- Equation of motion of the GF (Martin-Schwinger)

$$\left(\frac{i\,d}{dt_1} - H_0(1)\right) G_c(1,1') = \delta(1-1') + \underbrace{i\int d2\, V(1-2)\langle T(\psi_1\psi_2\psi_{2^+}^+\psi_{1'}^+)\rangle}_{\text{def. }\Sigma \ = \int d2\, \Sigma(1,2)\, G_c(2,1')}$$

- Compare to Dyson equation:

$$\overset{\text{full}}{G_c}(1,1') = \overset{\text{free}}{G_c^0}(1,1') + \int d2\, d3\, G_c^0(1,2)\, \Sigma(2,3)\, G_c(3,1').$$

$$\leadsto \left(i\frac{d}{dt_1} - H_0(1)\right) G_c(1,1') = \delta(1-1') + \int d2\,d3\, \delta(1-2)\, \Sigma(2,3)\, G_c(3,1')$$

$$= \delta(1-1') + \int d3\, \Sigma(1,3)\, G_c(3,1') \quad \checkmark$$

– Compare to Hartree-Fock approximation: full expect.

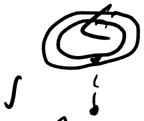
$$\langle J(\psi_1 \psi_2 \psi_{2^+}^+ \psi_{1'}^+) \rangle \approx \langle J(\psi_1 \psi_{1'}^+) \rangle \langle J(\psi_2 \psi_{2^+}^+) \rangle$$

$$- \langle J(\psi_1 \psi_2^+) \rangle \langle J(\psi_2 \psi_{1'}^+) \rangle$$

⇝ Plug into equation of motion:

$$\left(i \frac{d}{dt_1} - H_0(1) \right) G_c(1,1') = \delta(1-1') - i \underbrace{\int d2\, V(1-2) \left(G_c(1,1') G_c(2,2^+) - G_c(1,2) G_c(2,1') \right)}_{=: \int d2\, \Sigma(1,2)\, G_c(2,1')}$$

where $$\Sigma(1,2) = \underbrace{\delta(1-2)(-i) \int d3\, V(1-3)\, G_c(3,3^+)}_{= V_H(1)\, \delta(1-2)} + i V(1-2)\, G_c(1,2)$$

- Conclusion: Hartree-Fock approximation amounts to resummation of the proper self-energy diagrams , 

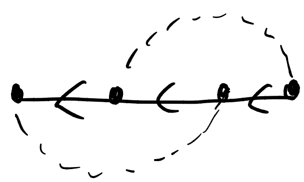
Hartree Fock

i.e. $_{HF}$ =  + , where  is the full propagator $\rightsquigarrow$ Equation for G_C is implicit and non-linear.

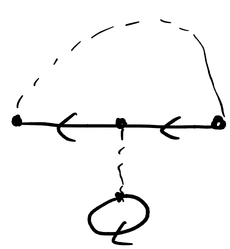
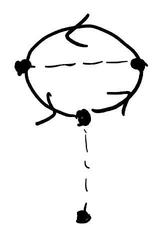
$\rightsquigarrow$ Iterative solution for self-consistency.

- Second Born approximation and skeleton diagrams

- Consider proper self-energy diagrams up to second order:

