REP: Z-Zahl-Messung

Res.-Obs. $\hat O = \sum_e O_e |e\rangle\langle e|$ Reserv.-EZ

$[\hat O, H_B] = 0$

$\bar\rho_B = \sum_e |e\rangle\langle e| \rho_B |e\rangle\langle e|$

· MGF(z)

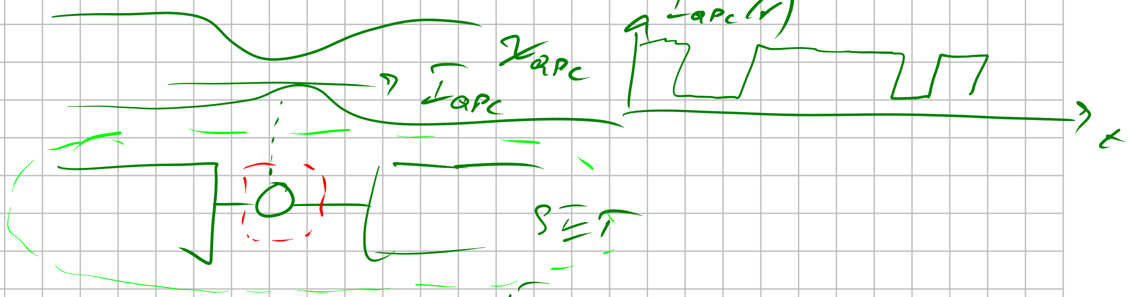
$\to$ allg. CF: $C^z_{AB}(\tau) = \mathrm{Tr}\{ e^{-i\hat O z} \tilde B_A(\tau) e^{+i\hat O z} B_B \bar\rho_B \} \to C_{AB}(\tau - z)$, $\quad \hat O = H_B$

$\gamma^z_{AB}(\omega) = \int C^z_{AB}(\tau)\, e^{+i\omega\tau} d\tau \to e^{+i\omega z} \gamma_{AB}(\omega)$

· SET

$$Z_{SET}(\chi) = \begin{pmatrix} -\Gamma_L(1-f_L) - \Gamma_R(1-f_R) & \Gamma_L f_L e^{i\chi_L} + \Gamma_R f_R \\ \Gamma_L(1-f_L) e^{-i\chi_L} + \Gamma_R(1-f_R) & -\Gamma_L f_L - \Gamma_R f_R \end{pmatrix}$$

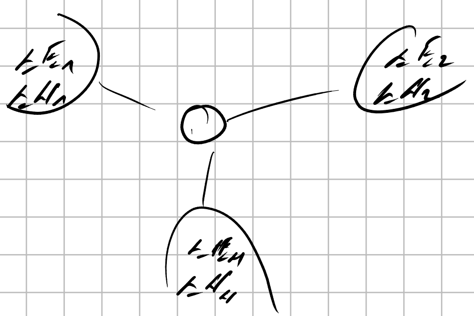
· QPC



$$Z_{QPC}(\chi) = \begin{pmatrix} 1 & \\ 0 & |1-\alpha|^2 \end{pmatrix} \left[(e^{-i\chi}-1) T_{R\to L} + (e^{+i\chi}-1) T_{L\to R} \right]$$

$Z_{QPC}(0) = 0$

3.3. Symmetrien in der Zählstatistik



BMS $\quad Z = \begin{pmatrix} Z_{pop}(\vec\chi, \vec\xi) & 0 \\ 0 & Z_{coh} \end{pmatrix}$ (Teilchen: $\vec\chi$, Energie: $\vec\xi$)

Pauli-ME $\quad \dot P_a = \sum_\nu \sum_{b \neq a} \gamma^{(\nu)}_{ab,ab}\, e^{i(N_a-N_b)\chi_\nu} e^{i(E_a-E_b)\xi_\nu} P_b - \sum_\nu \sum_{b\neq a} \gamma^{(\nu)}_{ba,ba} P_a$

$$\frac{\gamma^{(\nu)}_{ab,ab}}{\gamma^{(\nu)}_{ba,ba}} = e^{\beta_\nu [(E_b - E_a) - \mu_\nu (N_b - N_a)]}$$

