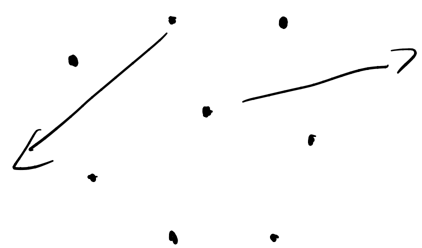
B.1.a Nesting and failure of perturbation theory in 1D

[For details, see Chapter 1 in T. Giamarchi, "Quantum Physics in one dimension"]

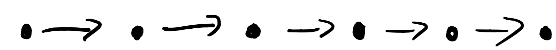
- Collective motion in $d=1$: Intuitive real space picture

$d \geq 2$



"individual" motion of electrons possible

$d=1$

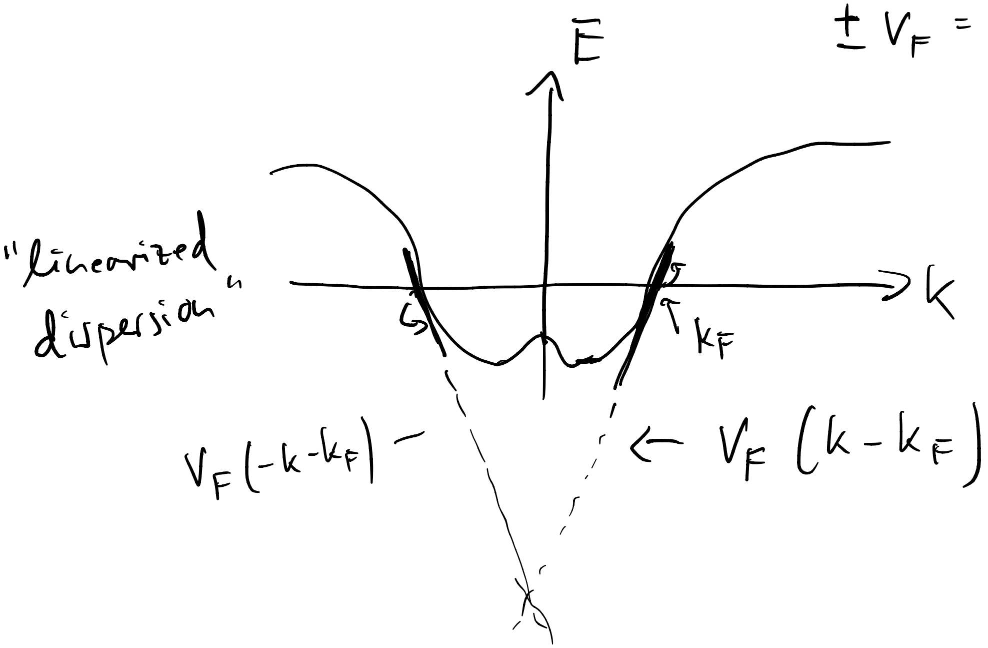


excitations necessarily collective

- Nesting property in momentum space
  - Nesting condition: $\boxed{\varepsilon(k+Q) = -\varepsilon(k)}$ for fixed Q in whole neighborhood of k.
  - In $d \geq 2$, "nesting" generically only at $\underbrace{\text{isolated points}}_{\hookrightarrow \text{ or at least zero-sets.}}$ in reciprocal space.

– Natural situation in $d=1$

$$\pm v_F = \frac{\partial \varepsilon(k)}{\partial k}\bigg|_{k=\pm k_F}$$



For all $k \approx -k_F$, we get with $Q = 2k_F$:

$$\varepsilon(k+2k_F) = v_F(k+k_F) = -v_F(-k-k_F) = -\varepsilon(k).$$

$\rightsquigarrow$ Nesting property generically satisfied in entire region around the Fermi surface

$$\left[\text{assuming } \frac{\partial \varepsilon(k)}{\partial k}\bigg|_{k_F} = -\frac{\partial \varepsilon(k)}{\partial k}\bigg|_{-k_F}\right]$$

