable only for dry foam and less than 10 bubble diameter thickness



[Monnereau & Vignes-Adler, 1998]

Optical tomography near the boundary

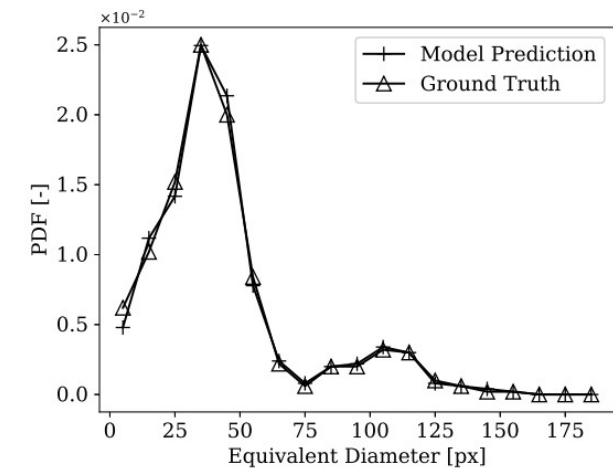
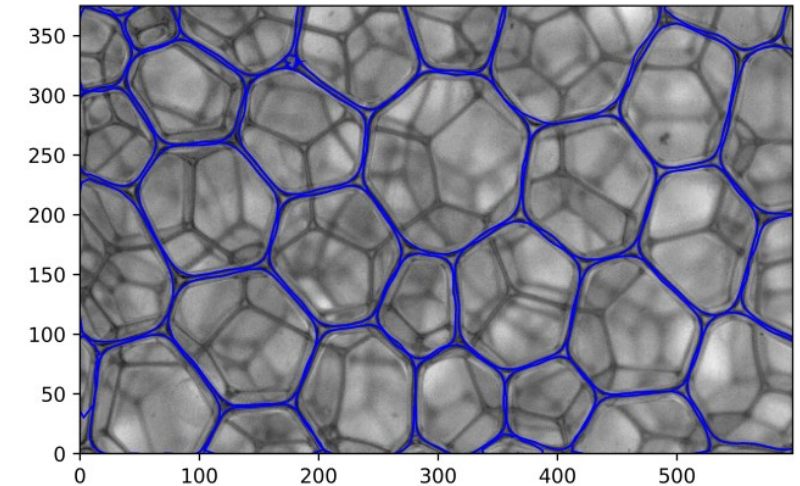
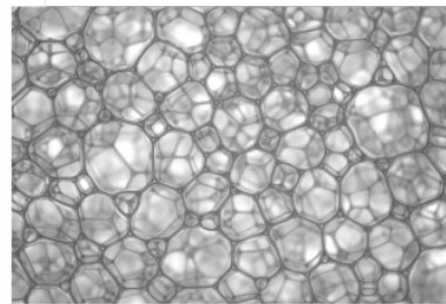
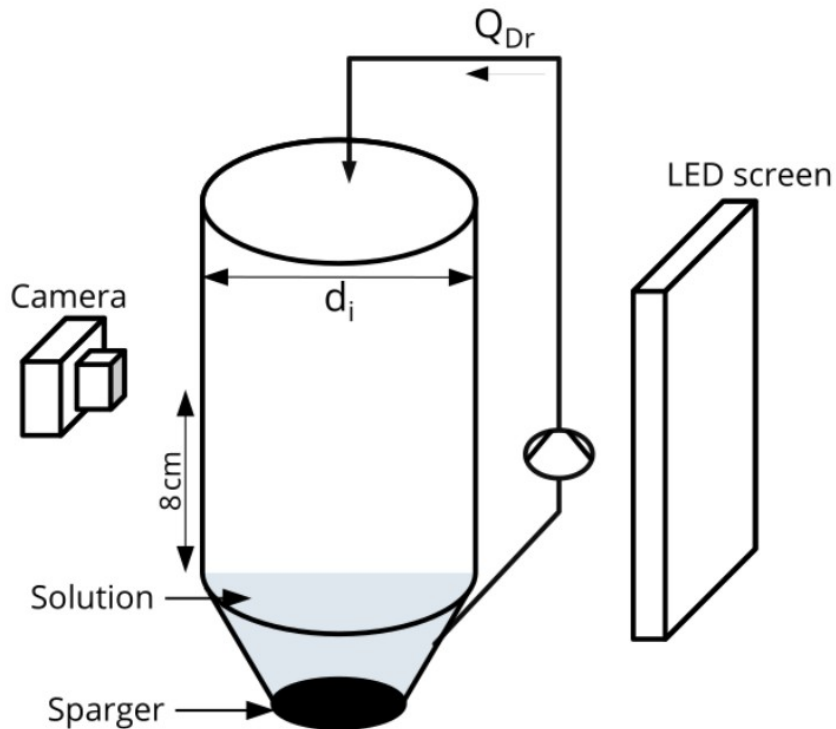
- Careful image processing yields crystalline structure
- Comparison with ray-tracing to tell apart FCC and HCP



[Van der Net et al. 2006]

Optical measurement of foam surfaces

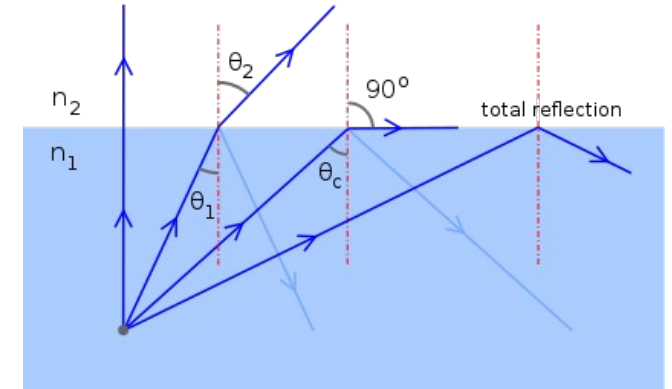
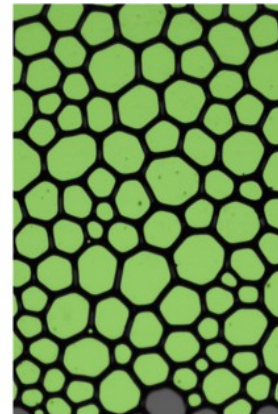
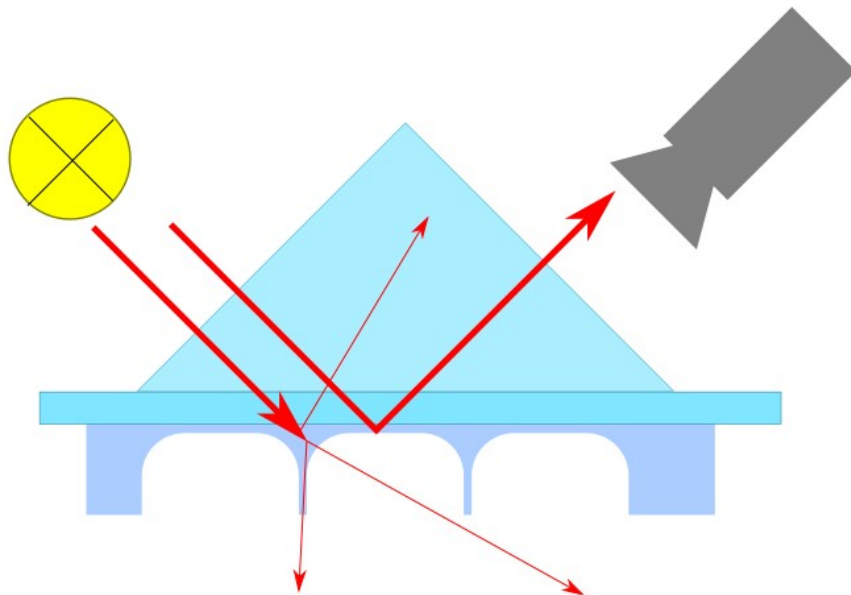
- Pictures of wall-bounded foam show a lot of structure
- Neural Network approach to segment bubbles



[Knüpfer & Heitkam 2022]

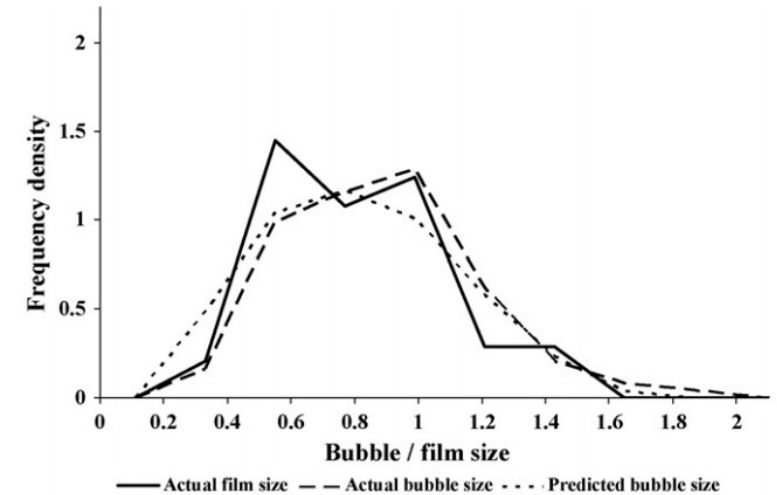
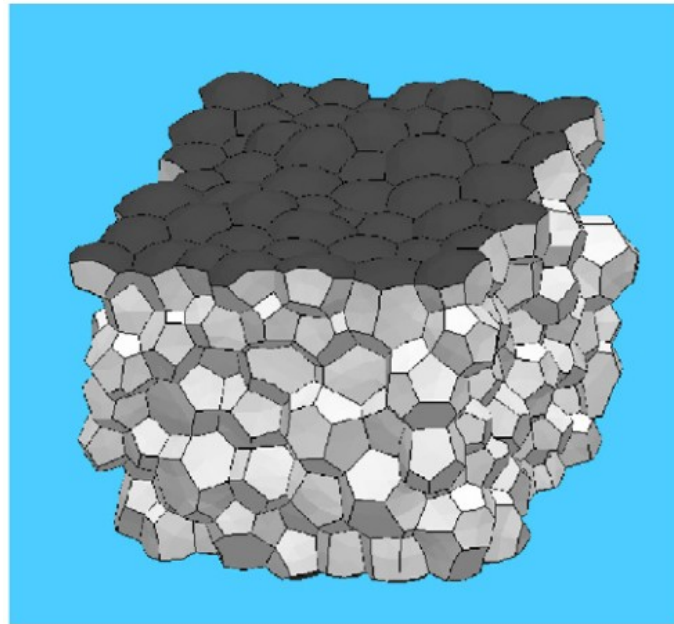
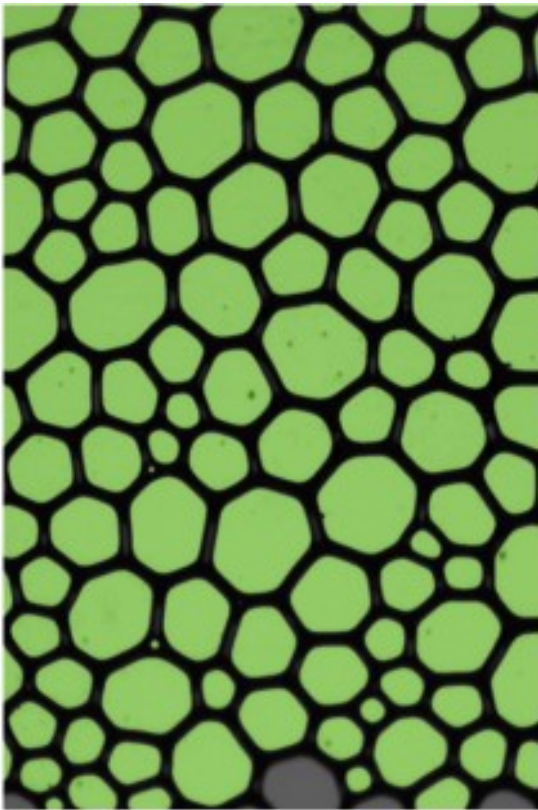
Optical measurement of foam surfaces

- Enhanced image contrast using total internal reflection
- Lamellas appear bright, Plateau borders appear dark