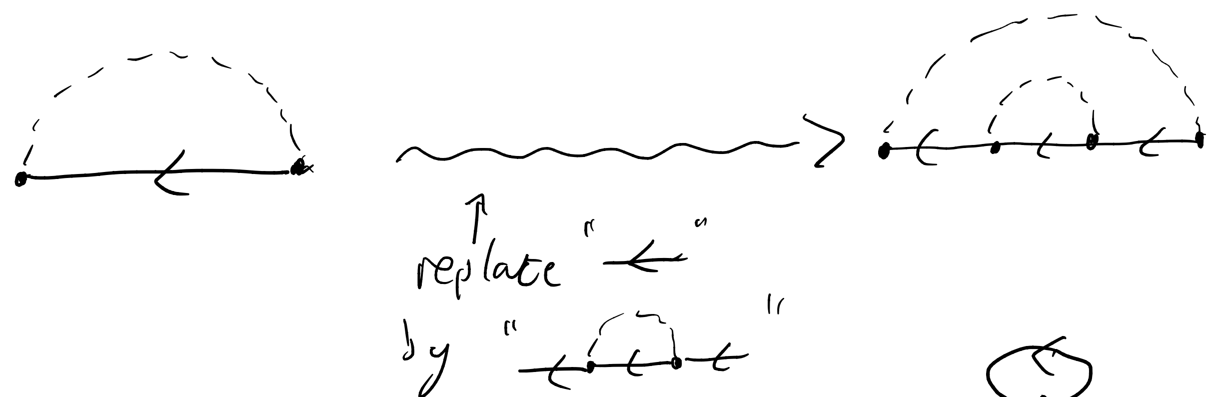
$\Sigma \approx$  +  +  +  +  +  +

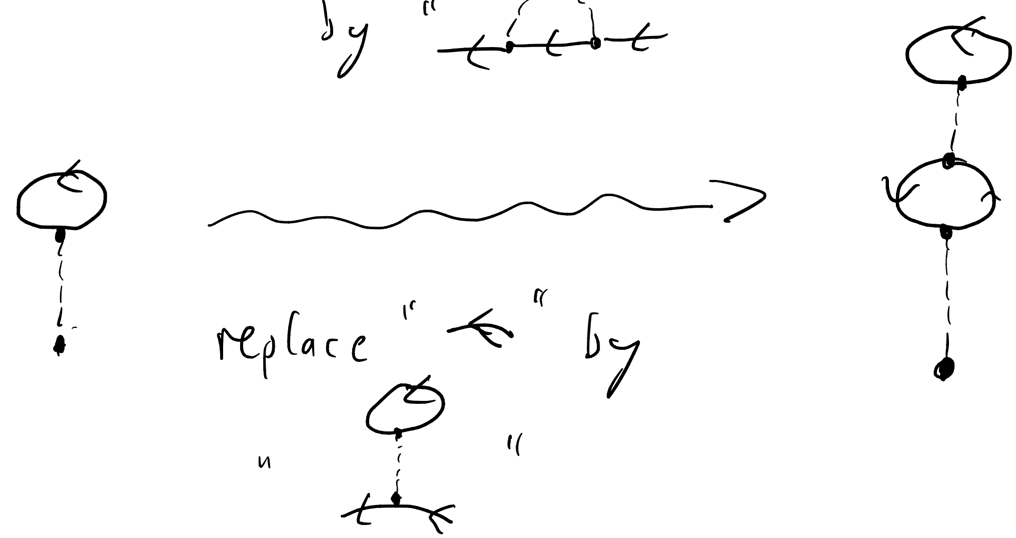
1st order

+  + 

- Observation: Some diagrams can be obtained from simpler ones by inserting a (lower order) self-energy diagram into a propagator, or a polarization into an interaction line.

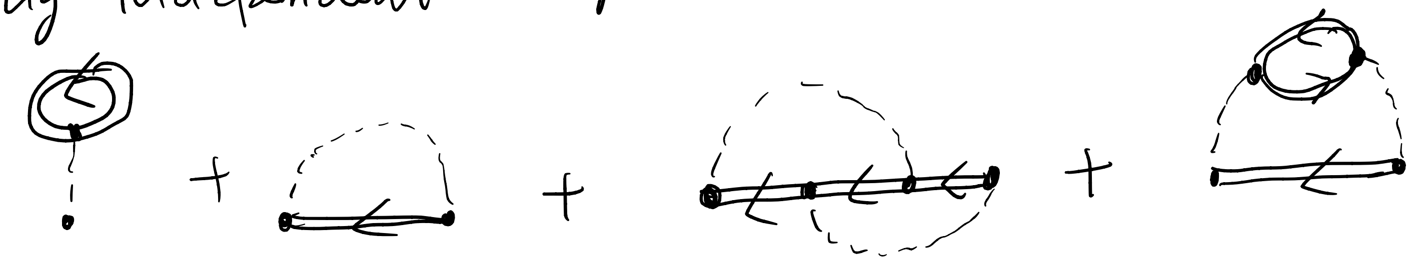
↳ Examples:





- Inspired by the self-consistent Hartree-Fock approximation we attempt to replace —←— by ═←═ in all self-energy diagrams (which comes at the cost of implicit non-linear equations for G_c! see A.3.c for further discussion)

↳ Remaining independent diagrams (G-skeleton diagrams)

$\Sigma \approx$ 

Hartree-Fock

2nd-Born

- Combination with interaction renormalization: W-skeleton diagrams with "•- - - -•" ⇒ "•═══•"

↳ $\Sigma \approx$ 

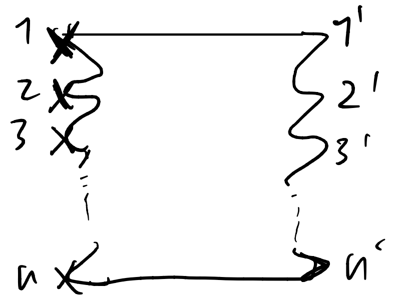
Fully dressed self-energy to second order.

