Rep

- Kraus map : pres. DM properties

$$\rho' = \sum_{\alpha\beta} \underset{\text{pos. semidefinite}}{\gamma_{\alpha\beta}} \tilde{K}_\alpha \rho \tilde{K}_\beta^\dagger \quad : \sum_{\alpha\beta} \gamma_{\alpha\beta} \tilde{K}_\beta^\dagger \tilde{K}_\alpha = \mathbb{1}$$

$$= \sum_\alpha K_\alpha \rho K_\alpha^\dagger \quad : \sum_\alpha K_\alpha^\dagger K_\alpha = \mathbb{1}$$

- Lindblad-LGKS form

$$\dot{\rho} = -i[\underset{H=H^\dagger}{H}, \rho] + \sum_{\alpha\beta} \underset{\text{pos. semidefinite}}{\gamma_{\alpha\beta}} \left(\tilde{L}_\alpha \rho \tilde{L}_\beta^\dagger - \tfrac{1}{2}\{\tilde{L}_\beta^\dagger \tilde{L}_\alpha, \rho\} \right)$$

$$= -i[H, \rho] + \sum_\alpha \left(L_\alpha \rho L_\alpha^\dagger - \tfrac{1}{2}\{L_\alpha^\dagger L_\alpha, \rho\} \right)$$

- example: HO

$$\dot{\rho} = -i[\Omega a^\dagger a, \rho] + \Gamma(1+n_B)\left[a\rho a^\dagger - \tfrac{1}{2}\{a^\dagger a, \rho\}\right] + \Gamma \cdot n_B \left[a^\dagger \rho a - \tfrac{1}{2}\{a a^\dagger, \rho\}\right]$$

- partial trace

$$\hat{O} = \sum_{ij} \sum_{\alpha\beta} O_{i\alpha j\beta} |i\rangle\langle j| \otimes |\alpha\rangle\langle\beta|$$

$$\mathrm{Tr}_B\{\hat{O}\} = \sum_\gamma \langle\gamma|\hat{O}|\gamma\rangle = \sum_{ij}\sum_\gamma O_{i\gamma j\gamma} |i\rangle\langle j|$$

red. DM $\quad \rho_A = \mathrm{Tr}_B\{\rho_{AB}\} \quad \rho_B = \mathrm{Tr}_A\{\rho_{AB}\}$

$$\langle\hat{A}\rangle = \mathrm{Tr}\{\hat{A}\otimes \mathbb{1}_B \, \rho_{AB}\} \overset{!}{=} \mathrm{Tr}_A\{\hat{A}\rho_A\}$$

- microscopic derivation (weak coupling)

$$H_n = \underbrace{\Omega a^\dagger a}_{\text{system}} + \underbrace{(a+a^\dagger)\sum_k (h_k b_k + h_k^* b_k^\dagger)}_{\text{int. } (H_I)} + \underbrace{\sum_k \omega_k b_k^\dagger b_k}_{\text{bath}}$$

H_I is small $\rightsquigarrow$ int. pic

$$\dot{\tilde{\rho}} = -i[\tilde{H}_I(t), \tilde{\rho}(t)]$$

$$\boxed{\tilde{H}_I(t) = e^{+i(H_S+H_B)t} H_I e^{-i(H_S+H_B)t}}$$

$[H_S, H_B] = 0 \qquad [a, H_B] = 0$

- $e^{+i\Omega t a^\dagger a} \cancel{e^{+iH_B t}}\, a\, \cancel{e^{-iH_B t}}\, e^{-i\Omega t a^\dagger a}$

$= e^{+i\Omega t a^\dagger a}\, a\, e^{-i\Omega t a^\dagger a} = \hat{a}(t)$

$$\frac{d}{dt}\hat{a} = e^{+i\Omega t a^\dagger a} \underbrace{[i\Omega a^\dagger a, a]}_{-i\Omega a} e^{-i\Omega t a^\dagger a}$$

$$= -i\Omega \cdot \hat{a}(t) \rightsquigarrow \boxed{\begin{aligned} \tilde{a}(t) &= e^{-i\Omega t} \cdot a \\ \tilde{a}^\dagger(t) &= e^{+i\Omega t} \cdot a^\dagger \end{aligned}}$$

bath: $e^{+iH_B t} b_k e^{+iH_B t} = e^{+i\omega_k b_k^\dagger b_k t} b_k e^{-i\omega_k b_k^\dagger b_k t}$