Bubble size measurement

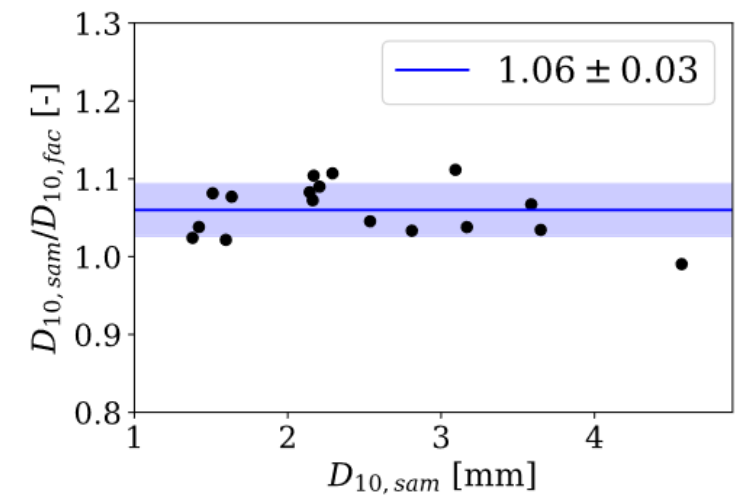
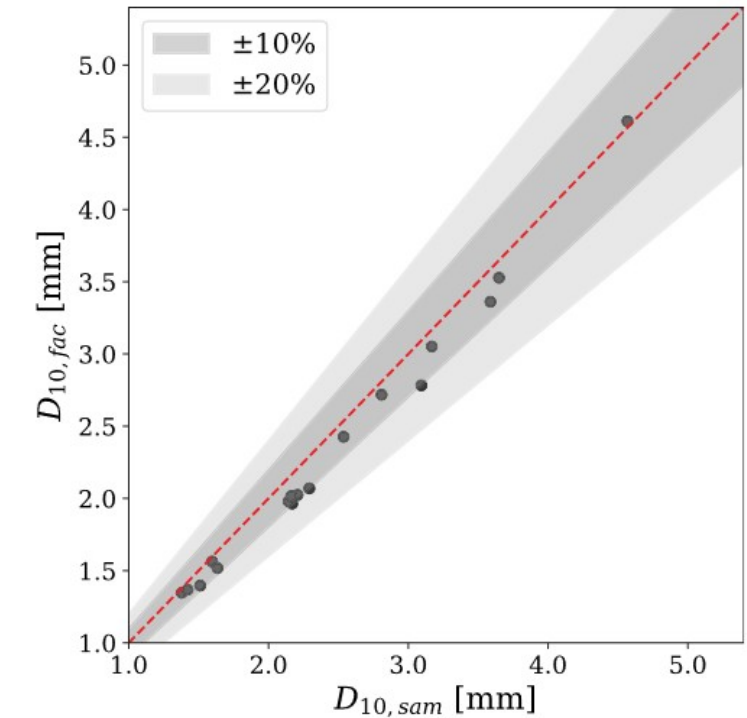
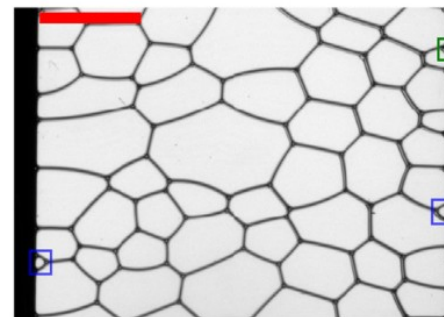
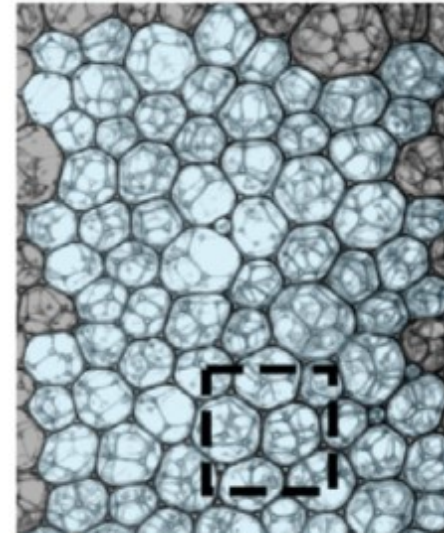
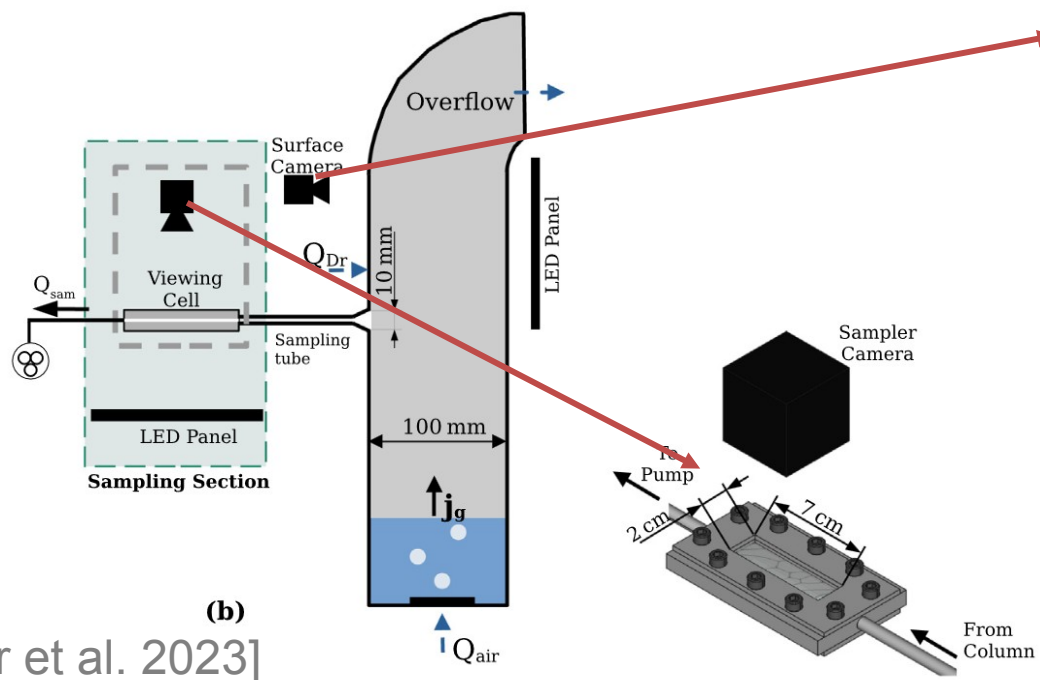
- Segmentation algorithm (discussed in lecture 3) → Lamella size
- Relation between lamella size and bubble size from Surface Evolver
- Recomputation algorithm in [Wang & Neethling 2008]



[Wang & Neethling 2008]

Bubble size measurement

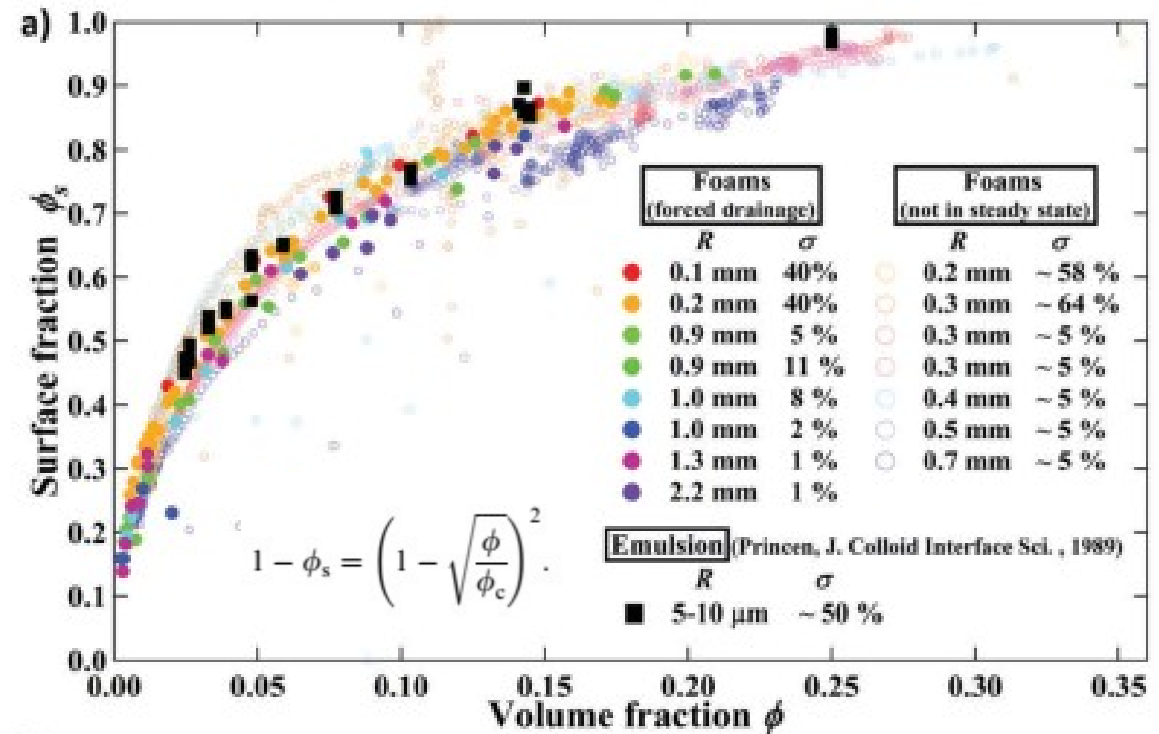
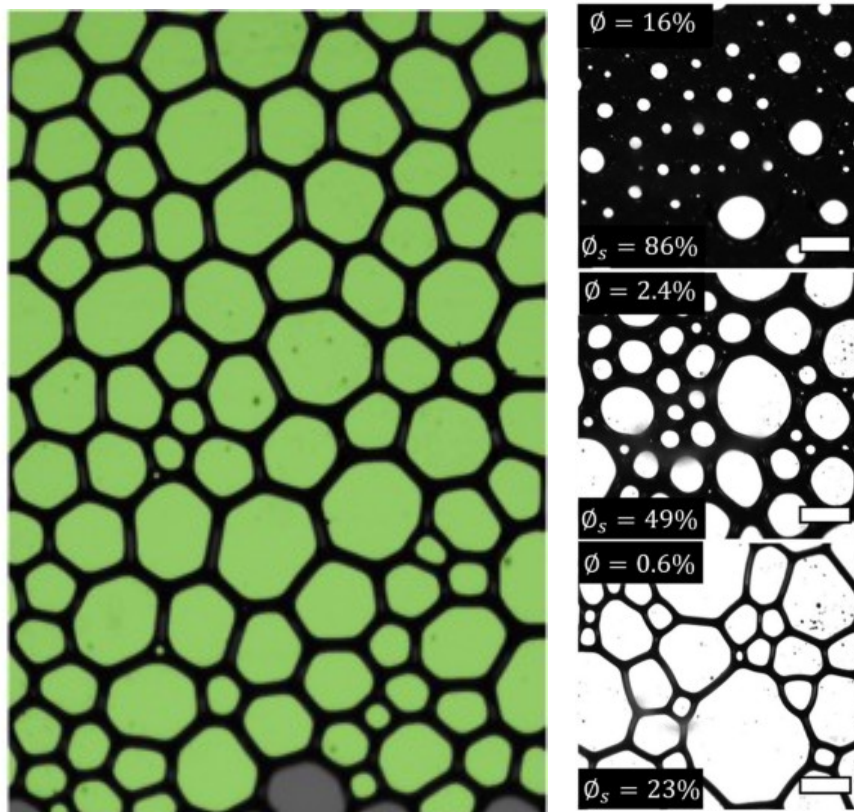
- Direct measurement with two methods:
- Surface segmentation vs bubble extraction
- Surface segmentation underestimates by 6%



[Knüpfer et al. 2023]

Liquid fraction measurement

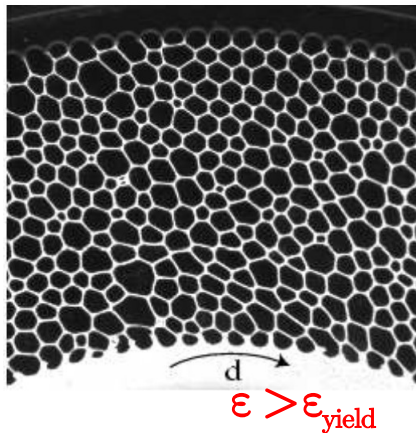
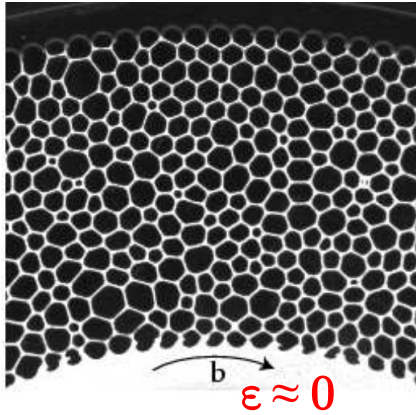
- Segmentation algorithm (discussed in lecture 3) → Lamella surface area fraction φ_s
- Relation between lamella size and bubble size from Surface Evolver



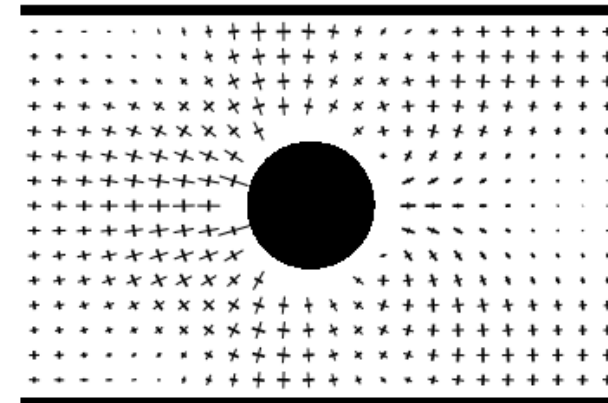
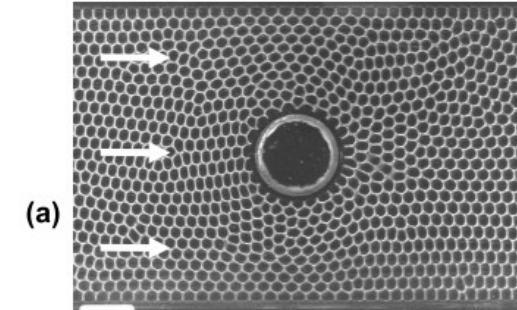
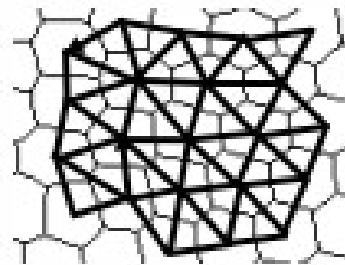
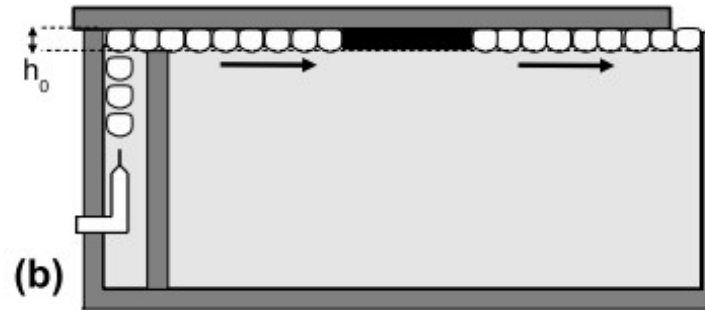
[Forel et al. 2016]

Stress (deformation)

- Segmentation algorithm (discussed in lecture 3) → Aspect ratio and orientation of bubbles
- Local anisotropic stress distribution



[Janiaud & Graner 2004]



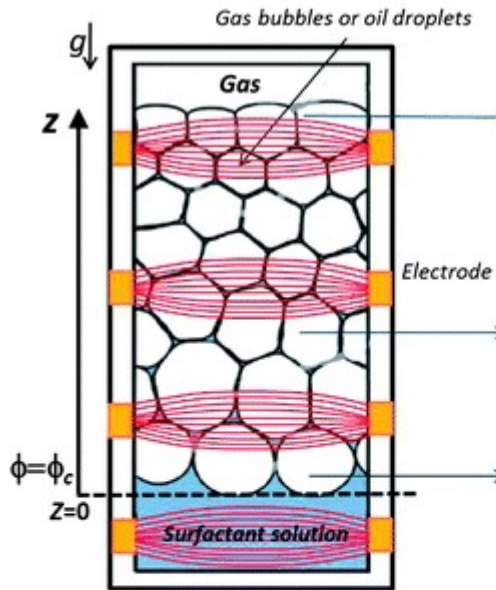
[Dollet & Graner 2006]

Structure

- Review lecture 9
- Measurement of foam rheology
- Optical measurement
- Resistivity measurement
- Radiography/Tomography
- Velocity measurement

