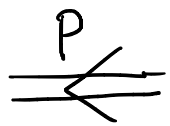
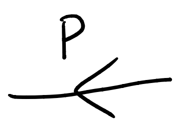
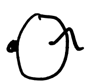
- Decoding of diagrams in momentum space

  - Main difference: Energy and (lattice) momentum conserved at every vertex.

  - Decoding table:

	$iG_c(P) \equiv iG_c(p,\omega)$	(Full causal GF)
	$iG_c^0(P) \equiv iG_c^0(p,\omega)$	(Free causal GF)
	$-iU(Q) \equiv -iU(q)$	(FT of interaction)
	$\langle \frac{N}{V} \rangle_0 = n_0(\mu)$	(unperturbed density)

Integration over all internal frequencies and momenta is implied, including a δ-function for energy-momentum conservation at every vertex.

↳ General structure when Fourier-transforming at a vertex Y:

$$\int dY \, G_c^0(\ldots - Y)\, G_c^0(Y - \ldots)\, U(Y - \ldots) =$$

$$= \int \frac{dK}{(2\pi)^{d+1}} \int \frac{dK_1}{(2\pi)^{d+1}} \int \frac{dK_2}{(2\pi)^{d+1}} \int dY \, e^{iK(\ldots - Y) + iK_1(Y - \ldots) + iK_2(Y - \ldots)}$$

$$\left[\int dY \, e^{iY(\ldots)} = (2\pi)^{d+1} \delta(\ldots) \right]$$

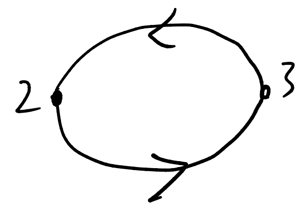
$$\times \, G_c^0(K)\, G_c^0(K_1)\, U(K_2) =$$

$$= \int dK \int \frac{dK_1}{(2\pi)^{d+1}} \int \frac{dK_2}{(2\pi)^{d+1}} \delta(-\omega + \omega_1 + \omega_2)\, \delta(-k + k_1 + k_2)\, G_c^0(K)\, G_c^0(K_1)\, U(K_2)$$

$\times$ - - - - [terms independent of Y].

A.1.c Self-Energy and Polarization Operator

- General observation: Mathematical expressions corresponding to building blocks of diagrams are independent of their specific context.

  - Example: 

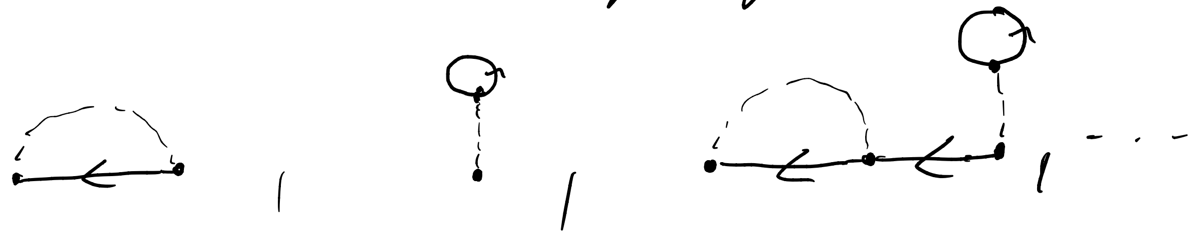
$\approx -i\, G_c^0(2,3)\, i\, G_c^0(3,2)$ wherever it appears in a diagram.

  ⇝ Basic idea: Account for more complex diagrams by systematically combining building blocks in an iterative fashion.

- Self-energy
  - A self-energy part of a diagram is any part

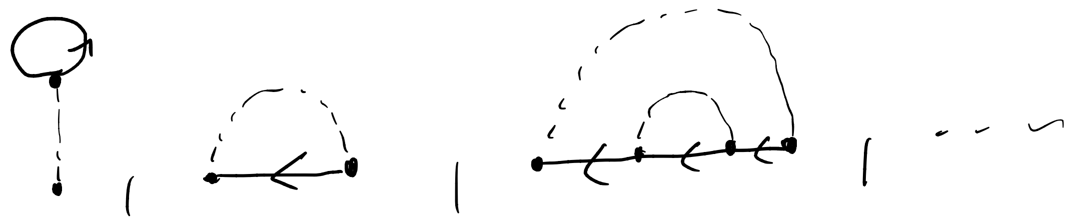
connected to the rest only by two particle lines.

↳ Examples



- Irreducible self-energy parts in addition cannot be further decomposed by cutting a <u>single</u> particle line.

