1. Probability conservation

Get the "Continuity equation for quantum mechanics"

\[ \frac{\partial |\psi(r)|^2}{\partial t} + \nabla \cdot \mathbf{f} = 0 , \]

and prove the conservation of probability flux density ("current")

\[ \mathbf{f} = \frac{i\hbar}{2m} [\psi(r)\nabla\psi^*(r) - \psi^*(r)\nabla\psi(r)] . \]

2. Potential step

![Potential Step Diagram]

Find \( T_{L\rightarrow R}(E) \) and \( R_{L\rightarrow R}(E) \), and compare with \( T_{R\rightarrow L}(E) \) and \( R_{R\rightarrow L}(E) \).

3. Rectangular barrier

![Rectangular Barrier Diagram]

Show that the probability for an electron to cross the rectangular barrier (see the picture) is

\[ T(E) = \frac{4E(V_0 - E)}{4E(V_0 - E) + V_0^2 \sinh^2 \left( \frac{\sqrt{2m(V_0 - E)} L}{\hbar} \right)} , \quad 0 < E < V_0 , \]

\[ T(E) = \frac{4E(E - V_0)}{4E(E - V_0) + V_0^2 \sin^2 \left( \frac{\sqrt{2m(E - V_0)} L}{\hbar} \right)} , \quad E > V_0 . \]
4. **Symmetry of the transmission**

Prove the relation $T_{R\rightarrow L}(E_z) = T_{L\rightarrow R}(E_z)$ for time-reversal systems, using the scattering matrix method.

5. **$\delta$-potential**

Find $T(E)$ and $R(E)$ for the $\delta$-potential $U(z) = \alpha \delta(z)$.

6. **Breit-Wigner formula**

Consider a double-barrier structure with two $\delta$-potentials

$$U(z) = \alpha \delta(z) + \alpha \delta(z - L),$$

$L$ is the distance between scatterers. Show that close to the resonance energies $E_n$ the transmission coefficient has a *Lorentzian* form

$$T(E) \approx \frac{\Gamma_n^2}{(E - E_n)^2 + \Gamma_n^2}.$$