

Exercises Algebraic Geometry

1. Let R be a Noetherian ring, and x an indeterminate. Prove that $\dim R[x, x^{-1}] = \dim R + 1$.
2. Let R be an arbitrary ring, show that $\dim R = \dim R_{\text{red}}$.
3. Let k be a field, and let A be a local k -algebra of finite type. Prove that $\text{Spec}(A)$ consists of a single point, and that A is finite-dimensional as a k -vector space. Conclude that A is a local Artin ring, and that $\kappa(A)$ is a finite field extension of k .