• Singularities in density-density correlation from nesting:

With $\chi_r(x-x', t-t') = -i\,\theta(t-t')\,\langle [n(x,t), n(x',t')] \rangle$,

susceptibility to density fluctuations

we get on Fourier transform:

$$\chi_r(q,\omega) = \frac{1}{L}\sum_k \frac{\langle n_k\rangle - \langle n_{k+q}\rangle}{\omega + (\varepsilon_k - \varepsilon_{k+q}) + i0^+}$$

. In the nesting scenario, this yields: $\chi_r(Q,\omega=0) = \frac{1}{L}\sum_k \frac{\frac{1}{e^{\varepsilon_k\beta}+1} - \frac{1}{e^{-\varepsilon_k\beta}+1}}{2\varepsilon_k + i0^+} =$

$$= -\frac{1}{L}\sum_k \frac{\tanh\left(\beta\frac{\varepsilon_k}{2}\right)}{2\varepsilon_k + i0^+}$$

$$\rightsquigarrow \operatorname{Re}\chi_r(Q,\omega=0) = -\beta\int d\varepsilon\, N(\varepsilon)\,\frac{\tanh\left(\frac{\beta\varepsilon}{2}\right)}{2\varepsilon} \approx$$

[density of states (DOS)]

[DOS at Fermi energy]

$$\approx -N(0)\log\left(\frac{v_F(k_{max}-k_F)}{T}\right)$$

, where k_{max}

defines the maximum deviation from k_F that satisfies the nesting condition.

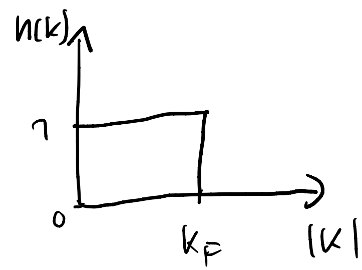
↳ Logarithmic divergence of χ_r for $\omega, T \to 0$.

↳ Strong indication for failure of perturbation theory and non-Fermi-liquid nature of the ground-state of the correlated 1D electron gas.

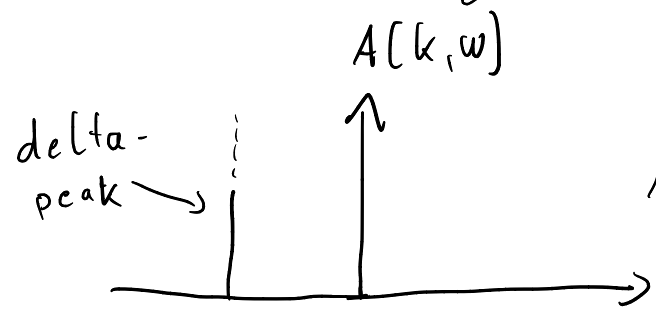
⇝ Central question: How does the ground-state in 1D deviate from a standard Fermi liquid?

