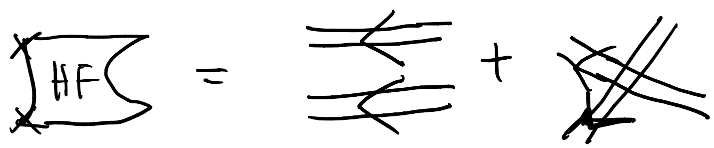
- All the "seemingly disconnected" (still not containing any pure vacuum amplitude component) diagrams can be encompassed by the Hartree-Fock approximation to the 2-particle GF:

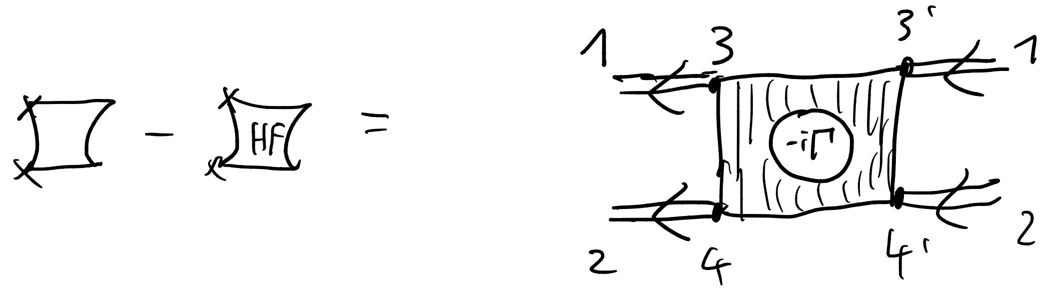
$\frac{1}{i^2} K_{HF} =$  , i.e. by replacing free single-particle propagators with full ones.

- The irreducible part is expressed through the vertex-function Γ:

$$\underbrace{K(1,2;1',2') - K_{HF}(1,2;1',2')}_{=:\ \tilde{K}(1,2;1',2')} = \int d3 \int d3' \int d4 \int d4'\, G_c(1,3)\, G_c(2,4)$$

$\times\ i\Gamma(3,4;3',4')\, G_c(3',1')\, G_c(4',2')$

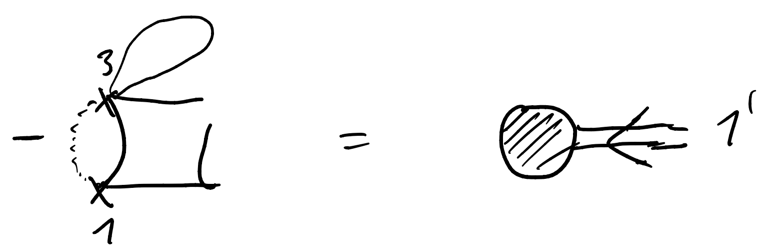
↳ Graphically:

