1. **2D step junction**

Find the scattering matrix $\hat{S}$ for a 2D step junction with $W_R = 2W_L$.

2. **Channel mixing in adiabatic junctions**

Estimate the corrections $\Delta_n$ in the equation

$$\left\{ -\frac{\hbar^2}{2m} \frac{\partial^2}{\partial z^2} + E_n(z) \right\} \psi_{nE}(z) = E\psi_{nE}(z) + \Delta_n,$$

leading to mixing of the channels with different $n$.

3. **Landauer formula from scattering states**

Use the current density expression

$$j = \frac{i e \hbar}{2m} \left[ \psi(r) \nabla \psi^*(r) - \psi^*(r) \nabla \psi(r) \right],$$

and the expression for the scattering state

$$\psi_{LnE}(r) = \sum_m t_{mn}(E) \sqrt{\frac{2}{\pi \hbar \nu_{Rm}}} \phi_{Rm}(x, y) e^{ik_m z}, \quad z > z_R,$$

to get the multichannel Landauer formula.

4. **$G_{sp}$ and $G'_{sp}$ conductance matrices**

The current can be written through conductance matrices $G_{sp}$ or $G'_{sp}$

$$I_s = \sum_p \left[ G_{ps} V_s - G_{sp} V_p \right] = \sum_p G'_{sp} V_p,$$

Prove the following relation at $T = 0$

$$G'_{sp} = \frac{e^2}{\hbar} N_s \delta_{sp} - G_{sp}.$$
5. 4-terminal junction

Consider the 4-terminal junction and assume that electrodes 3 and 4 are weakly coupled voltage probes. Prove the relation

\[ V_{34} = \frac{T_{31}T_{42} - T_{32}T_{41}}{(T_{31} + T_{32})(T_{41} + T_{42})} V_{12}. \]