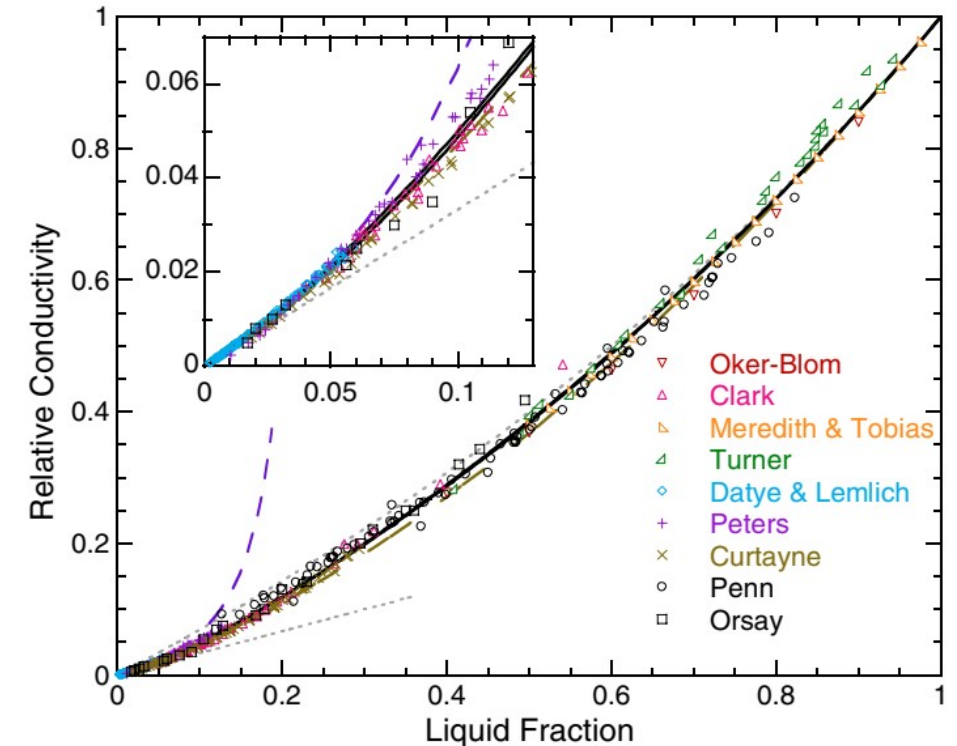
Liquid fraction $\leftrightarrow$ resistivity

- Gas has zero conductivity – current is transported by Plateau border network
- Conductivity is a good measure for liquid fraction



$$\frac{\sigma_{foam}}{\sigma_{liquid}} = \varphi_l \frac{2(1 + 12\varphi_l)}{6 + 29\varphi_l - 9\varphi_l^2}$$

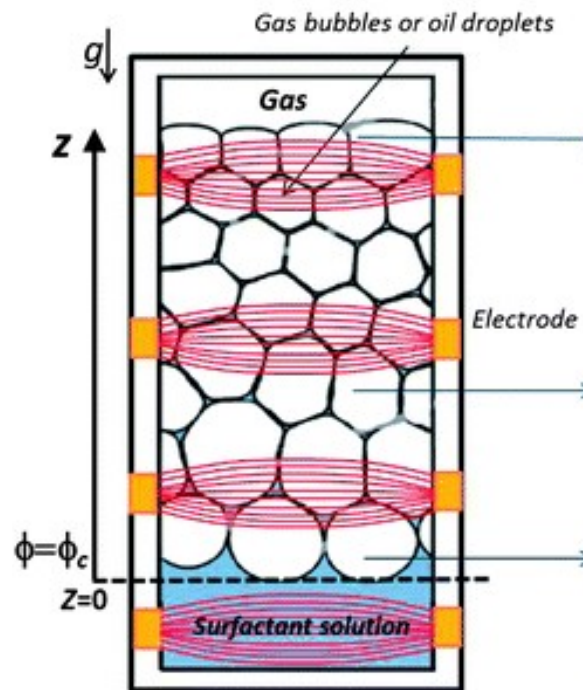
$$\varphi_l = \sigma \frac{3(1 + 11\sigma)}{1 + 25\sigma + 10\sigma^2}, \sigma = \frac{\sigma_{foam}}{\sigma_{liquid}}$$



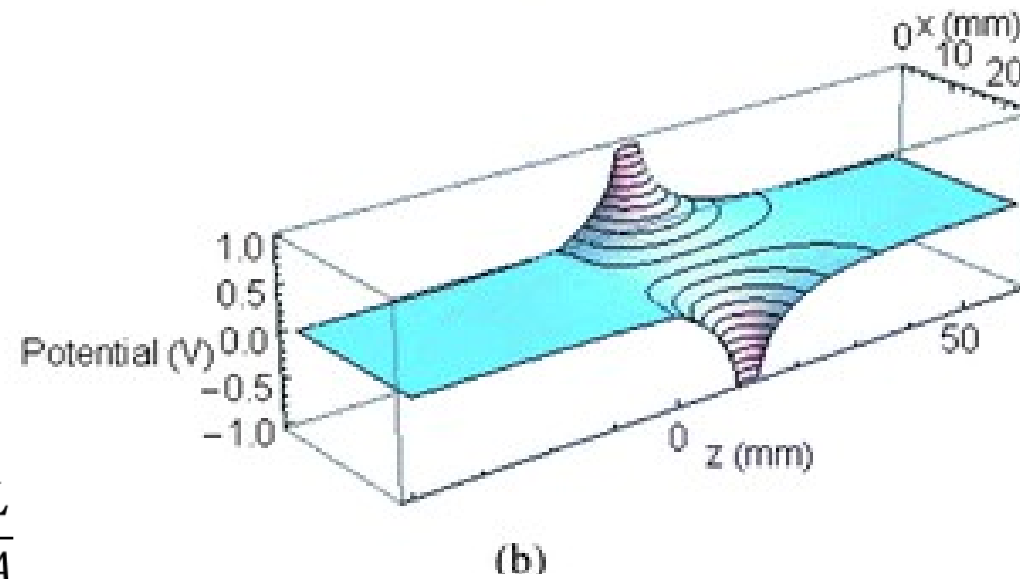
[Feitosa et al. 2005]

Liquid fraction $\leftrightarrow$ resistivity

- Shape of electric field over-weights volume close to electrodes
- Larger electrodes are preferred



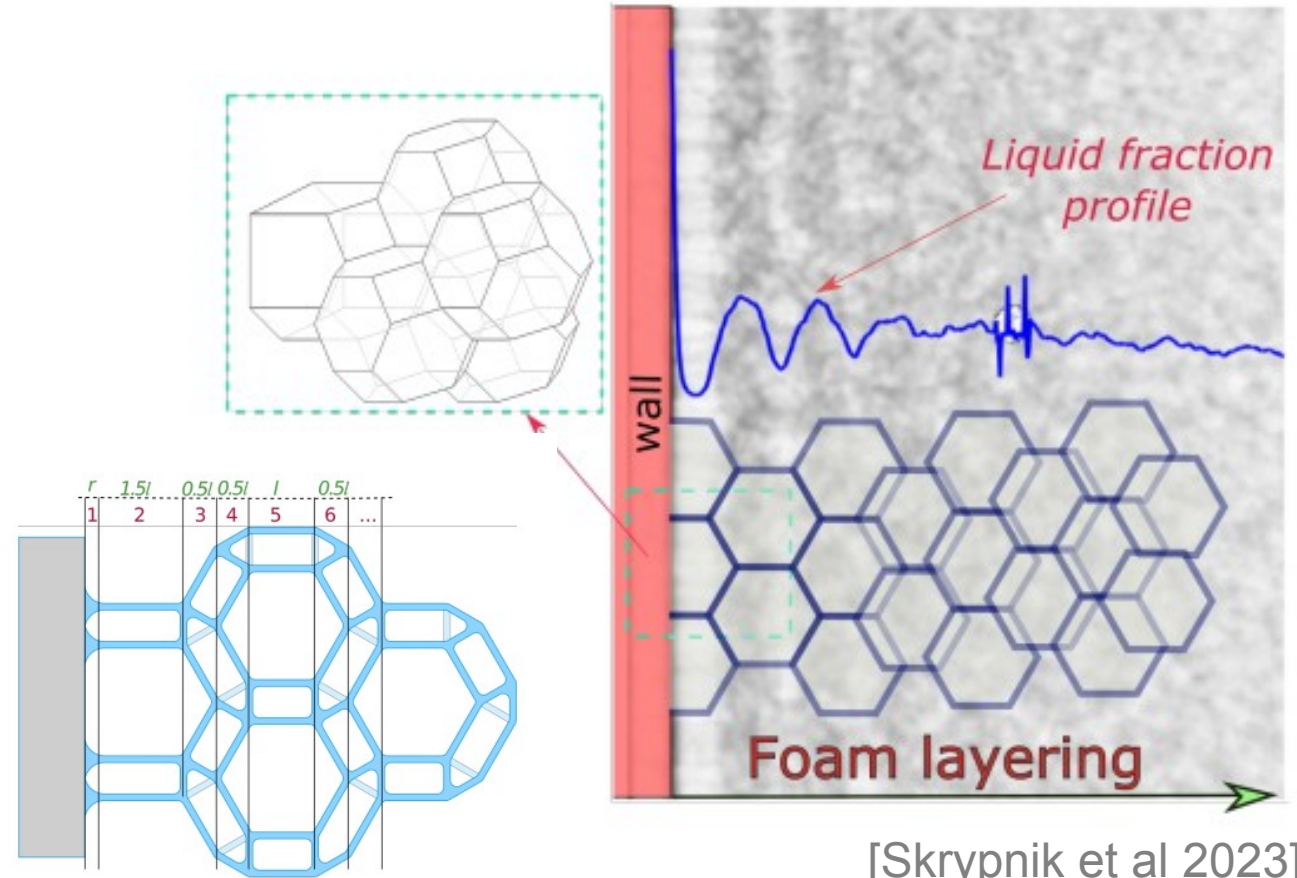
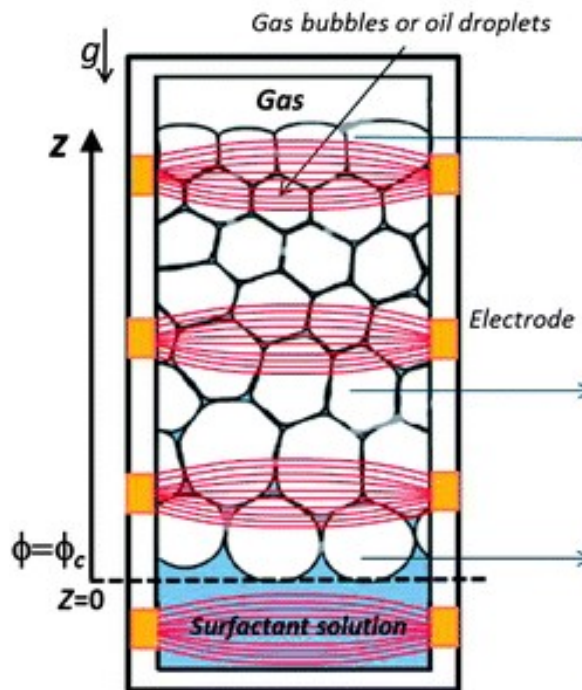
$$R \sim \frac{1}{\sigma} \frac{L}{A}$$



[Maestro et al. SM 2013]

Liquid fraction $\leftrightarrow$ resistivity

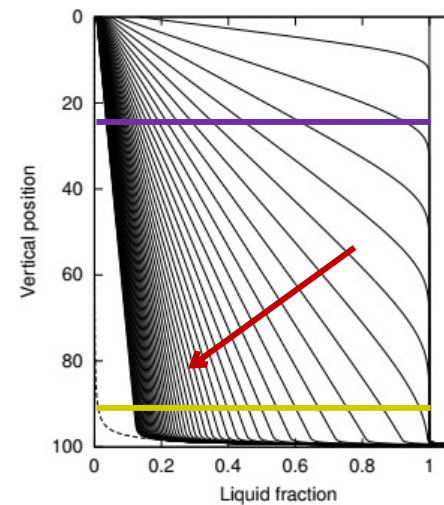
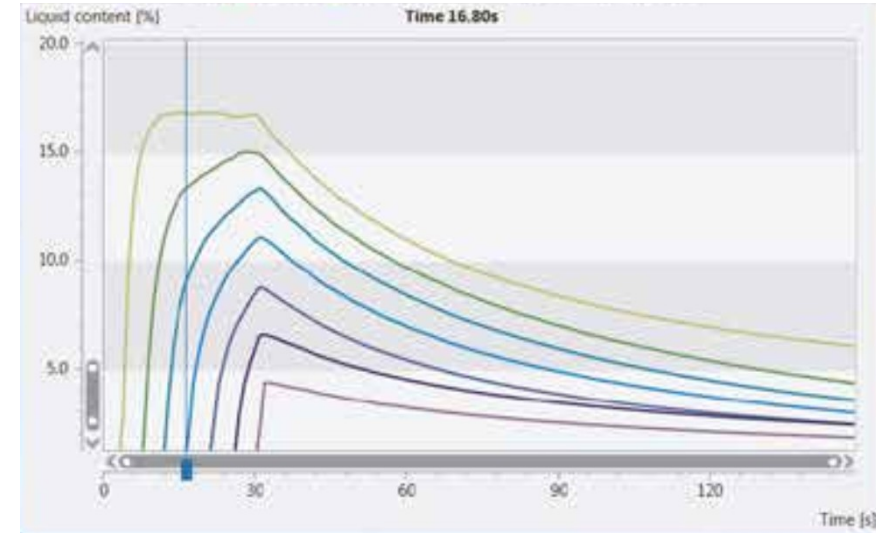
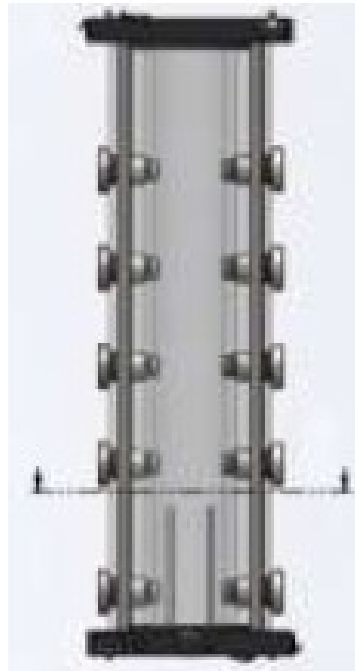
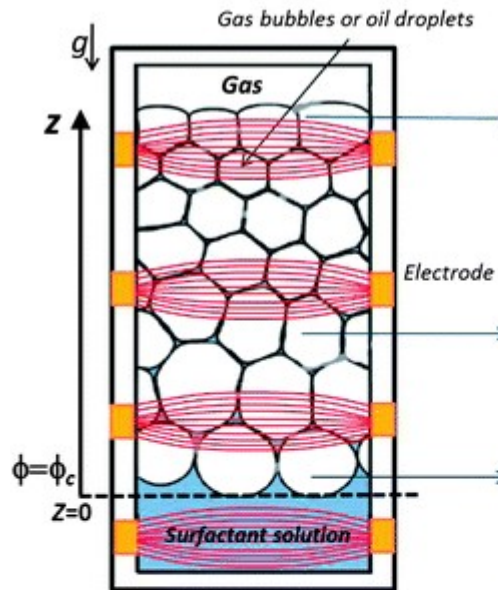
- Liquid fraction layers at wall
- Influence of electric conductivity???