$H_B = \sum_k \omega_k b_k^\dagger b_k$ $\qquad \boxed{\begin{aligned} \tilde{b}_k(t) &= e^{-i\omega_k t} b_k \\ \tilde{b}_k^\dagger(t) &= e^{+i\omega_k t} b_k^\dagger \end{aligned}} \leftarrow \dfrac{e^{-\beta\sum_k \omega_k b_k^\dagger b_k}}{Z} = \bar{\varrho}_B$

$\dot{\tilde{\varrho}} = -i[\tilde{H}_I(t), \tilde{\varrho}]$ $\qquad \tilde{H}_I = (a \cdot e^{-i\Omega t} + h.c.) \sum_k (h_k b_k \cdot e^{-i\omega_k t} + h.c.)$

$\int_0^t dt'$ $\qquad$ h_k small

$$\tilde{\varrho}(t) - \varrho_0 = -i \int_0^t [\tilde{H}_I(t'), \tilde{\varrho}(t')]\, dt'$$

full universe

$$\dot{\tilde{\varrho}} = -i[\tilde{H}_I(t), \varrho_0] - \int_0^t dt' [\tilde{H}_I(t), [\tilde{H}_I(t'), \tilde{\varrho}(t')]]$$ still exact

$\tilde{\varrho}_S = Tr_B\{\tilde{\varrho}\}$

$$\dot{\tilde{\varrho}}_S = -i\, \underline{Tr_B\{[\tilde{H}_I(t), \varrho_0]\}} - \int_0^t dt'\, Tr_B\{[\tilde{H}_I(t), [\tilde{H}_I(t'), \tilde{\varrho}(t')]]\}$$

not closed $\longrightarrow$ $\boxed{\text{Born approximation} \quad \varrho(t) = \varrho_S(t) \otimes \bar{\varrho}_B + \mathcal{O}(h_k)}$

$\varrho_0 = \varrho_S^0 \otimes \bar{\varrho}_B$

$Tr_B\{[\tilde{H}_I(t), \varrho_0]\} = 0$ $\qquad \bar{\varrho}_B = \dfrac{e^{-\beta H_B}}{Z}$ $\qquad [H_B, \bar{\varrho}_B] = 0$

$\tilde{a} + \tilde{a}^\dagger$

$\tilde{H}_I = \tilde{S} \otimes \tilde{B}$ $\qquad Tr_B\{B \cdot \bar{\varrho}_B\} = 0 \leftarrow$ can always be fulfilled

system $\quad$ bath $\qquad \tilde{B} = \sum_k (h_k \tilde{b}_k + h.c.)$

$$H_n = H_S + H_B + \sum_\alpha S_\alpha \otimes B_\alpha \qquad Tr_B\{B_\alpha \cdot \bar{\varrho}_B\} \neq 0$$

$$= H_S + \sum_\alpha g_\alpha \cdot S_\alpha + H_B + \sum_\alpha S_\alpha \otimes (B_\alpha - g_\alpha \cdot \mathbb{1})$$

$$\boxed{\dot{\tilde{\varrho}}_S = -Tr_B\left\{\int_0^t [\tilde{H}_I(t), [\tilde{H}_I(t'), \tilde{\varrho}_S(t') \otimes \bar{\varrho}_B]]dt'\right\} \quad \text{NMME}}$$

- ME: $\varrho_S(t')$ needs to be known (integro-diff. eqn.)
- violations of positivity possible
- 1st order DE for DM

example: $\dot{\tilde{\varrho}}_S = -\int_0^t [(a e^{-i\Omega t} + h.c.), (a \cdot e^{-i\Omega t'} + h.c.) \cdot \tilde{\varrho}_S(t')] \times$

$$\times \sum_n |k_n|^2 [e^{-i\omega_n(t-t')}(1+n_B(\omega_n)) + e^{+i\omega_n(t-t')} n_B(\omega_n)] + h.c.$$