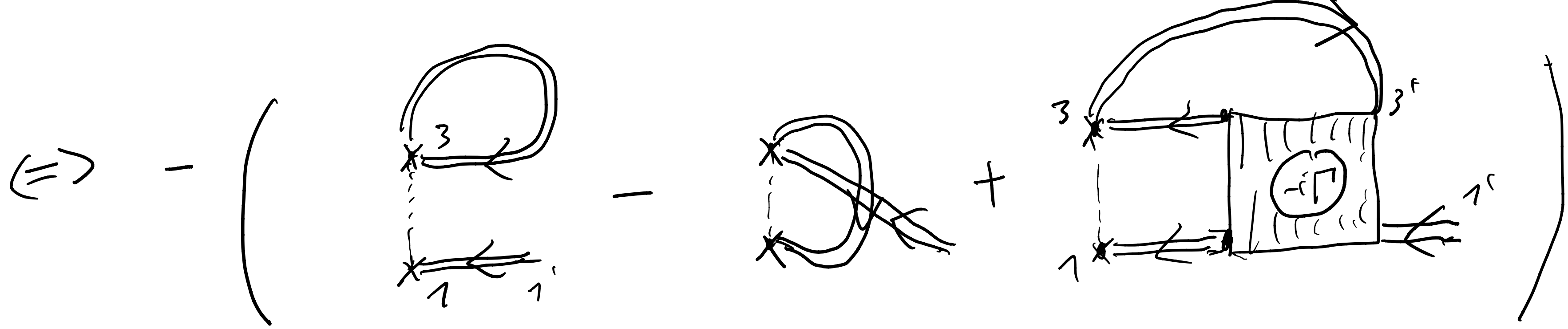


- Relation between self-energy and vertex function (see Zagoskin, Chapter 2.2.4 for more details)
  - Definition of the self-energy from Martin-Schwinger equation of motion:

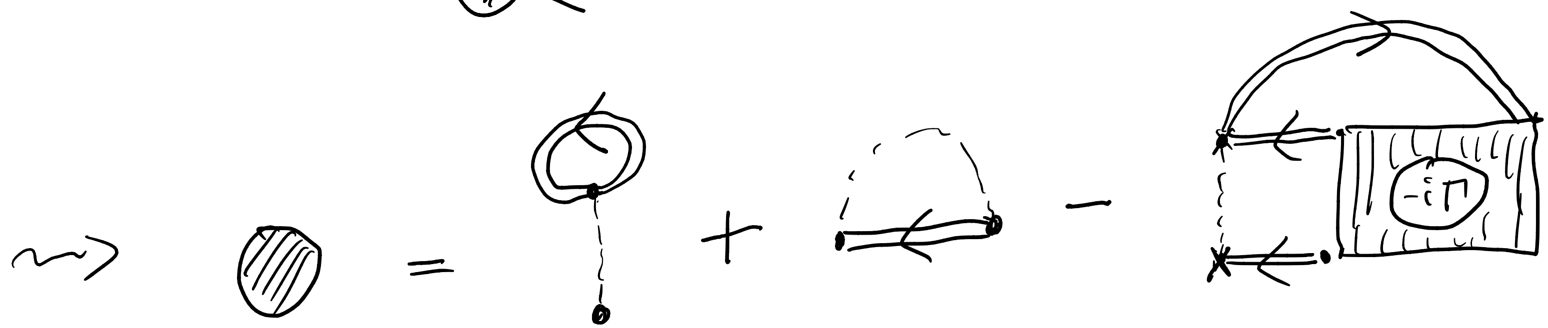
$$-i\int d3\, U(1-3)\, K(1,3;1',3^+) = \int d3\, \Sigma(1,3)\, G_c(3,1')$$

  - Graphically:





remove in all diagrams



[reversing direction of propagator ⇒ gives "−" sign]

$$\Sigma(P) = U(0)\langle n\rangle_\mu - i\int dP_1\, U(P-P_1)\, G_c(P_1) +$$

$$\int dP_1 \int dP_2\, G_c(P_1)\, G_c(P_2)\, \Gamma(P_1, P_2; P, P_1+P_2-P) \times$$

$$\times\, G_c(P_1+P_2-P)\, U(P-P_1)$$

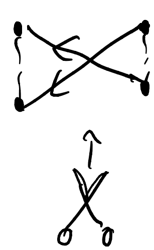
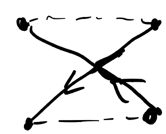
(Generalized Dyson equation in reciprocal space)

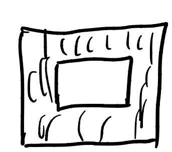
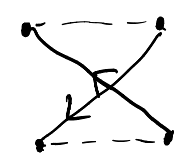
$\rightsquigarrow$ The self-energy is composed of the Hartree-Fock part and the vertex function part, where the above "short-circuited" vertex function is sufficient to uniquely determine Σ.