[Skrypnik et al 2023]

Liquid fraction $\leftrightarrow$ resistivity

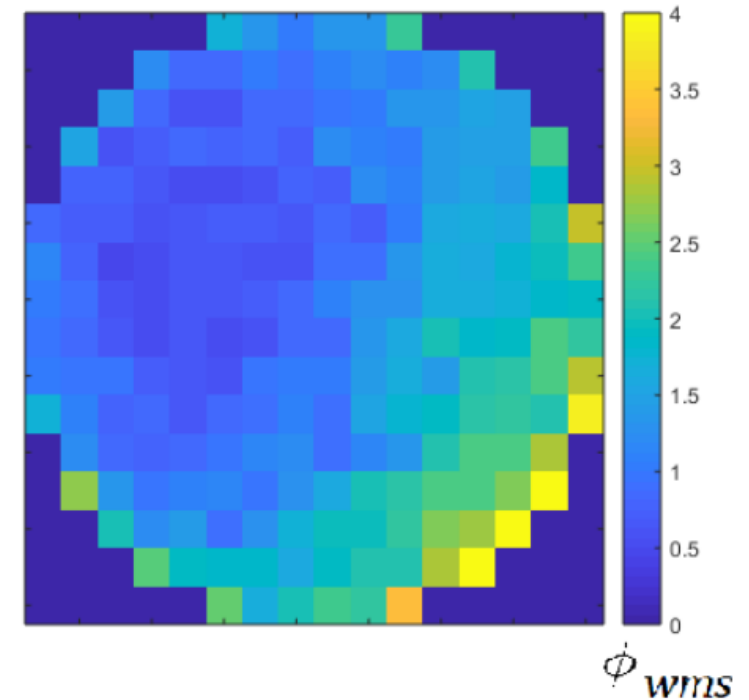
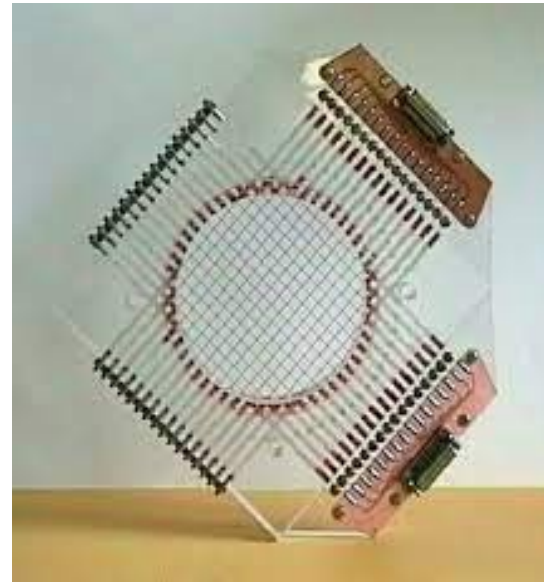
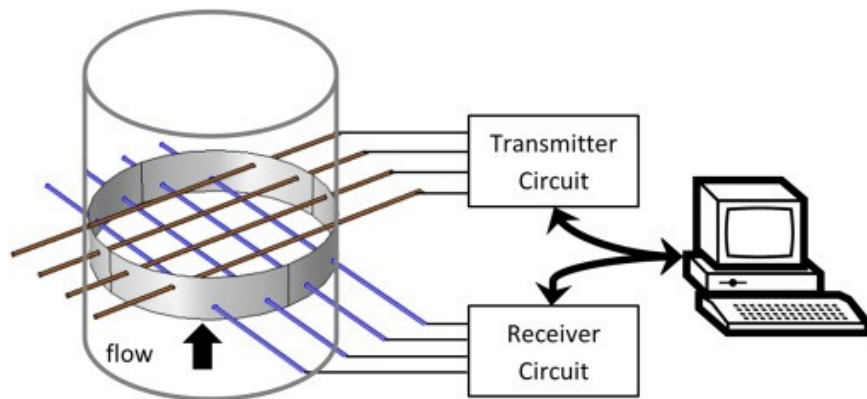
- Commercialized for liquid profile measurement
- Liquid profile yields drainage resistance



[Hoang et al. 2020]

Wire-Mesh sensor

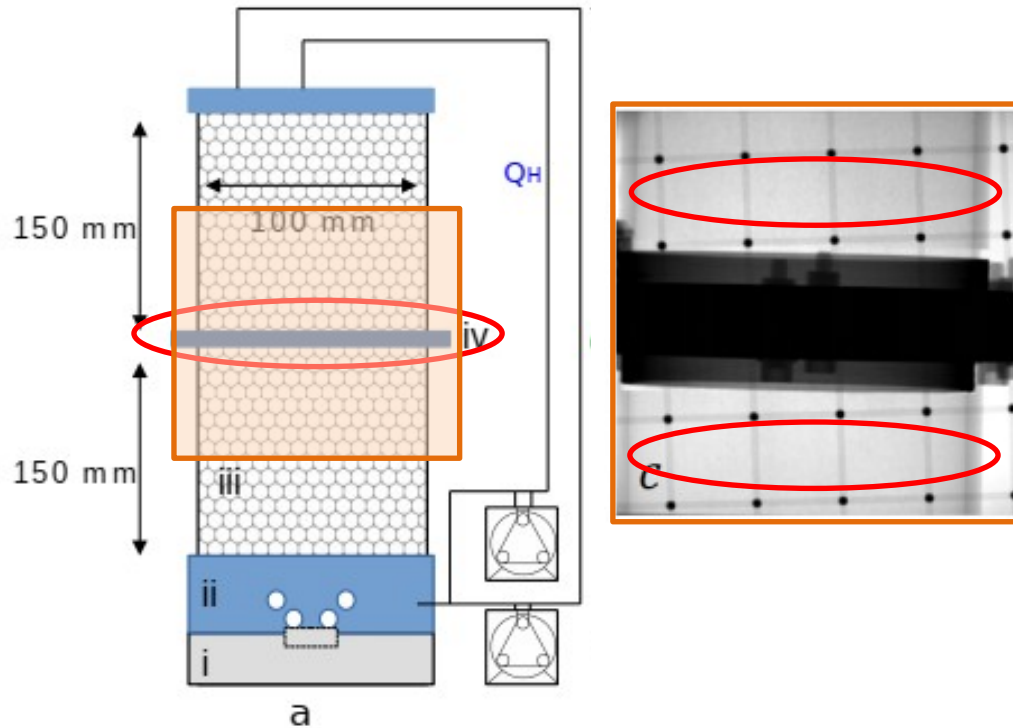
- Measurement at crossing points between two arrays of wires
- Current injection in one electrode pair $\rightarrow$ el. Potential on all electrodes



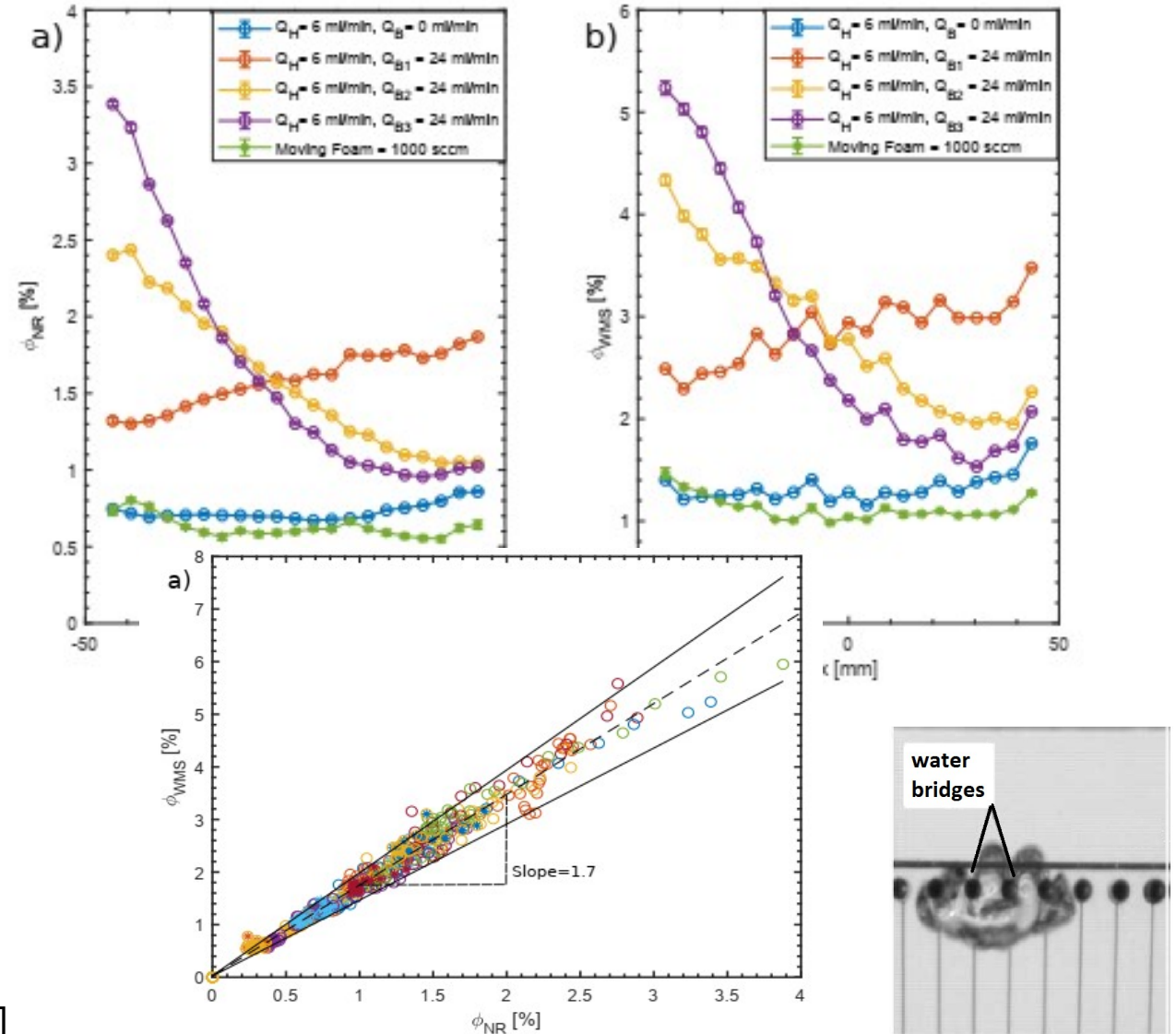
[Ziauddin et al. 2022]

Calibration of Wire-Mesh

- Wire-Mesh sensor for liquid fraction in foam

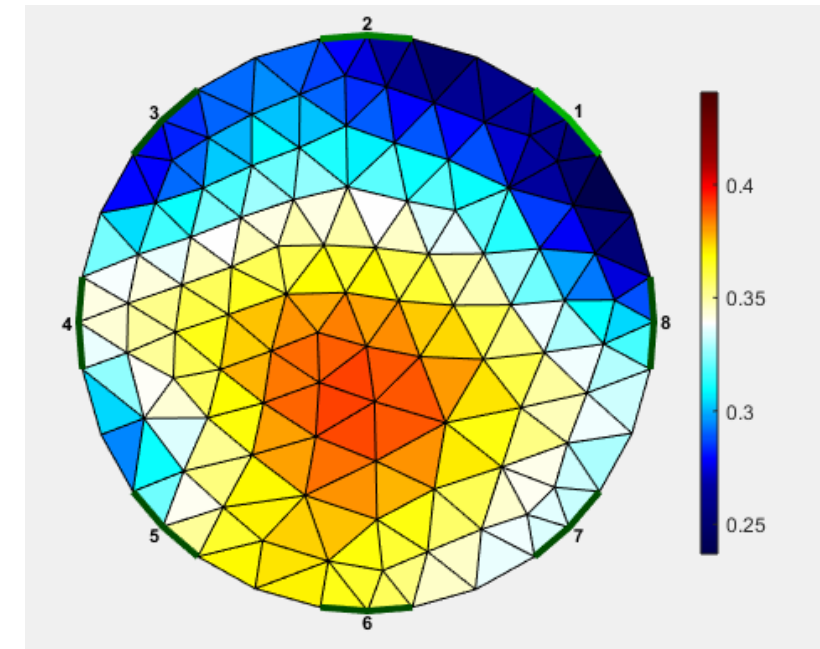
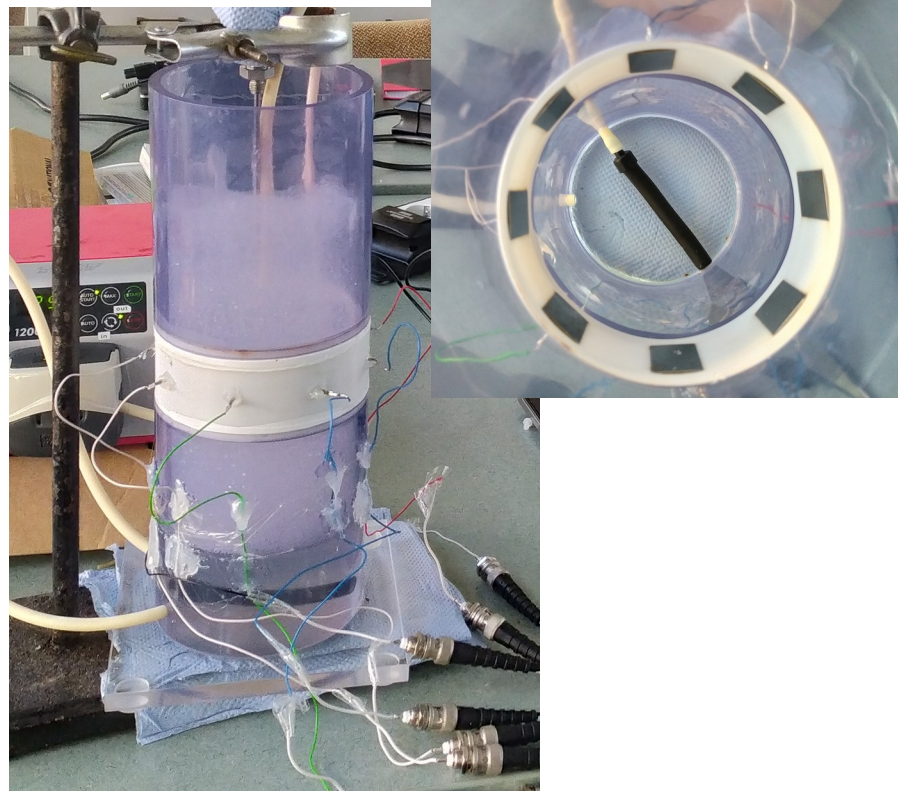
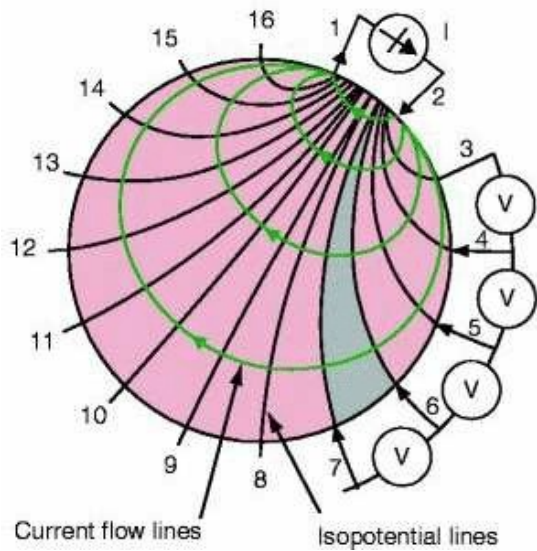