$$Tr_B\{b_n^+ b_q \bar{\varrho}_B\} = \delta_{nq} \cdot n_B(\omega_n)$$

$$Tr_B\{b_n^+ b_q^+ \bar{\varrho}_B\} = 0$$

trick

$$\boxed{\text{spectral function} \quad \Gamma(\omega) = 2\pi \sum_n |k_n|^2 \delta(\omega - \omega_n) \quad \omega_n > 0 \leadsto \omega > 0}$$

res. frequencies; coupling strengths

$$\sum |k_n|^2 [\ldots] = \frac{1}{2\pi}\int_0^\infty \{\Gamma(\omega)[1+n_B(\omega)]e^{-i\omega(t-t')}d\omega + \Gamma(\omega) n_B(\omega) e^{+i\omega(t-t')}\}d\omega$$

analyt. cont. $\Gamma(-\omega) = -\Gamma(+\omega)$

$$n_B(-\omega) = -[1+n_B(+\omega)]$$

$$\leadsto \frac{1}{2\pi}\int_{-\infty}^{\infty} \underbrace{\Gamma(\omega)[1+n_B(\omega)]}_{\gamma(\omega):\ FT\ of\ C(\tau)} e^{-i\omega(t-t')}d\omega = C(t-t') \quad \text{res. corr. function}$$

if $\gamma(\omega)$ is flat $\leadsto$ $C(\tau)$ is highly peaked if $C(t-t') \approx C_0\, \delta(t-t')$

$$\boxed{\begin{array}{l}\text{Markov approximation} \\ \rho_S(t') \approx \rho_S(t) \quad \mathrm{I} \\ \int_0^t dt' [\dots] \approx \int_0^\infty dt [\dots] \quad \mathrm{II}\end{array}}$$

Redfield-II ME

drop all oscillating terms in t

$$\dot{\tilde{\rho}}_S(t) = \int_0^\infty [(a e^{-i\Omega t} + h.c.), (a e^{-i\Omega(t-\tau)} + h.c.)\, \tilde{\rho}_S(t)]\, C(\tau)\, d\tau + h.c.$$

not yet LGKS form!

need another approx. in general

$\rightsquigarrow$ "secular approximation" (analogous to rotating wave approx.)

$$\dot{\tilde{\rho}}_S = -\int_0^\infty d\tau \left([a, a^+ \tilde{\rho}_S(t)] e^{-i\Omega\tau} + [a^+, a \tilde{\rho}_S(t)] e^{+i\Omega\tau} \right) C(\tau)\, d\tau + h.c.$$

Lindblad form

to see that: insert $C(\tau) = \frac{1}{2\pi} \int \overbrace{\Gamma(\omega)\,[1 + n_B(\omega)]}^{\gamma(\omega)} e^{-i\omega\tau}\, d\omega$

$$\frac{1}{2\pi} \int_0^\infty e^{\pm i\omega\tau}\, d\tau = \frac{1}{2} \delta(\omega) \pm \frac{i}{2\pi} \mathcal{P} \frac{1}{\omega}$$

$$\dot{\tilde{\rho}}_S = \gamma(+\Omega)\left[a \tilde{\rho}_S a^+ - \tfrac{1}{2}\{a^+ a, \tilde{\rho}_S\}\right]$$
$$\gamma(-\Omega)\left[a^+ \rho_S a - \tfrac{1}{2}\{a a^+, \rho_S\}\right]$$
$$- \frac{i}{2\pi} \int \mathcal{P} \frac{\gamma(\omega)}{\Omega - \omega}\, d\omega\, [a^+ a, \tilde{\rho}_S] + \frac{i}{2\pi} \int \mathcal{P} \frac{\gamma(\omega)}{\omega + \Omega} [a a^+, \tilde{\rho}_S]$$

Lamb-shift $= \mathcal{O}(h_a^2)$

$\rightsquigarrow$ example derived

+ $\tilde{\Omega} = \Omega + \Delta\Omega$

+ $\gamma(+\Omega) = \Gamma(\Omega)\,[1 + n_B(+\Omega)]$

$\gamma(-\Omega) = -\Gamma(+\Omega)\,(-[n_B(+\Omega)]) = \Gamma(\Omega) \cdot n_B(\Omega)$

$$\frac{\gamma(\Omega)}{\gamma(+\Omega)} = \frac{n_B(\Omega)}{1 + n_B(\Omega)}$$

$e^{-\beta\Omega}$