- Bethe - Salpeter equations

- Basic idea : Define a two-particle counterpart of the proper self-energy

↳ The particle-particle irreducible vertex function $\tilde{\Gamma}_{p-p}$ contains all diagrams that cannot be further decomposed by cutting <u>two</u> particle lines between incoming and outgoing ends (horizontally co-propagating)

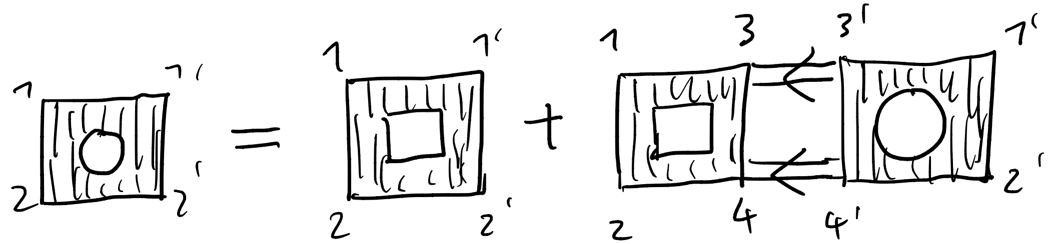
( reducible ,  irreducible ...)

↳ Graphically : $-i\tilde{\Gamma}_{p-p}$ =  =  +  + ...

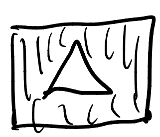
- Bethe-Salpeter equation:

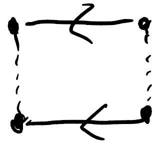
$$-i\Gamma(1,2;1',2') = -i\tilde{\Gamma}_{p-p}(1,2;1',2') + \int d3 \int d3' \int d4 \int d4' \, \tilde{\Gamma}_{p-p}(1,2;3,4)$$
$$\times\, G_c(3,3')\, G_c(4,4')\, \Gamma(3',4';1',2').$$

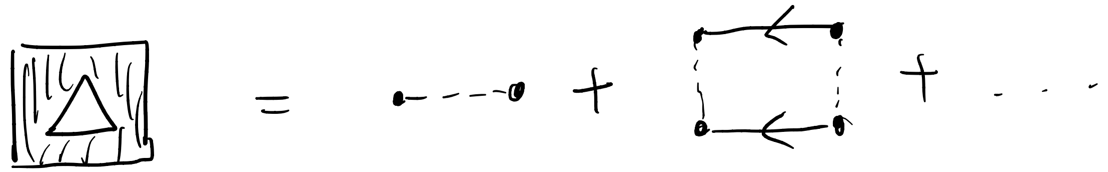
↳ Graphically



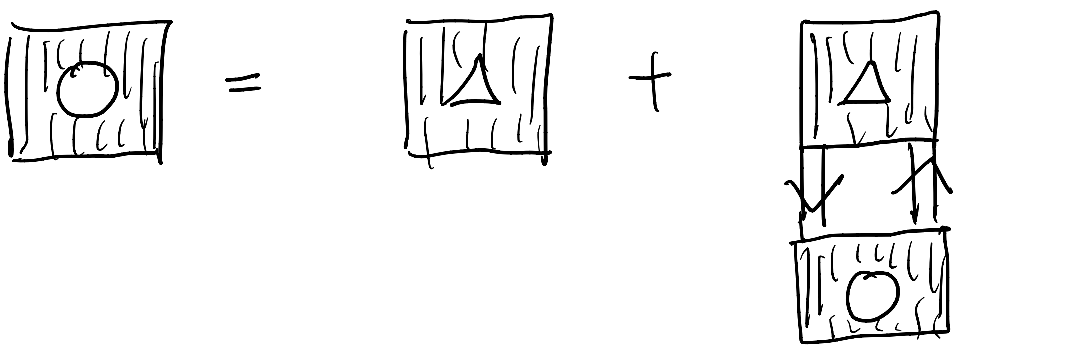
- A mathematically equivalent version can be formulated for the particle-hole irreducible vertex function

$-i\tilde{\Gamma}_{p-h}$ =  , defined as the sum of all vertex-function diagrams that cannot be decomposed by cutting two vertically counterpropagating particle lines.

( p-h irreducible ,  p-h reducible)

$\rightsquigarrow$ 

↳ Corresponding Bethe-Salpeter equation:



Full p-H and p-p approach are equivalent, but can yield different quality of approximation depending on the physical situation.