[Ziauddin et al., Journal of Physics: Condensed Matter , 2022]



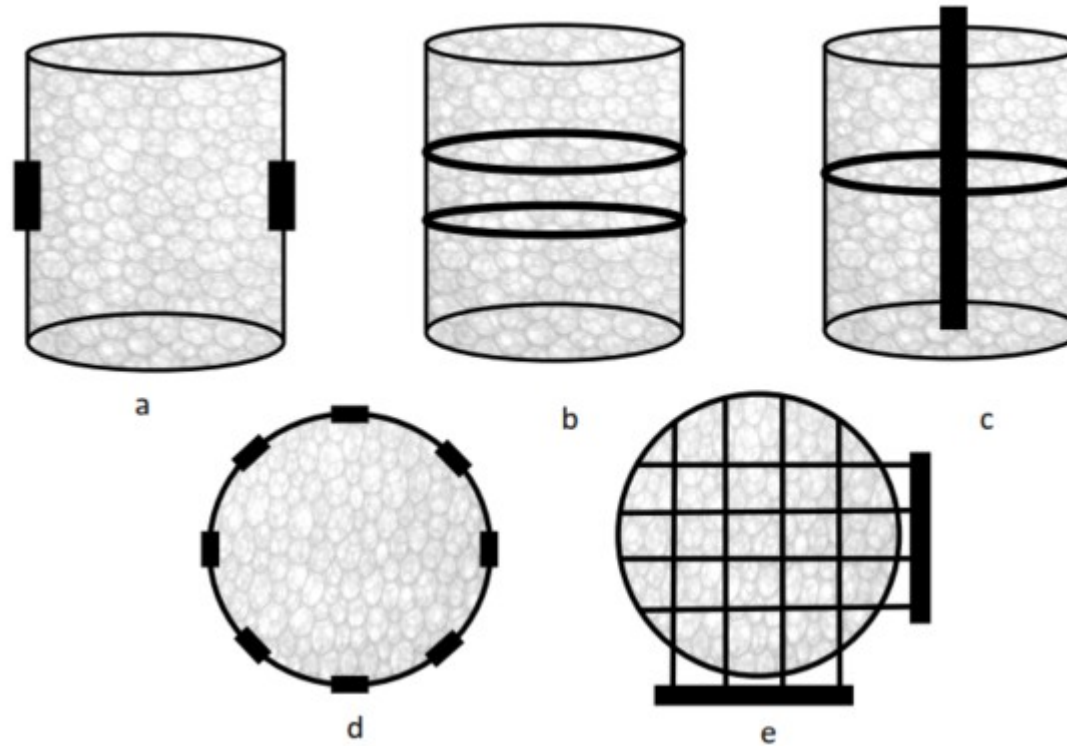
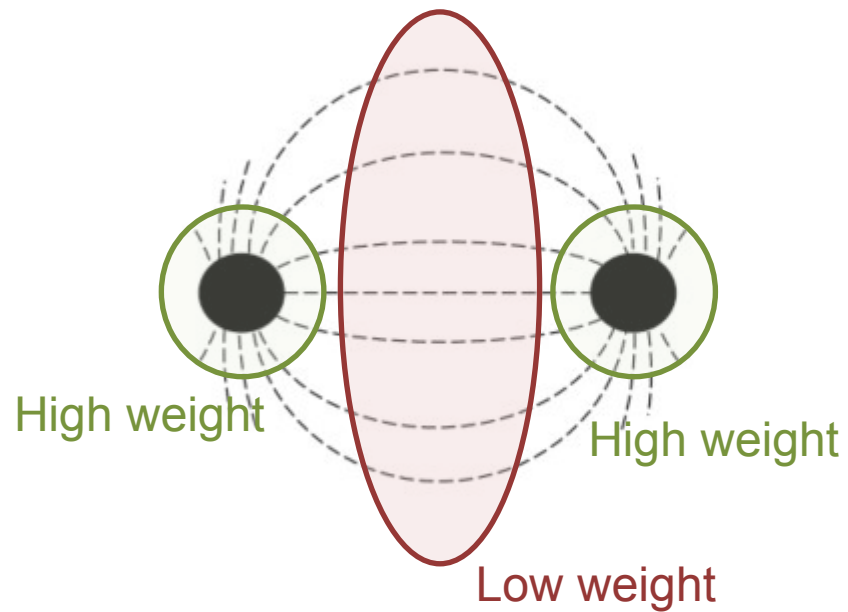
Electric resistance tomography

- Set of electrodes along the circumference
- Current injection in one electrode pair $\rightarrow$ el. Potential on all electrodes
- $\rightarrow$ Highly nonlinear, complex reconstruction process



Electrode Arrangements

- Many arrangements possible
- Focus on different zones, weighted according to local current density

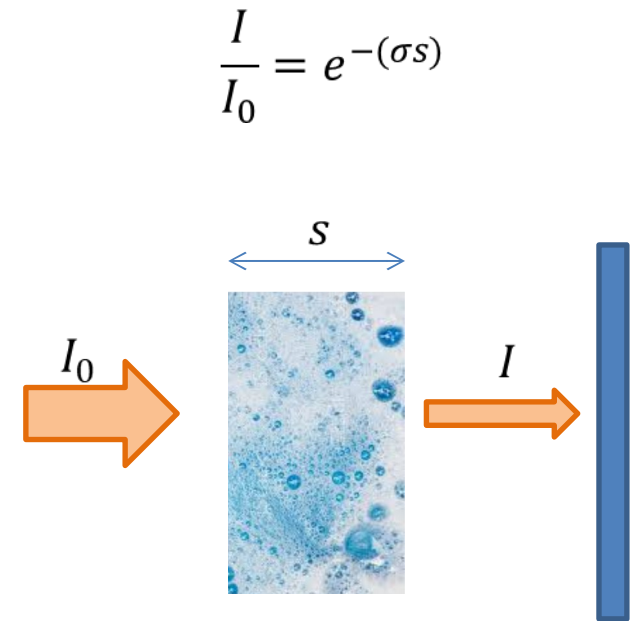
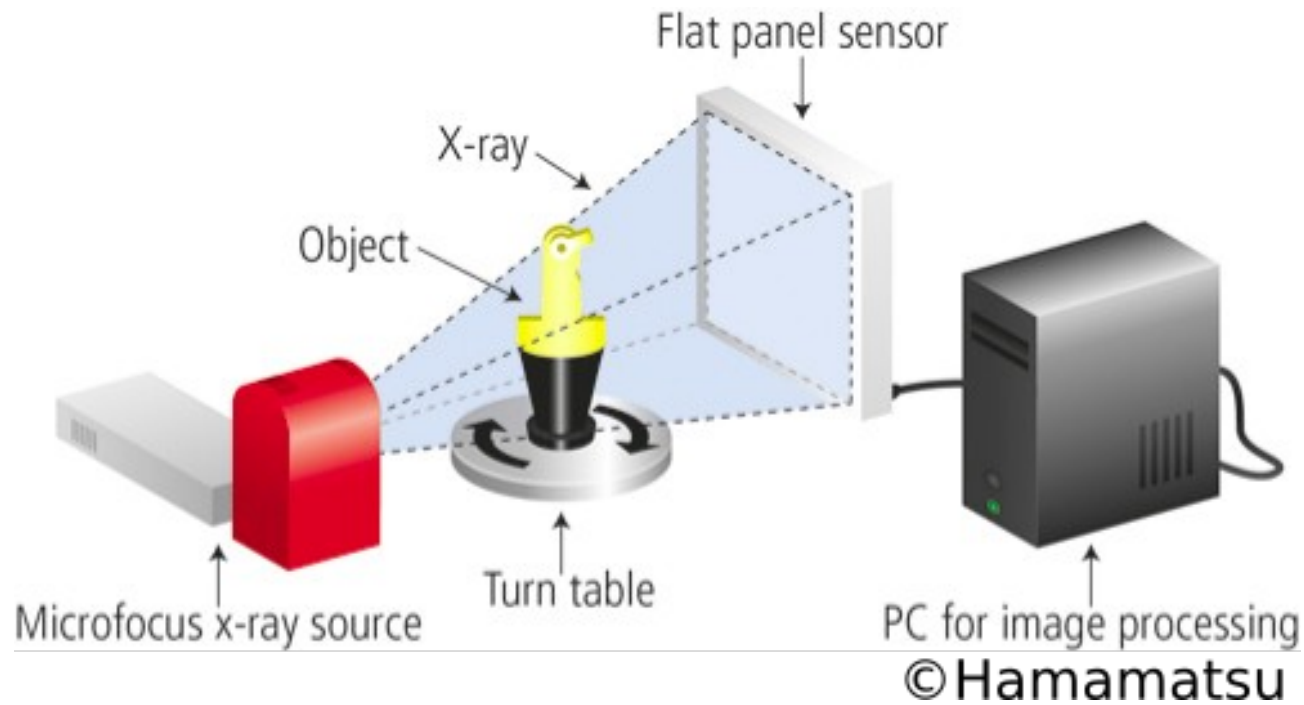


Structure

- Review lecture 9
- Measurement of foam rheology
- Optical measurement
- Resistivity measurement
- Radiography/Tomography
- Velocity measurement

Radiography

- Radiation source and detector
- Radiation is attenuated by measurement object → Beer-Lambert law



[Wang & Cilliers 1999]

Exercise: Attenuation

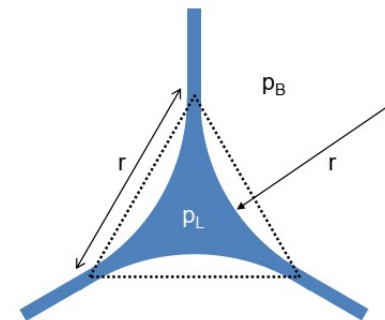
- How strong is the relative attenuation of different foam samples

- X-ray, 150 keV: "NIST Standard Reference Database 126" $\rightarrow \frac{\sigma}{\rho} = 0.0575 \frac{\text{cm}^2}{\text{g}}$ $\frac{I}{I_0} = e^{-(\sigma s)}$

- (a) Block of shaving foam, 10 cm thickness, $D_b=0.1\text{mm}$, $\varphi_l=10\%$

- (b) Plateau border (transversal oriented) of soap foam, $D_b=5\text{mm}$, $\varphi_l=0.1\%$

Decoration theorem: $\varphi_l \approx 0.72 \frac{r^2}{R_e^2}$



Exercise: Attenuation

- X-ray, 150 keV: "NIST Standard Reference Database 126" → $\frac{\sigma}{\rho} = 0.0575 \frac{\text{cm}^2}{\text{g}} \rightarrow \sigma = 0.0575 \frac{1}{\text{cm}}$

- (a) Shaving foam, 10 cm thickness, $D_b=0.1\text{mm}$, $\varphi_l=10\%$

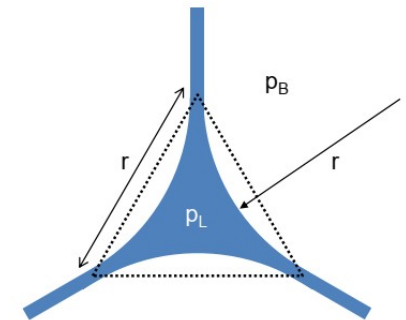
$$\frac{I}{I_0} = e^{-(\sigma s)} = e^{-(0.0575 / \text{cm} \cdot 10 \text{ cm} \cdot 10\%)} = 94.4\% \quad \frac{\Delta I}{I_0} = 5.6\%$$

- (b) Plateau border (transversal oriented) of bath tub foam, $D_b=5\text{mm}$, $\varphi_l=0.1\%$

$$r = R_e \sqrt{\frac{\varphi_l}{0.72}} \approx 0.093 \text{ mm}$$

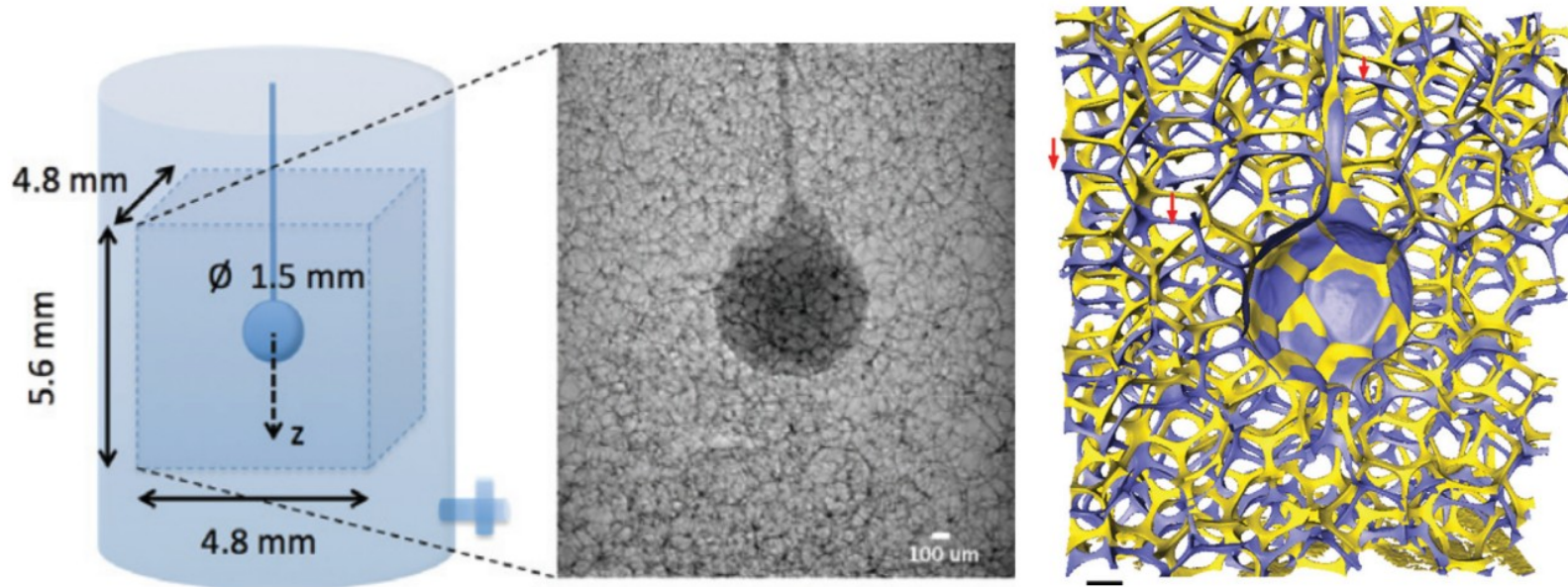
$$\text{Decoration theorem: } \varphi_l \approx 0.72 \frac{r^2}{R_e^2}$$

$$\frac{I}{I_0} = e^{-(\sigma r)} = e^{-(0.0575 / \text{cm} \cdot 0.0093 \text{ cm})} = 99.946\% \quad \frac{\Delta I}{I_0} = 0.054\%$$



