

- All the "seemingly disconnected" (still not containing any pure vacuum amplitude component) diagrams can be encompassed by the Hartree-Fock approximation to the 2-particle GF:

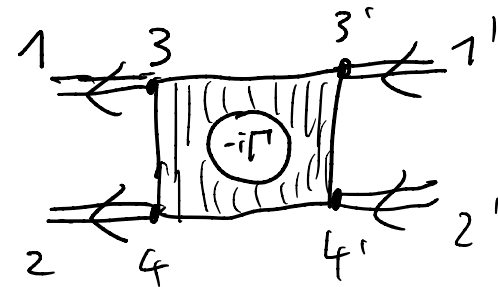
$$\frac{1}{i^2} K_{\text{HF}} = \text{[diagram: two parallel lines with a box labeled HF in the middle]} = \text{[diagram: two parallel lines]} + \text{[diagram: two crossed lines]}, \text{ i.e. by replacing free single-particle propagators with full ones.}$$

- The irreducible part is expressed through the vertex-function Γ :

$$\underbrace{K(1,2;1',2') - K_{\text{HF}}(1,2;1',2')}_{=: \tilde{K}(1,2;1',2')} = \int d3 \int d3' \int d4 \int d4' G_c(1,3) G_c(2,4)$$

$$\times i \Gamma(3, 4; 3', 4') G_c(3', 1') G_c(4', 2')$$

↳ Graphically: $\square - \square_{\text{HF}} =$



- Relation between self-energy and vertex function
(see Zagoskin, Chapter 2.2.4 for more details)

– Definition of the self-energy from Martin-Schwinger equation of motion:

$$-i \int d3 U(1-3) K(1, 3; 1', 3') = \int d3 \Sigma(1, 3) G_c(3, 1')$$

– Graphically: $-\text{[Diagram of a loop with a shaded circle]} = \text{[Diagram of a shaded circle with an incoming line labeled 1']}$

$$\Rightarrow - \left(\begin{array}{c} \text{Diagram 1: Loop with external lines 1, 1', 3} \\ \text{Diagram 2: Crossed loop with external lines 1, 1', 3} \\ \text{Diagram 3: Box diagram with internal lines 1, 1', 3, 3' and a circle containing } -i\Gamma \end{array} \right)$$

$$= \begin{array}{c} \text{Diagram 4: Shaded circle with external lines 1, 1'} \\ \text{Diagram 5: Loop with external lines 1, 1'} \\ \text{Diagram 6: Box diagram with internal lines 1, 1', 3, 3' and a circle containing } -i\Gamma \end{array}$$

remove in all diagrams

$$\Rightarrow \text{Diagram 4} = \begin{array}{c} \text{Diagram 5} \\ \text{Diagram 6} \\ \text{Diagram 7: Box diagram with internal lines 1, 1', 3, 3' and a circle containing } -i\Gamma \end{array}$$

