

$$i G_c(r, r') \approx \uparrow \theta(u^2)$$

$$i G_c^0(r, r') + \frac{(-i)^{\frac{1}{2}} \int d1 \int d2 U(1-2) \langle 0 | T(\psi_1^+ \psi_2^+ \psi_2 \psi_1 \psi_r \psi_{r'}) | 0 \rangle}{1 + (-i)^{\frac{1}{2}} \int d1 \int d2 U(1-2) \langle 0 | T(\psi_1^+ \psi_2^+ \psi_2 \psi_1) | 0 \rangle}$$

$$(i) : \langle 0 | T(\psi_1^+ \psi_2^+ \psi_2 \psi_1 \psi_r \psi_{r'}) | 0 \rangle \stackrel{\text{Wick}}{=} \overbrace{\psi_1^+ \psi_1} \overbrace{\psi_2^+ \psi_2} \overbrace{\psi_r \psi_{r'}} + (-1) \overbrace{\psi_1^+ \psi_2} \overbrace{\psi_2^+ \psi_1} \overbrace{\psi_r \psi_{r'}} +$$

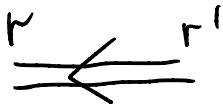
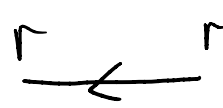
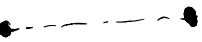
$$(-1) \overbrace{\psi_1^+ \psi_r} \overbrace{\psi_1 \psi_2^+} \overbrace{\psi_2 \psi_{r'}} + \overbrace{\psi_1^+ \psi_2} \overbrace{\psi_2^+ \psi_r} \overbrace{\psi_1 \psi_{r'}} +$$

$$(-1) \overbrace{\psi_1^+ \psi_r} \overbrace{\psi_2^+ \psi_2} \overbrace{\psi_1 \psi_{r'}} + (-1) \overbrace{\psi_1^+ \psi_1} \overbrace{\psi_2^+ \psi_r} \overbrace{\psi_2 \psi_{r'}}.$$

$$(ii) : \langle 0 | T(\psi_1^+ \psi_2^+ \psi_2 \psi_1) | 0 \rangle \stackrel{\text{Wick}}{=} \overbrace{\psi_1^+ \psi_1} \overbrace{\psi_2^+ \psi_2} + (-1) \overbrace{\psi_1^+ \psi_2} \overbrace{\psi_2^+ \psi_1}$$

(vacuum amplitude)

- Elementary building blocks of Feynman diagrams for our model:

	$i G_c(r, r')$	(Full propagator (causal GF))
	$i G_c^0(r, r')$	(Free causal GF)
	$-i U(1-2)$	(Interaction potential)
	$\langle n \rangle_0 = \overline{\Psi_1^+ \Psi_1}$	(Free density)

(i) Graphical representation of $-\frac{i}{2} \int d1 \int d2 U(1-2) \langle 0 | T(\Psi_1^+ \Psi_2^+ \Psi_2 \Psi_1 \Psi_r \Psi_{r'}) | 0 \rangle$

$$= \left(\text{Diagram 1} \right) - \left(\text{Diagram 2} \right) +$$

Diagram 1: Two circles, each with a dot on its right side labeled 1 and 2 respectively. A dashed line connects the two dots. To the right of the circles is a horizontal line with an arrow pointing left, labeled r and r' at the ends.

Diagram 2: A single circle with a dot on its right side labeled 1. To the right of the circle is a horizontal line with an arrow pointing left, labeled r and r' at the ends.

$$+ \text{Diagram 3} + 1 \leftrightarrow 2$$

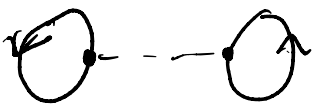
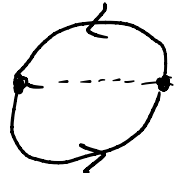
Diagram 3: A horizontal line with an arrow pointing left, labeled r and r' at the ends. Two dots are on the line, labeled 1 and 2. A dashed arc connects the two dots above the line.

$$+ \text{Diagram 4} + 1 \leftrightarrow 2$$

Diagram 4: A horizontal line with an arrow pointing left, labeled r and r' at the ends. A dot is on the line. A dashed line connects the dot to a circle below it, which has a dot on its right side labeled 1.