A.1.d Many-particle Green's functions

- The n-particle ($2n$-point) causal Green's function is defined as:

$$G_{(n)}(1,2,\dots n;1',2',\dots n') = \frac{1}{(i)^n}\left\langle T(\psi_1\psi_2\dots\psi_n\psi^\dagger_{n'}\dots\psi^\dagger_{1'})\right\rangle$$

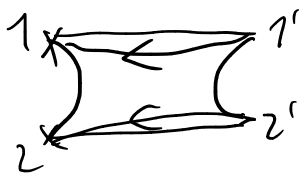
– Diagrammatic representation



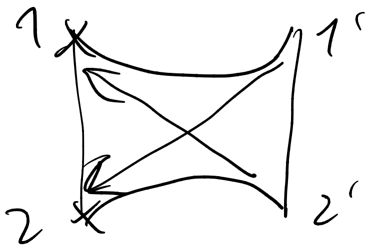
"x" denotes outgoing lines (annihilation operator)

– Diagrammatic rules analogous to $n=1$ case with one extra rule: When using Wicks theorem, add to each diagram the sign of the permutation that maps the incoming labels $(1',\dots,n')$ to the outgoing labels $(1,\dots,n)$

↳ Examples ($n=2$):

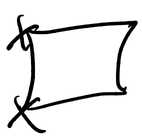


$\sim \; (1', 2') \longrightarrow (1, 2)$ (positive sign)

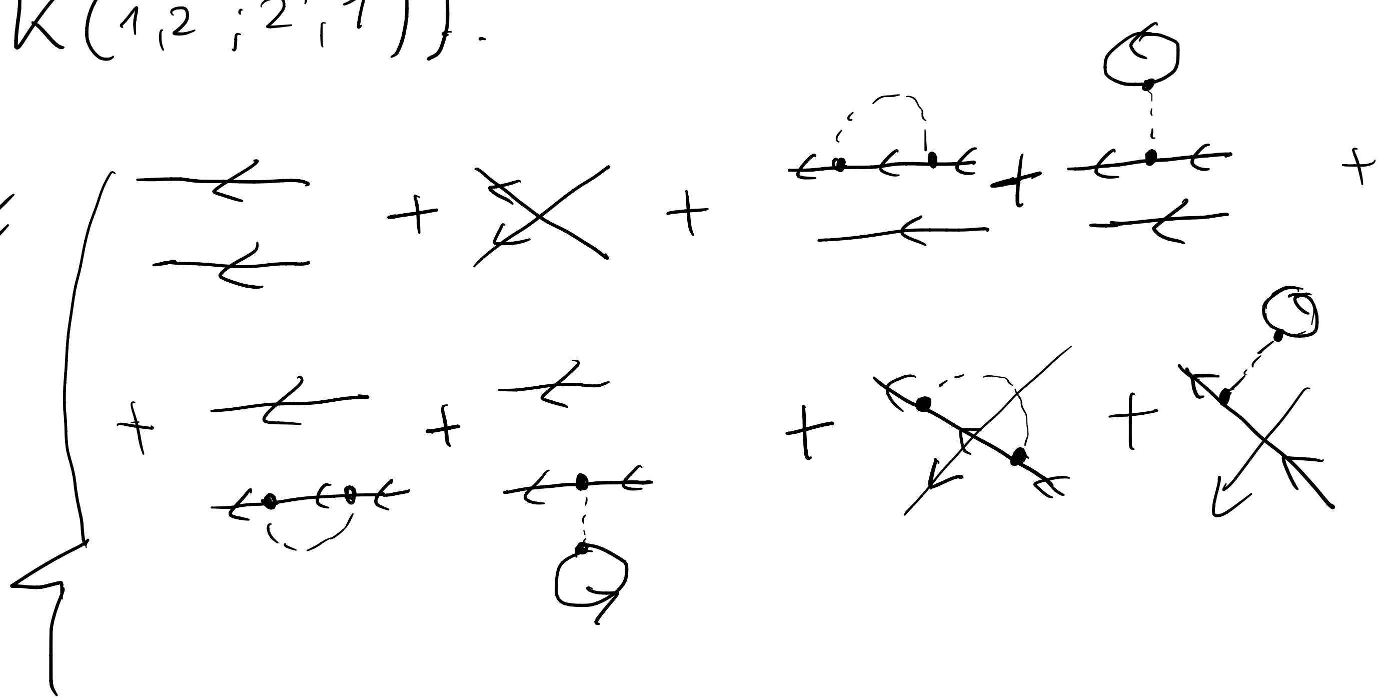


$\sim \; (1', 2') \longrightarrow (2, 1)$ (negative sign)

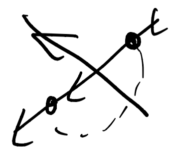
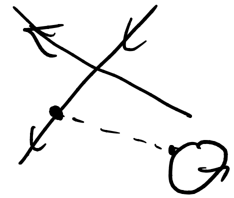
– First order expansion of the two-particle GF (called $K(1,2;2',1')$).

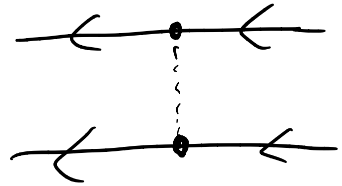


$\approx$



"seemingly disconnected"

+  +  +

Direct interactions (irreducible) +  +