X-ray tomography

- High-Intensity radiation from synchrotron source -> better contrast
- Very precise measurement requires long exposure
- → Small measurement volume, small velocities

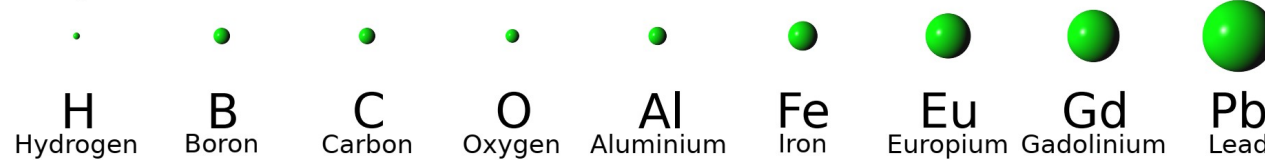


[Raufaste et al. EPL 2015]

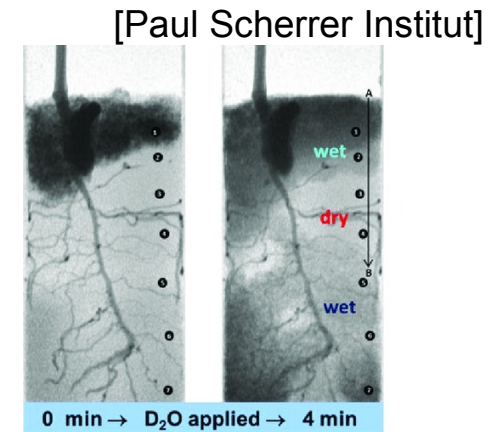
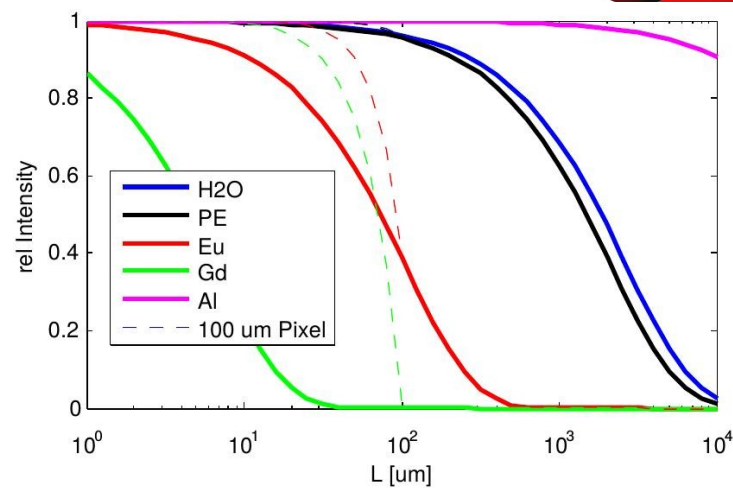
Neutron radiography

- Neutron sources with sufficient neutron flux are rare
- Unique attenuation properties

X-ray (150kV)



Neutrons (25meV)



Exercise: Attenuation

- X-ray, 150 keV: "NIST Standard Reference Database 126" → $\frac{\sigma}{\rho} = 0.0575 \frac{\text{cm}^2}{\text{g}} \rightarrow \sigma = 0.0575 \frac{1}{\text{cm}}$
 $\sigma_{\text{neutron}} = 3.74 \frac{1}{\text{cm}}$

- (a) Shaving foam, 10 cm thickness, $D_b=0.1\text{mm}$, $\varphi_l=10\%$

$$\frac{I}{I_0} = e^{-(\sigma s)} = e^{-(0.0575 / \text{cm} \cdot 10 \text{ cm} \cdot 10\%)} = 94.4\%$$

$$\frac{\Delta I}{I_0} = 5.6\%$$

$$\frac{\Delta I}{I_0} = 97.6\%$$

- (b) Plateau border (transversal oriented) of soap foam, $D_b=5\text{mm}$, $\varphi_l=0.1\%$

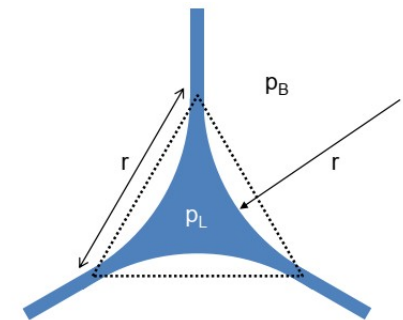
$$r = R_e \sqrt{\frac{\varphi_l}{0.72}} \approx 0.093 \text{ mm}$$

$$\text{Decoration theorem: } \varphi_l \approx 0.72 \frac{r^2}{R_e^2}$$

$$\frac{I}{I_0} = e^{-(\sigma r)} = e^{-(0.0575 / \text{cm} \cdot 0.0093 \text{ cm})} = 99.946\%$$

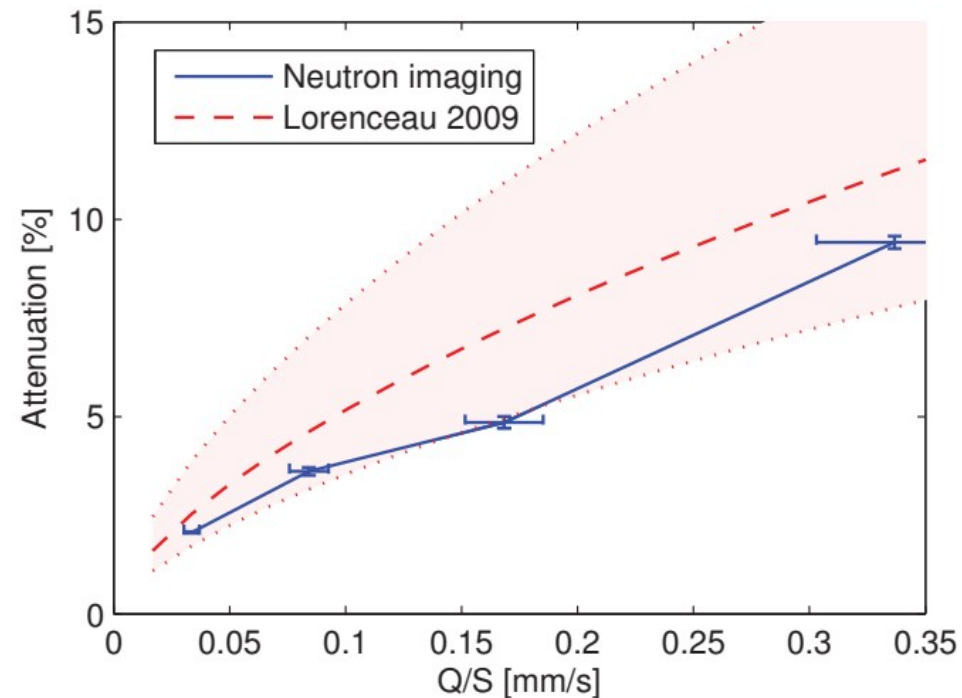
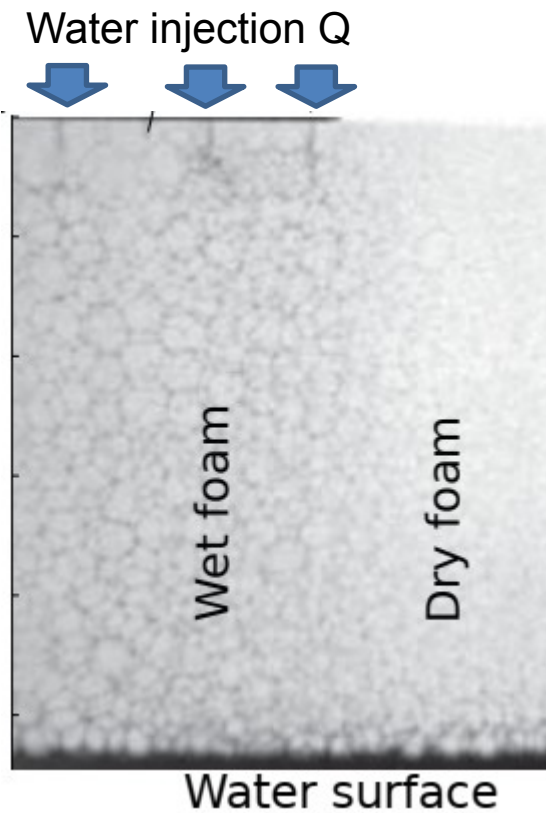
$$\frac{\Delta I}{I_0} = 0.054\%$$

$$\frac{\Delta I}{I_0} = 3.5\%$$



Neutron radiography of liquid fraction

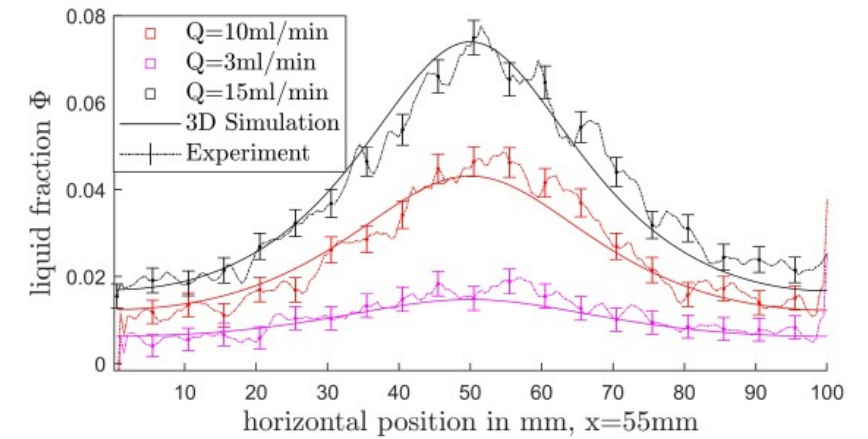
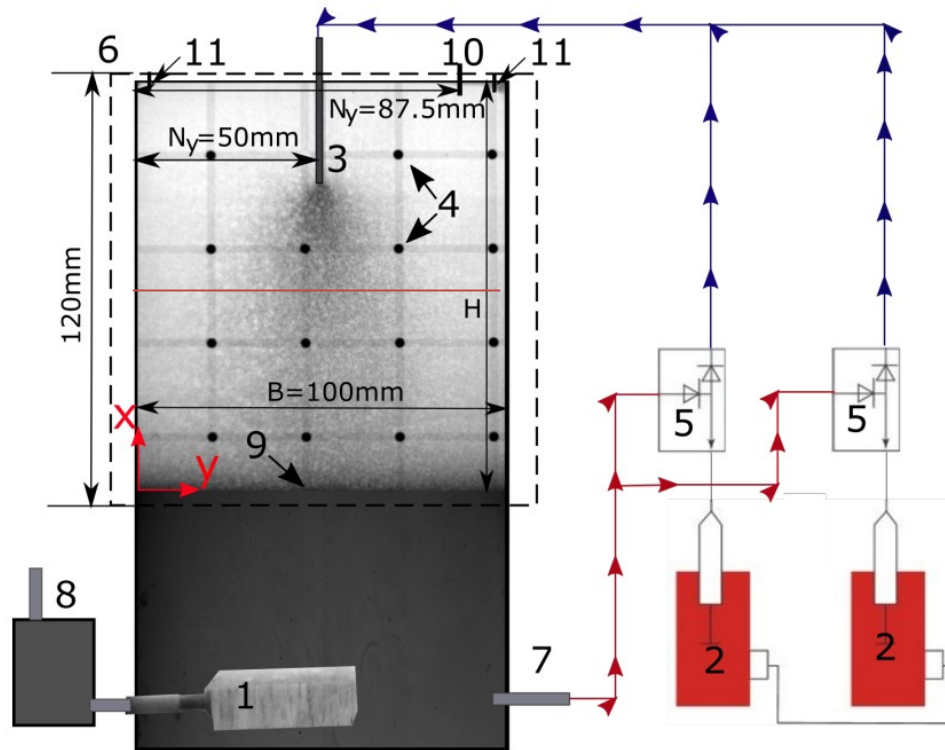
- Forced drainage setup in neutron radiography
- Plateau borders visible
- Liquid fraction can be quantified



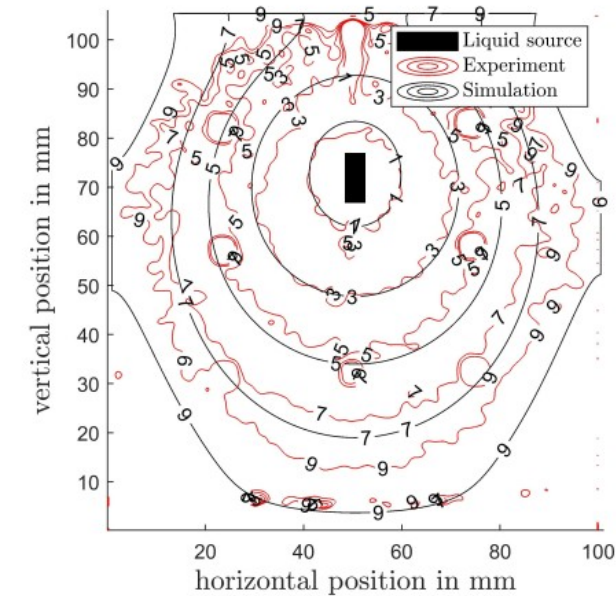
[Heitkam et al. 2018]