↳ Examples:



- The (proper) self energy is the sum of all irreducible self-energy parts:

$-i\Sigma =$

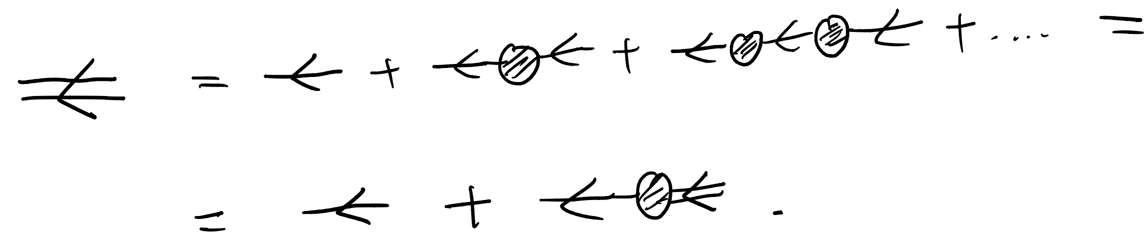
$+ \ldots$

- Dyson equation: Resumming arbitrary combinations of self-energy diagrams

$$iG_c = iG_c^0 + iG_c^0 \Sigma G_c^0 + iG_c^0 \Sigma G_c^0 \Sigma G_c^0 + \dots\dots =$$

$$= iG_c^0 + iG_c^0 \Sigma G_c$$

- Graphically:



- Practical use: Approximating the self-energy up to a given order, the Dyson equation amounts to resumming an infinite series of concatenations of these diagrams.

- Explicit formal solution of Dyson equation

$$iG_c = i\sum_{n=0}^{\infty} (G_c^0 \Sigma)^n G_c^0 \underset{\substack{\uparrow\\ \text{geometric}\\ \text{series}}}{=} i(1-G_c^0\Sigma)^{-1} G_c^0 \underset{\substack{\uparrow\\ \boxed{(AB)^{-1} = B^{-1}A^{-1}}}}{=}$$

$$= i\left(G_c^{0^{-1}}(1-G_c^0\Sigma)\right)^{-1} = i\left(G_c^{0^{-1}} - \Sigma\right)^{-1} =$$

$$= i\,\frac{1}{G_c^{0^{-1}} - \Sigma}$$

↳ Simplest case with scalar GF in momentum space:

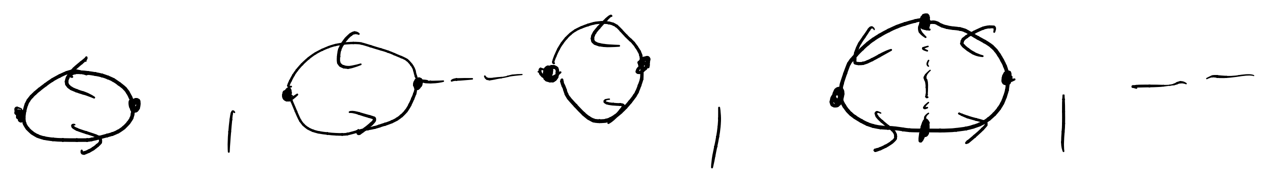
$$G_c(p,\omega) = \frac{1}{\omega - \varepsilon_p - \Sigma(p,\omega)} \,.$$

- Renormalization of interaction

  - Polarization operator $i\Pi \simeq$  analogous to self-energy, where the role of particle lines $\leftarrow$ is replaced by interaction lines •----• .

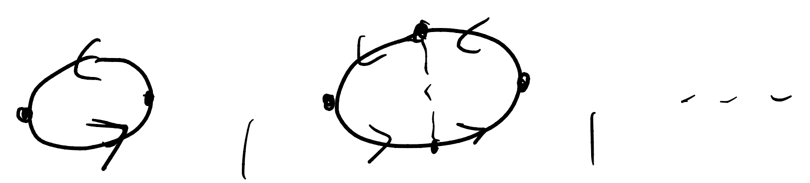
  - Polarization insertion is a part of a diagram that is connected to the rest only by two interaction lines

    ↳ Examples :

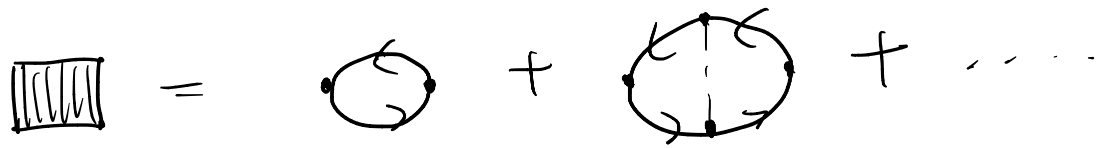


  - Irreducible polarization insertion cannot be reduced by cutting a single interaction line

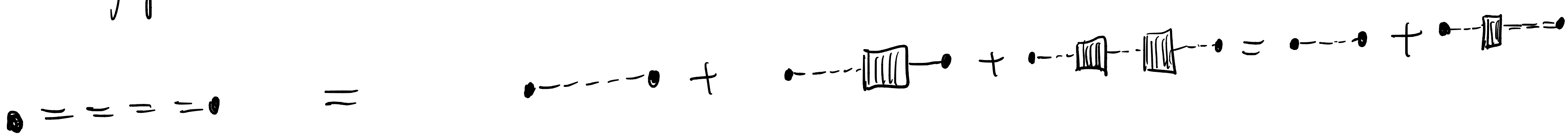
    ↳ Examples :



- The polarization operator is the sum of all irreducible polarization insertions



- Effective interaction (analog of solution to Dyson equation)



$$U_{eff}(p) = U(p) + U(p)\Pi(p)U_{eff}(p) = \frac{U(p)}{1 - U(p)\Pi(p)}.$$

- Mathematical issue: Self-energy and polarization operator amount to changing the order of terms in the perturbative series ⇝ Mathematically justified only for absolutely converging series (not the case for perturbative series in general).
⇝ Physical justification by convergent and physically plausible results.