Recap: Fermi liquids

- Non-interacting Fermi gas at $T=0$

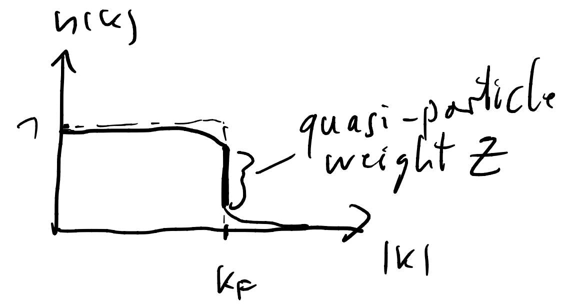


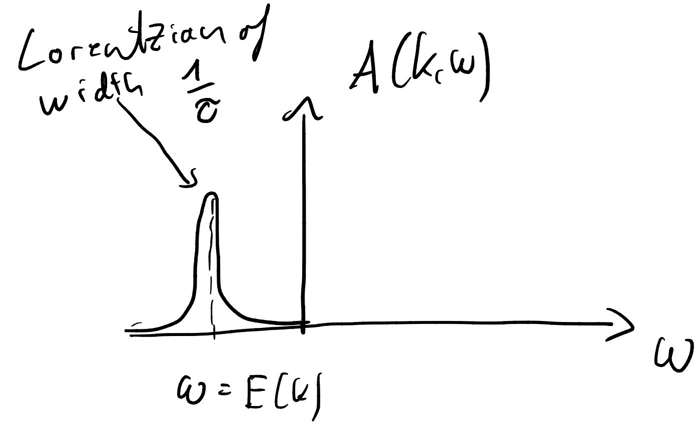
spectral function



$A(k,\omega) = \delta(\omega - \varepsilon_k)$

- Typical Fermi-Liquid at $T=0$





time-dependence of quasi-particle excitation $\sim e^{-iE(k)t} \, e^{-t/\tau}$

$\uparrow$ life-time

- For $d=3$, Landau showed that $\tau \sim \frac{1}{(E(k))^2}$, i.e. the quasi-particle life-time diverges at the Fermi surface such that quasi-particles live through many oscillation periods $\frac{2\pi}{E(k)}$.

- Typical Fermi temperatures in metals $\sim 10^4\,K \rightsquigarrow$ most experiments dominated by excitations very close to the Fermi surface.

- Effective dispersion $E(k) \simeq \underbrace{E_F}_{=0} + \frac{k_F}{m^*}(k-k_F)$

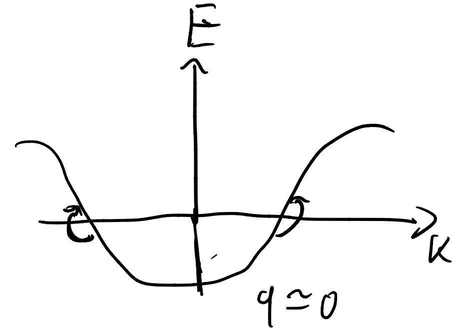
[effective mass]

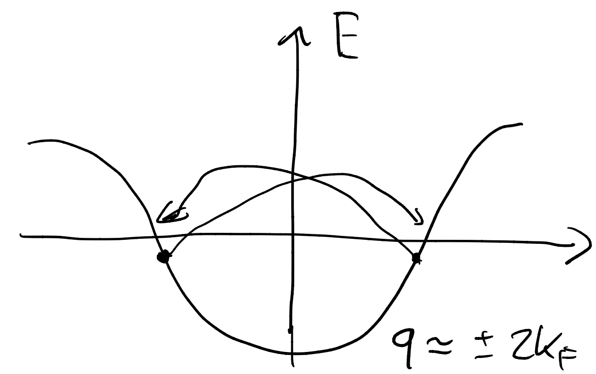
[Fermi-momentum k_F, unchanged by interactions (Luttinger theorem)]

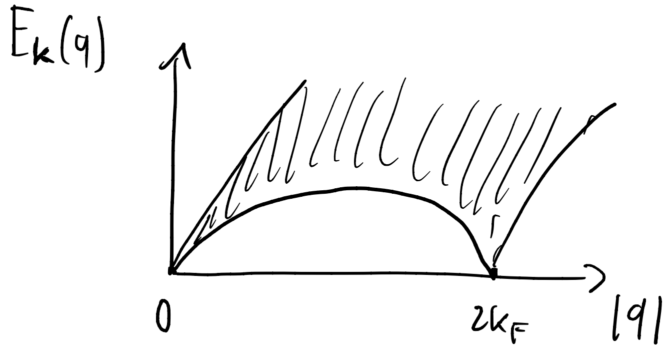
[- Quasi-particles behave similar to electrons but with effective parameters]

B.1.b Particle-hole excitations as quasi-particles

- Particle is taken from below the Fermi surface and placed above
  - Two scenarios for $E_k(q) = \varepsilon(k+q) - \varepsilon(k) \approx 0$







– Continuum of low-energy excitations in $d \geq 2$