Neutron radiography of liquid fraction

- Measurement of liquid fraction distribution
- Unsteady, 2-dimensional distribution investigated
- Comparison to drainage equation



(a) Front positions with $Q = 6\text{ml/min}$



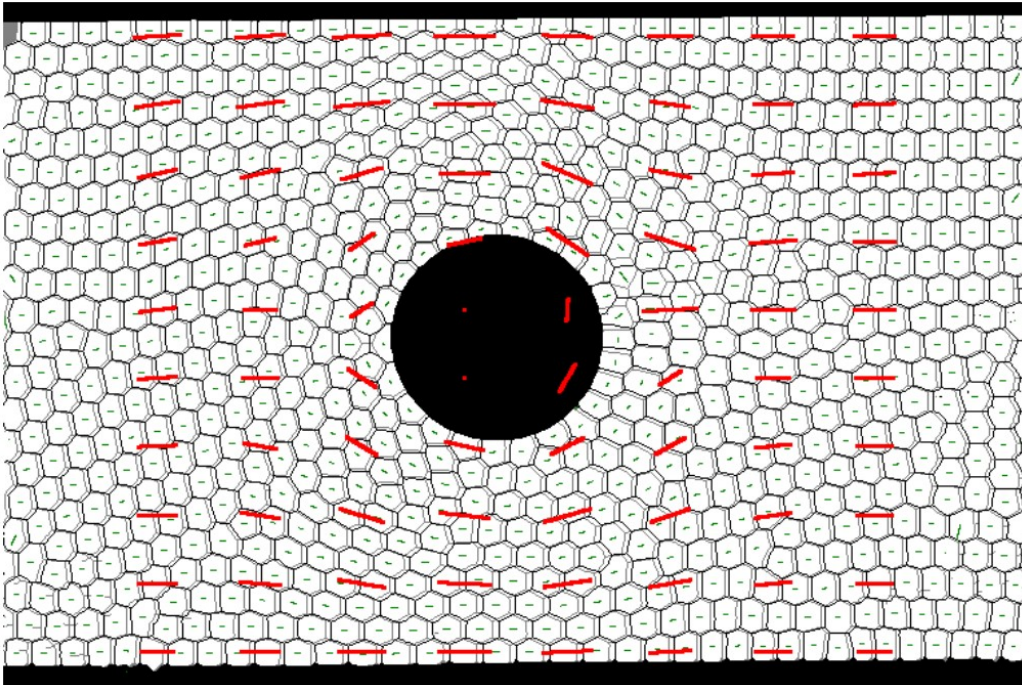
[Walzel et al. 2022]

Structure

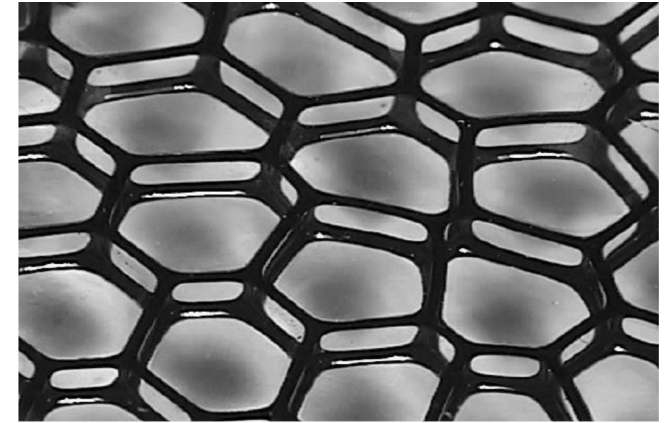
- Review lecture 9
- Measurement of foam rheology
- Optical measurement
- Resistivity measurement
- Radiography/Tomography
- Velocity measurement

Optical velocity measurement

- Consecutive images with defined time shift Δt analysed
- A: Segmentation $\rightarrow$ shift of each bubble center position $\Delta x_i \rightarrow$ velocity $u_i = \Delta x_i / \Delta t$



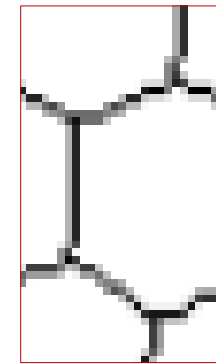
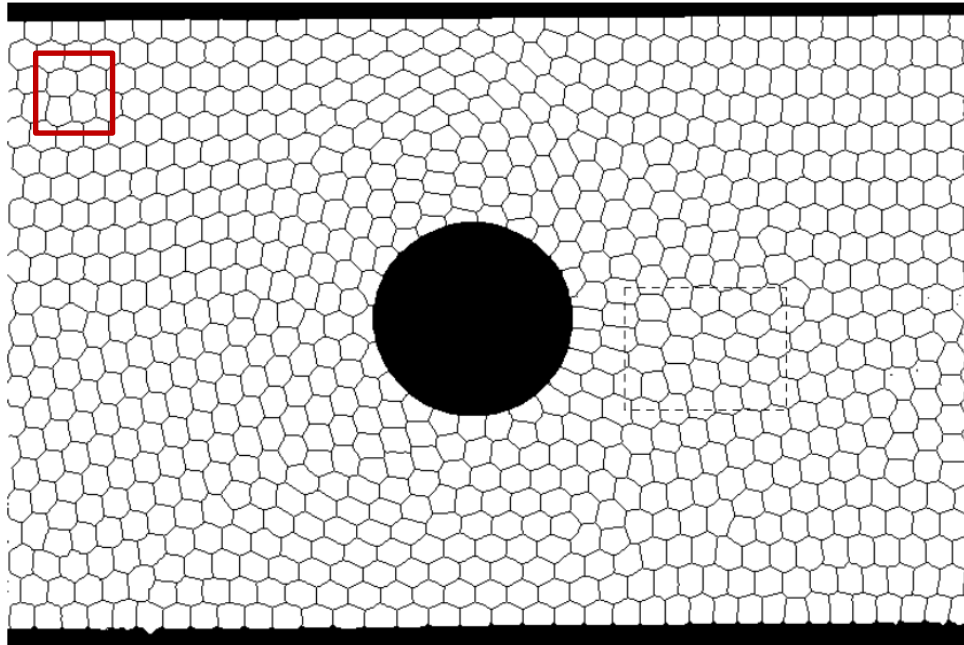
[B. Dollet]



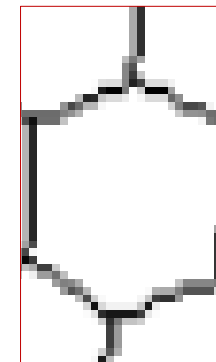
(c) Eric Janiaud

Optical velocity measurement

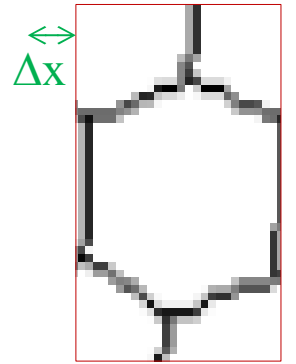
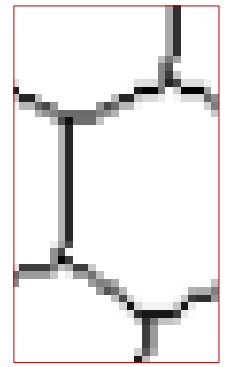
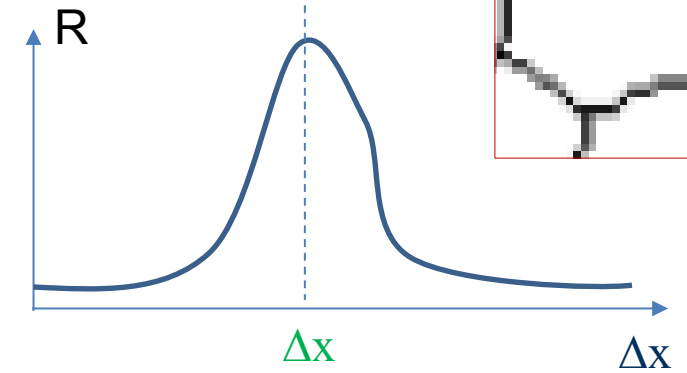
- Consecutive images with defined time shift Δt analysed
- B: PIV-Algorithm $\rightarrow$ **interrogation windows**
- $\rightarrow$ cross-correlation of consecutive images yields $\Delta x(x,y)$, $\rightarrow$ velocity $u = \Delta x / \Delta t$



t_1

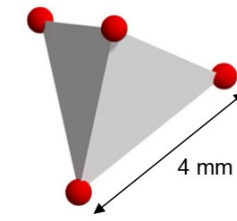


$t_1 + \Delta t$

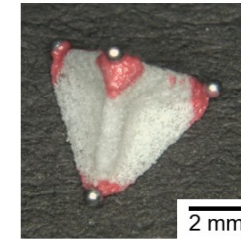


Xray-Particle Tracking Velocimetry

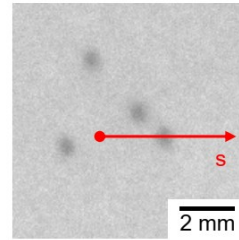
- 3D-printed particles with metal content, sticking in foam
- Tracking of particles yields velocity and vorticity of the foam flow



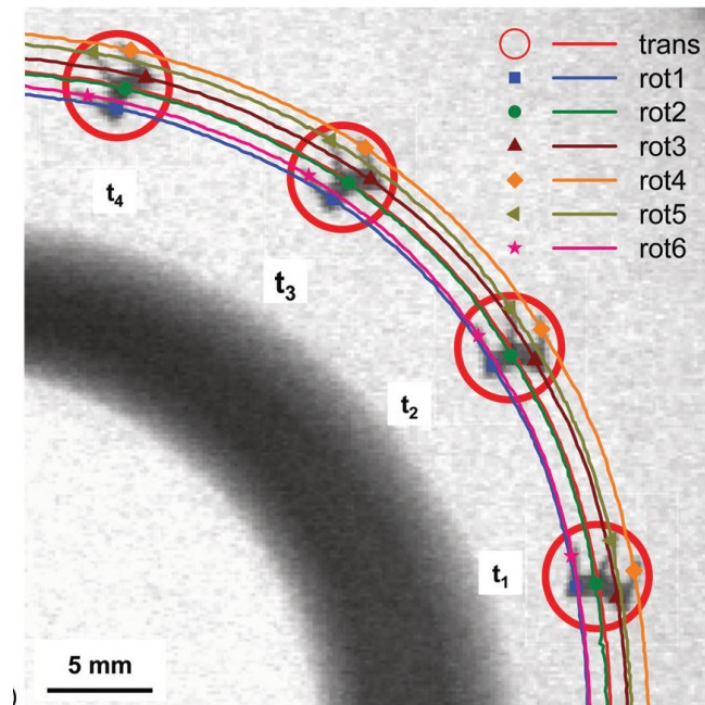
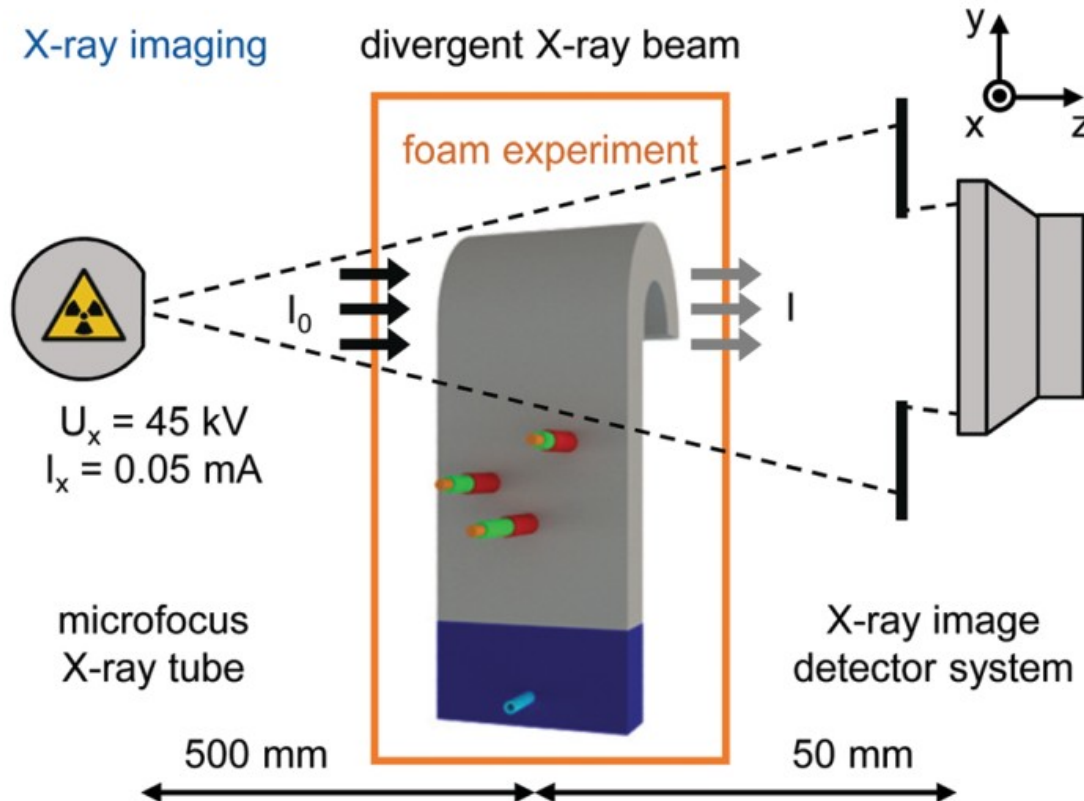
(a) CAD concept



(b) close-up image



(c) X-ray image of tracer T1



[Lappan et al. 2020]

