TECHNISCHE UNIVERSITÄT DRESDEN
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Relativistic Quantum Field Theory

Quiz

Problem 1: (*Green function*)

Consider the inhomogeneous wave equation

$$\left(\frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \nabla^2\right) G(\mathbf{r}, t) = 4\pi\delta^3(\mathbf{r})\delta(t) \tag{1}$$

a) Utilizing Fourier decomposition of the Green function $G(\mathbf{r}, t)$ and the delta distributions with respect to spatial and temporal dimensions, derive an integral representation for $G(\mathbf{r}, t)$.

b) The integrals feature simple poles along the real axis that necessitate treatment using complex analysis. Introduce a small parameter $\mathrm{i}\epsilon$ to displace the pole into the complex plane and proceed with the evaluation of the integrals. Utilize the residue theorem for evaluation.

c) Depending on whether the pole is shifted into the lower or upper half-plane, distinct outcomes for $G(\mathbf{r}, t)$ are obtained. What is the physical interpretation of these differing results?

Problem 2: (*Perturbation theory*)

Consider the Hamiltonian

$$\hat{H} = \hat{H}_0 + \hat{H}_I \tag{2}$$

with $\hat{H}_0 = (\hat{p}^2 + \omega^2\hat{x}^2)/2$ and $\hat{H}_I = \lambda\hat{x}^4$. The operators $\hat{x}$ and $\hat{p}$ satisfy the canonical commutation relations.

a) Diagonalize the operator $\hat{H}_0$ by expressing the operators $\hat{x}$ and $\hat{p}$ in terms of appropriate creation and annihilation operators $\hat{a}^\dagger$ and $\hat{a}$, satisfying the commutation relation $[\hat{a}, \hat{a}^\dagger] = 1$.

b) Apply stationary perturbation theory up to the leading order in λ to determine the ground state energy E_0 and the ground state for the total Hamiltonian $\hat{H}$.

c) At $t = 0$, the system is in the vacuum state $|0\rangle$, and the coupling constant λ is suddenly increased from 0 to a finite value. Transform the Schrödinger equation into the interaction picture and determine the leading order correction to the wave function.

Problem 3: (*Free particle*)

Find the Lagrangian for a free particle in three dimensions

a) Obtain the from the corresponding Hamilton function the Hamiltonian using the correspondence principle.

b) What is the form of the wavefunction $\psi(x, y, z, t)$ that satisfies the Schrödinger equation for the given system?