1. Estimate the density of “electron gas” in metals assuming that each atom contributes \(Z\) electrons (usually 1 to 3), and the density of metals is about 1 g/cm\(^3\). Compare with the density of a classical gas at normal temperature and pressure.

2. Use the equation \(m \frac{d\nu}{dt} + \frac{mv}{\tau} = -eE\) to show that the complex Drude conductivity \(\sigma(\omega)\) is

\[
\sigma(\omega) = \sigma(0) \left[ \frac{1 + i\omega\tau}{1 + (\omega\tau)^2} \right],
\]

where \(\sigma(0) = ne^2\tau/m\).

\(\sigma(\omega)\) is defined as \(j(\omega) = \sigma(\omega)E(\omega), j(t) = \text{Re} \left( j(\omega)e^{-i\omega t} \right), E(t) = \text{Re} \left( E(\omega)e^{-i\omega t} \right)\).

3. Show that the chemical potential of a Fermi gas in two dimensions is given by

\[
\mu(T) = k_B T \ln[\exp(E_F/k_B T) - 1] = k_B T \ln[\exp(\pi n \hbar^2 / mk_B T) - 1],
\]

for \(n\) electrons per unit area. \(E_F = \pi n \hbar^2 / m\) is defined at \(T = 0\).

4. Consider the Dirac-Kronig-Penney model

\[
U(x) = U_0 \sum_n \delta(x - an).
\]

a) Find the equation for \(\epsilon_n(k)\).

b) Consider the limit \(\frac{mU_0 a}{\hbar^2} \gg 1\) and find the lowest energy band \(\epsilon_1(k)\).

c) Consider the effective mass approximation near the bottom of the band.

d) Consider semi-classical dynamics of electrons in the constant electric field (Bloch oscillations).