$\hookrightarrow \dot{\vec P} = W(\vec\chi, \vec\xi) \cdot \vec P$

$$\boxed{W^T(-\vec\chi - i\vec A, -\vec\xi - i\vec B) = W(\vec\chi, \vec\xi)}$$

$\vec A = \begin{pmatrix} -\mu_1 \cdot \beta_1 \\ \vdots \\ -\mu_M \cdot \beta_M \end{pmatrix} \quad \vec B = \begin{pmatrix} \beta_1 \\ \vdots \\ \beta_M \end{pmatrix}$ "Affinitäten"

$$\bar{\sigma}_i = -\sum_\nu \beta_\nu \left[\bar{I}_E^\nu - \mu_\nu \bar{I}_\mu^{(\nu)}\right] = -\sum_\nu \left(B_\nu I_E^\nu + A_\nu I_\mu^\nu\right)$$

$$C(z,t) \approx \lambda_{dom}(z)\cdot t$$

charakt. Polynom: $0 = |W(\vec{E},\vec{q}) - \lambda(\vec{E},\vec{q})|$

evaluate@shifted values $\longrightarrow$ $= |W(-\vec{E}-i\vec{A}, -\vec{q}-i\vec{B}) - \lambda(-\vec{E}-i\vec{A}, \vec{q}-i\vec{B})|$

transpose $\longrightarrow$ $= |W^T(-\vec{E}-i\vec{A}, -\vec{q}-i\vec{B}) - \lambda(\ldots)\cdot 1|$

symmetry $\longrightarrow$ $= |W(\vec{E},\vec{q}) - \lambda(-\vec{E}-i\vec{A}, -\vec{q}-i\vec{B})|$

$$\leadsto \boxed{\lambda(-\vec{E}-i\vec{A}, -\vec{q}-i\vec{B}) = \lambda(\vec{E},\vec{q})}$$

1D Bsp $\lim_{t\to\infty} P_n(t) = \frac{1}{2\pi}\int_{-\pi}^{+\pi} e^{C(z,t)}\, e^{-inz}\, dz \;\hat{=}\; FT$

$$C(-z-i\sigma,t) = C(+z,t)$$

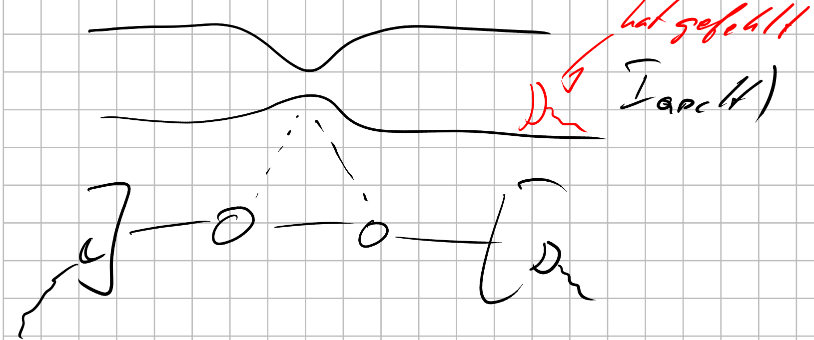
$$\lim_{t\to\infty} \frac{P_n(t)}{P_{-n}(t)} = e^{-n\cdot\sigma}$$

$$\hookrightarrow \lim_{t\to\infty} \frac{P_{(+t\vec{E},\, +t\vec{q})}(t)}{P_{(-t\vec{E},\, -t\vec{q})}(t)} = e^{-(t\vec{E}\cdot\vec{B} + t\vec{q}\cdot\vec{A})} = e^{-\sum \beta_\nu[tE_\nu - \mu_\nu t q_\nu]}$$

$t\,S \;\hat{=}$ Langzeit Entropie-Pr.

$$\boxed{\text{Crooks-Fluktuations-Theorem (quantitativer als 2. HS)} \quad \lim_{t\to\infty} \frac{P_{+tS}(t)}{P_{-tS}(t)} = e^{+tS}}$$

vorher: $\frac{d}{dt} S_{system}(t) + \frac{d}{dt}\sum_\nu S_\nu(t) \geq 0$ Aussage über Mittelwerte



4. Exakte Lösungen

+ spezielle Modelle / nicht-WW Modelle $\rightarrow$ für benchmarks + allg. Aussagen

+ sehr weitr. Aussagen aber nicht tiefgehend (nur glob. Eigenschaften)

4.1. Nichtperturbative Entropie-Produktion

setting

$$\rho(t) = U(t)\rho(0)U^\dagger(t) \qquad \dot U = -iH(t)U(t)$$

$$H(t) = H_S(t) + \sum_\nu H_I^{(\nu)}(t) + \sum_\nu H_B^{(\nu)}$$

$$\rho(0) = \rho_S(0) \bigotimes_\nu \bar\rho_\nu \qquad \bar\rho_\nu = \frac{e^{-\beta_\nu(H_B^\nu - \mu_\nu N_B^\nu)}}{Z_\nu}$$