reversing direction
of propagator $\Rightarrow$
gives "-" sign

$$\Sigma(P) = U(0) \langle n \rangle_m - i \int dP_1 U(P-P_1) G_c(P_1) +$$

$$\int dP_1 \int dP_2 G_c(P_1) G_c(P_2) \Gamma(P_1, P_2; P, P_1+P_2-P) \times$$

$$\times G_c(P_1+P_2-P) U(P-P_1)$$

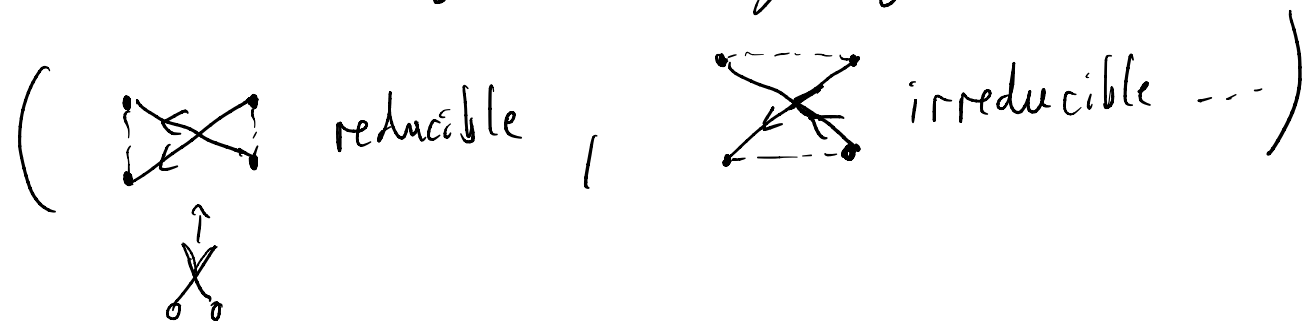
(Generalized Dyson equation in reciprocal space)

$\leadsto$ The self-energy is composed of the Hartree-Fock part and the vertex function part, where the above "short-circuited" vertex function is sufficient to uniquely determine Σ .

- Bethe-Salpeter equations

- Basic idea : Define a two-particle counterpart of the proper self-energy

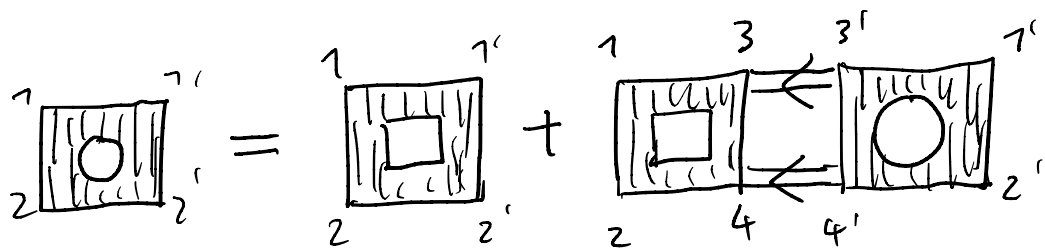
↳ The particle-particle irreducible vertex function $\tilde{\Gamma}_{p-p}$ contains all diagrams that cannot be further decomposed by cutting two particle lines between incoming and outgoing ends (horizontally co-propagating)



↳ Graphically : $-i \tilde{\Gamma}_{p-p} = \text{[Diagram of a box with internal lines]} = \text{[Diagram of a vertical line]} + \text{[Diagram of an irreducible vertex]} + \dots$

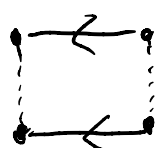
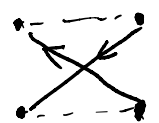
- Bethe-Salpeter equation:

$$-i \tilde{\Gamma}(1, 2, 1', 2') = -i \tilde{\Gamma}_{P-P}(1, 2, 1', 2') + \int d^3 \int d^3' \int d^4 \int d^4' \tilde{\Gamma}_{P-P}(1, 2, 3, 4) \\ \times G_c(3, 3') G_c(4, 4') \Gamma(3', 4'; 1', 2').$$

↳ Graphically 

- A mathematically equivalent version can be formulated for the particle-hole irreducible vertex function

$-i \tilde{\Gamma}_{P-H} =$ , defined as the sum of all vertex-function diagrams that cannot be decomposed by cutting two vertically counterpropagating particle lines.

( p-h irreducible ,  $\leftarrow$  p-h reducible)

$$\leadsto \boxed{\text{triangle}} = \text{---} + \boxed{\text{square loop}} + \dots$$

$\hookrightarrow$ Corresponding Bethe-Salpeter equation:

$$\boxed{\text{circle}} = \boxed{\text{triangle}} + \boxed{\text{triangle}} \begin{array}{c} \downarrow \uparrow \\ \boxed{\text{circle}} \end{array}$$

Full P-H and P-P approach are equivalent, but can yield different quality of approximation depending on the physical situation.