(only 4 of 6 diagrams are topologically distinct)

(ii) Graphical rep. of $-\frac{i}{2} \int d_1 \int d_2 U(1-2) \langle 0 | T(\Psi_1^\dagger \Psi_2^\dagger \Psi_2 \Psi_1) | 0 \rangle$

" = "  + (-1) .

- Crucial observation: Disconnected diagrams in numerator (i) contain identical components to denominator (ii).

• Cancellation theorem: Vacuum diagrams in the denominator of G_c exactly cancel all disconnected diagrams in the numerator of G_c .

- In pictures for our example:

$$\begin{array}{c} \text{Diagram 1} + \text{Diagram 2} + \text{Diagram 3} \end{array} \left(1 + \frac{1}{2} \text{Diagram 4} + \frac{-1}{2} \text{Diagram 5} \right)$$

$$\text{Diagram 1} + \frac{1}{2} \text{Diagram 4} + \frac{-1}{2} \text{Diagram 5}$$

- General proof: Consider ν -th order in $W(t)$ term in the numerator:

$$\langle 0 | T(e^{-i \int_{-\infty}^{+\infty} d\tau W(\tau)} \Psi_r \Psi_r^\dagger) | 0 \rangle^{(\nu)} = \sum_{n=0}^{\infty} \sum_{m=0}^{\infty} \delta_{m+n, \nu} (-i)^{\nu} \frac{1}{\nu!} \left(\frac{\nu!}{m! n!} \right) \times$$

$$\times \int_{-\infty}^{+\infty} dt_1 \dots dt_m \langle 0 | T(W(t_1) \dots W(t_m) \Psi_r \Psi_r^\dagger) | 0 \rangle_{\text{conn.}} \int dt_{m+1} \dots dt_{\nu} \langle 0 | T(W(t_{m+1}) \dots W(t_{\nu})) | 0 \rangle$$

only connected
contributions

$$\sum_{\nu=0}^{\infty} \dots \langle 0 | T(e^{-i \int_{-\infty}^{+\infty} d\tau W(\tau)} \Psi_r \Psi_r^\dagger) | 0 \rangle = \left(\sum_{m=0}^{\infty} \frac{(-i)^m}{m!} \int_{-\infty}^{+\infty} dt_1 \dots dt_m \langle 0 | T(W(t_1) \dots W(t_m) \Psi_r \Psi_r^\dagger) | 0 \rangle_{\text{conn.}} \right)$$

$$\times \left(\sum_{n=0}^{\infty} \frac{(-i)^n}{n!} \int_{-\infty}^{+\infty} dt_1 \dots dt_n \langle 0 | T(W(t_1) \dots W(t_n)) | 0 \rangle \right)$$

$$= \langle 0 | e^{-i \int_{-\infty}^{+\infty} d\tau W(\tau)} | 0 \rangle, \text{ cancelled by denominator. } \square$$

$\leadsto$ The causal GF is the sum of the connected diagrams.

$$\not\approx \approx \leftarrow + \leftarrow \text{---} \leftarrow + \leftarrow \text{---} \leftarrow$$

- Diagrammatic rules

- (1) Draw all topologically distinct Feynman diagrams
- (2) Translate them into expressions with free causal GFs (all internal variables $1, \dots, n$ are integrated over)
- (3) Multiply every diagram by $(-1)^F$, where F is the number of closed propagator loops with more than one vertex "•" (i.e. free densities $\langle n \rangle_0 \sim \odot$ do not count, and neither do interaction lines)