$\uparrow$ unk. bei $t=0$

$\rightarrow$ leite 2. HS ab Esposito 2010 NJP

$$\Sigma(t) = -\mathrm{Tr}\{\rho(t)\ln\rho(t)\} \overset{\text{konst.}}{=} -\mathrm{Tr}\{\rho(0)\ln\rho(0)\}$$
$$= -\mathrm{Tr}_S\{\rho_S(0)\ln\rho_S(0)\} - \sum_\nu \mathrm{Tr}_\nu\{\bar\rho_\nu \ln\bar\rho_\nu\}$$

exact red. DM $\rho_S(t) = \mathrm{Tr}_{\{\nu\}}\{\rho(t)\}$ $\rho_\nu(t) = \mathrm{Tr}_{S,\nu'\neq\nu}\{\rho(t)\}$

$$S(t) = -\mathrm{Tr}_S\{\rho_S(t)\ln\rho_S(t)\}$$

$$\rightarrow S(0) = \Sigma(t) + \sum_\nu \mathrm{Tr}_\nu\{\bar\rho_\nu \ln\bar\rho_\nu\}$$

$$\Delta S(t) = S(t) - S(0) = -\mathrm{Tr}_S\{\rho_S(t)\ln\rho_S(t)\} - \Sigma(t) - \sum_\nu \mathrm{Tr}_\nu\{\bar\rho_\nu\ln\bar\rho_\nu\}$$
$$= -\mathrm{Tr}\{\rho(t)\ln\rho_S(t)\} + \mathrm{Tr}\{\rho(t)\ln\rho(t)\} - \sum_\nu \mathrm{Tr}_\nu\{\bar\rho_\nu\ln\bar\rho_\nu\}$$
$$= \mathrm{Tr}\{\rho(t)[\ln\rho(t) - \ln\rho_S(t)\bigotimes_\nu\bar\rho_\nu]\} + \sum_\nu \mathrm{Tr}_\nu\{[\rho_\nu(t) - \bar\rho_\nu]\ln\bar\rho_\nu\}$$
$$= \underbrace{D(\rho(t)\,\|\,\rho_S(t)\otimes\bar\rho_\nu)}_{\geq 0} \underbrace{- \sum_\nu \beta_\nu \mathrm{Tr}\{[\rho_\nu(t) - \bar\rho_\nu][H_B^{(\nu)} - \mu_\nu N_B^{(\nu)}]\}}_{+\sum_\nu \beta_\nu \Delta Q_\nu(t)}$$

$D(\rho,\sigma) = \mathrm{Tr}\{\rho\ln\rho - \rho\ln\sigma\} \geq 0$; $D(\rho,\rho) = 0$

$\Rightarrow$ $\Delta_i S(t) = D(\rho(t)\,\|\,\rho_S(t)\otimes\bar\rho_\nu) \geq 0$ aber $\frac{d}{dt}\Delta_i S(t) < 0$ ist möglich

misst die System-Res.-Korrelation

iii.) Austausch-Entropie

$$\Delta_e S = \sum_\nu \beta_\nu \Delta Q_\nu(t) \qquad \Delta Q_\nu = \langle H_B^\nu - \mu_\nu N_B^\nu \rangle_0 - \langle H_B^\nu - \mu_\nu N_B^\nu \rangle_t$$

iii.) $$\frac{d}{dt} \Delta_i S(t) = \frac{d}{dt}\left[S(t) - S(0) - \sum_\nu \beta_\nu \Delta Q_\nu \right]$$

$$= \frac{d}{dt} S(t) - \sum_\nu \beta_\nu \frac{d}{dt} \Delta Q_\nu$$

Lindblad

$$\xrightarrow{t\to\infty} 0 \boxed{- \sum_\nu \beta_\nu \left(I_E^\nu - \mu_\nu I_N^\nu \right) \geq 0}$$

(Lindblad) Verletzung $\stackrel{?}{\hat{=}}$ nicht-Markov-Dynamik

iv.) $$S_\nu(t) = -\mathrm{Tr}_\nu\{\rho_\nu(t) \ln \rho_\nu(t)\}$$

$$S_c(t) = \Sigma(t) - S(t) - \sum_\nu S_\nu(t) \leq 0$$

$$D(\rho(t) \| \rho_S(t) \otimes \rho_\nu(t)) = -\Sigma(t) + S(t) + \sum_\nu S_\nu(t) = -S_c(t) \geq 0$$

$$\Delta_i S(t) + S_c(t) = \sum_\nu D(\rho_\nu(t) \| \bar{\rho}_\nu) \geq 0$$

$$\Delta_i S(t) \geq -S_c(t) \geq 0$$