Velocity U

$$\mathbf{x}^i = \frac{1}{4} \sum_{j=1}^4 \mathbf{x}_j^i$$

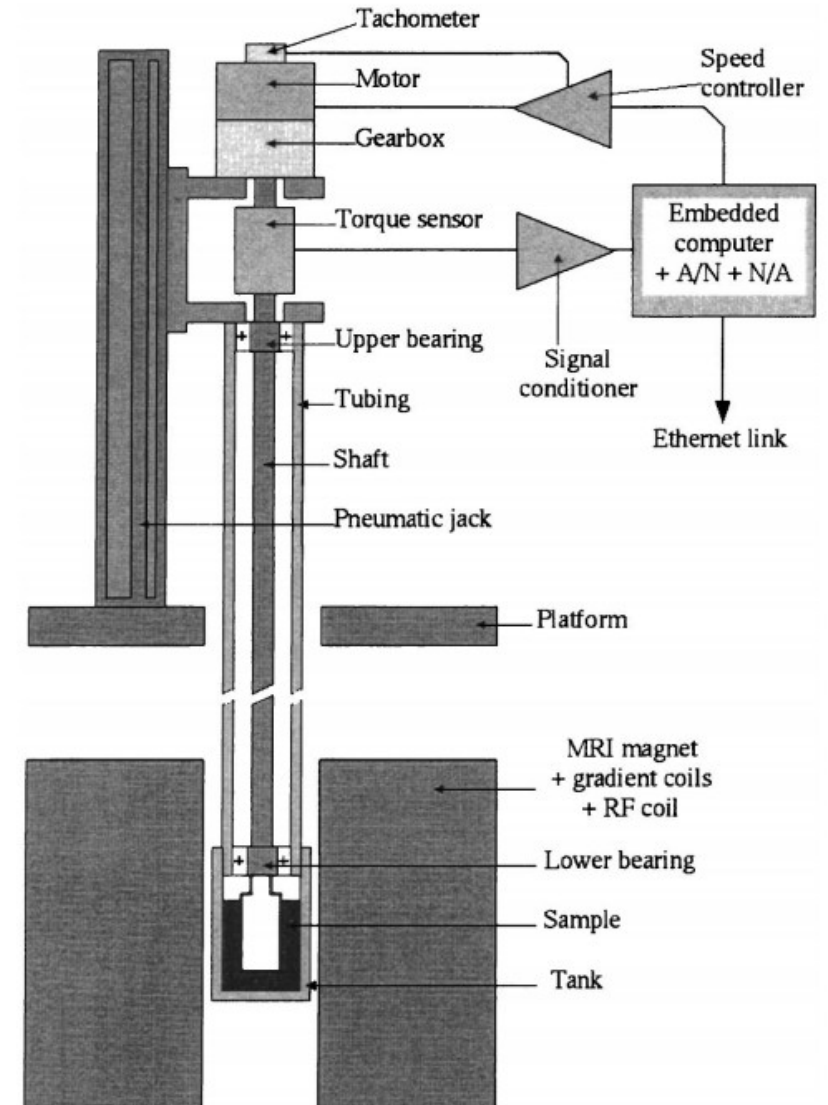
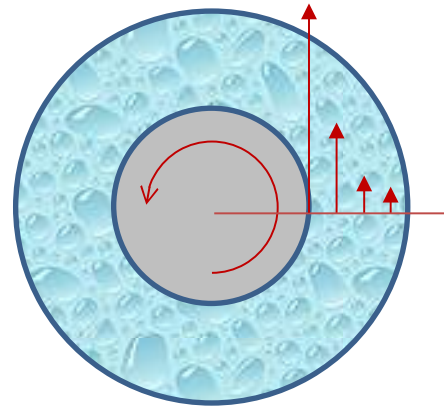
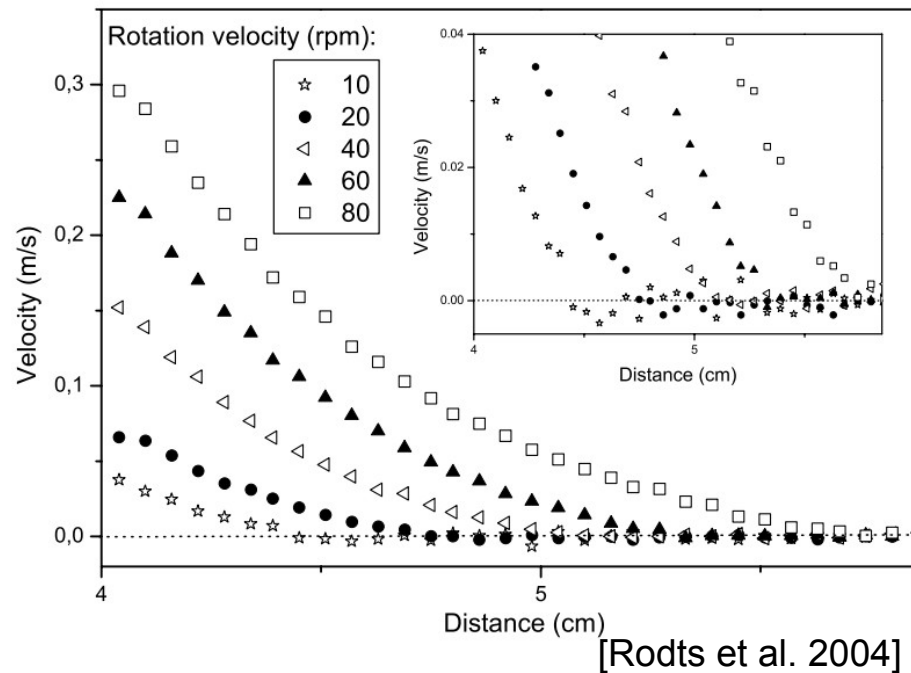
$$\mathbf{U}^i = \frac{\mathbf{x}^{i+1} - \mathbf{x}^{i-1}}{2 \cdot \Delta t}$$

Rotation ω

$$\omega_g^i = \frac{\alpha_g^{i+1} - \alpha_g^{i-1}}{2 \cdot \Delta t}$$

Magnetic Resonance Imaging (MRI)

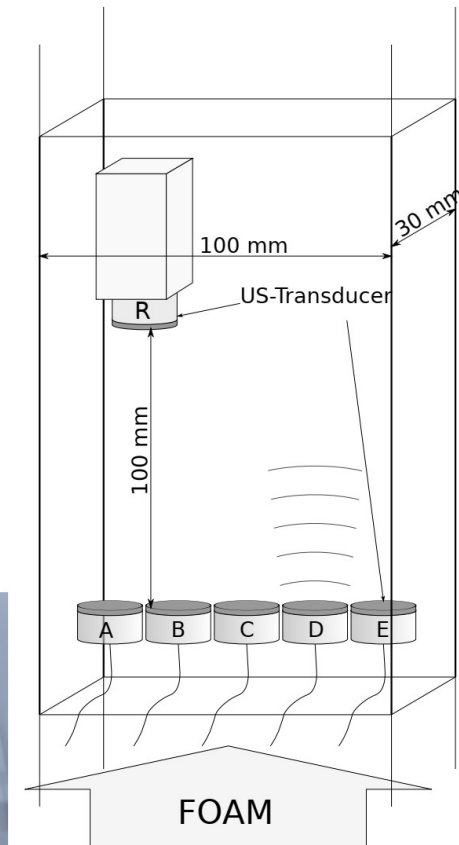
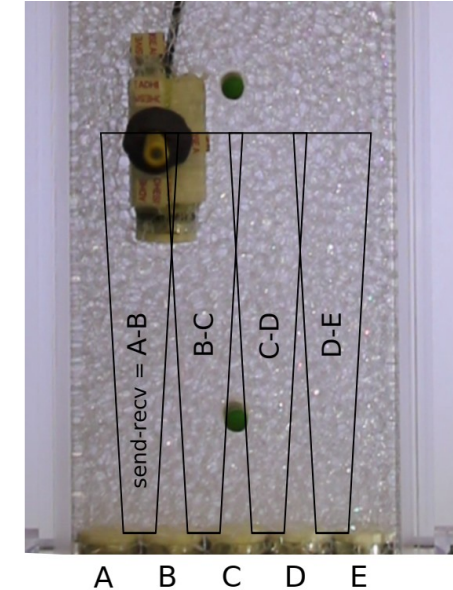
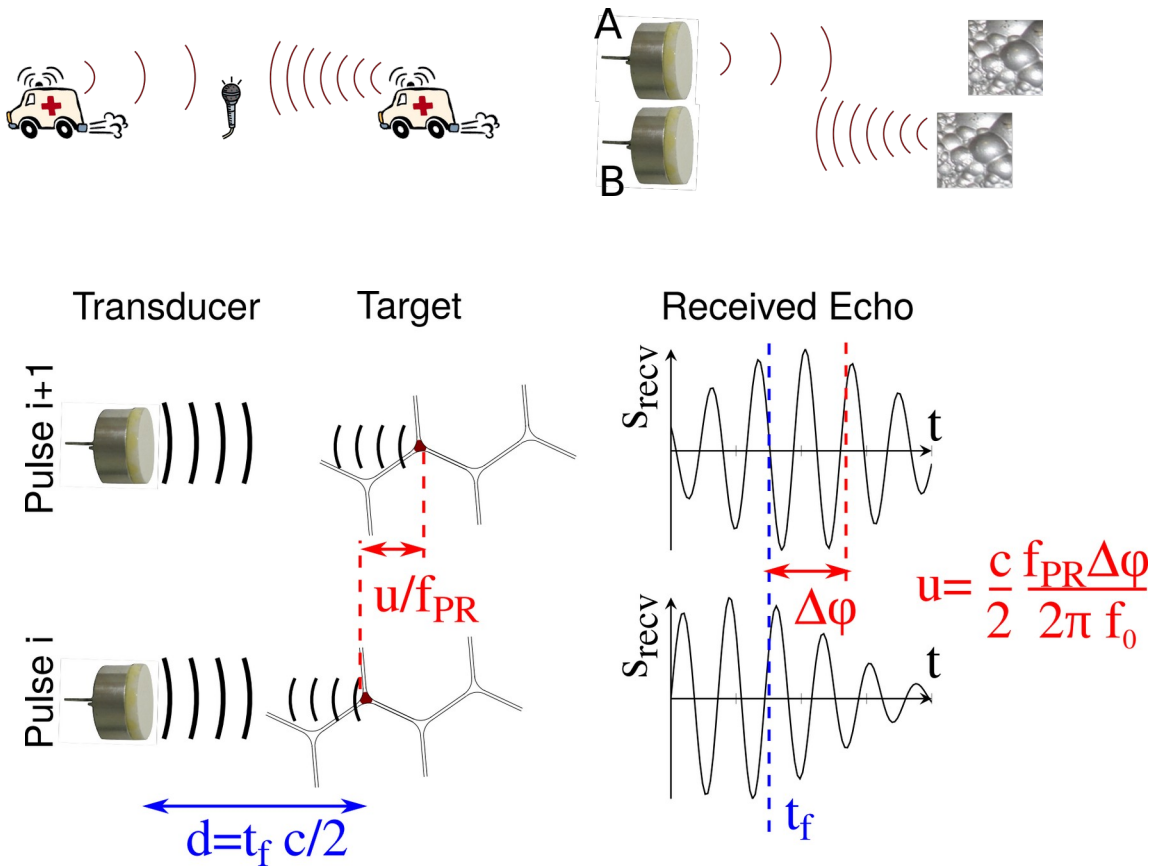
- Couette rheometer placed inside NMR, filled with shaving foam
- Velocity (of liquid phase) vs position measured by NMR
- 90 sec measurement time



[Raynaud & Moucheron 2002]

Ultrasound Doppler velocimetry

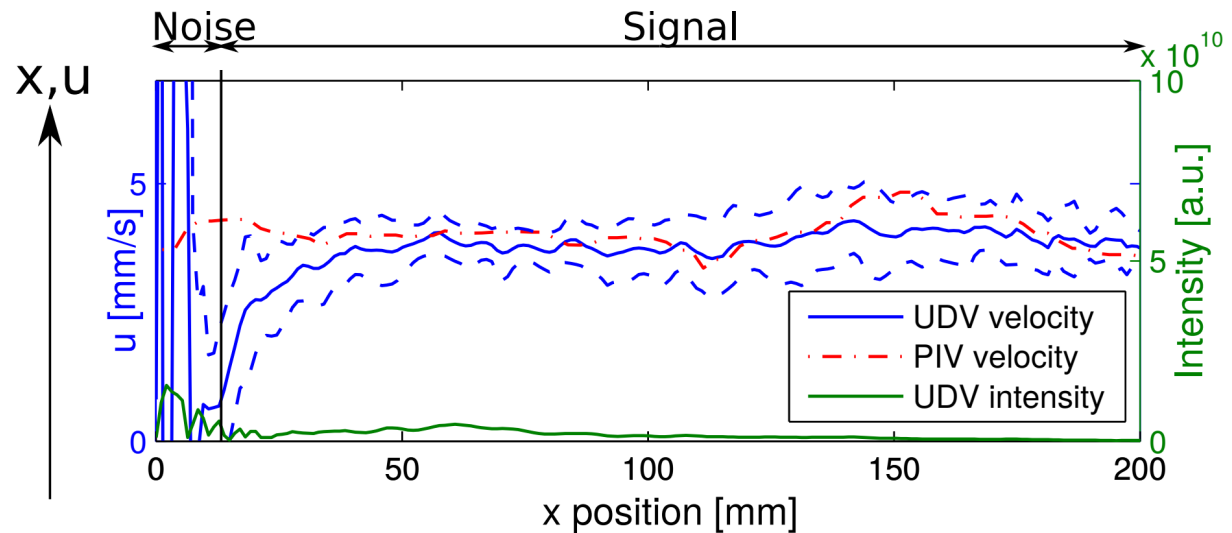
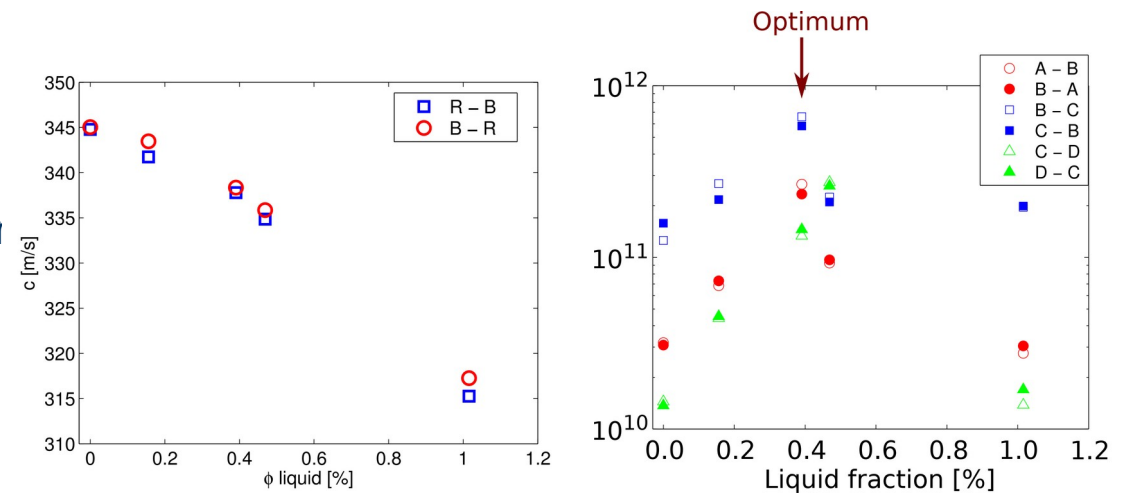
- Pulse reflection at moving target → Doppler frequency shift
- Time of flight yields longitudinal position



[Nauber et al. 2018]

Ultrasound Doppler velocimetry

- Liquid fraction influences speed of sound and attenuation
- Velocity measurement with 1fps and 10 % uncertainty



[Nauber et al. 2018]