TECHNISCHE UNIVERSITÄT DRESDEN
HZDR HELMHOLTZ ZENTRUM DRESDEN ROSSENDORF
Faculty of physics
Prof. Dr. R. Schützhold
Dr. F. Queisser

Relativistic Quantum Field Theory

Exercise 3

Problem 1: (*Complex scalar field*)

Given is the Lagrangian of a complex scalar field

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi) \cdot (\partial^\mu \phi)^* - \frac{m^2}{2}|\phi|^2 .$$

a) Find the global symmetry of the Lagrange density and the corresponding Noether current.

b) How does the Lagrange density change when the symmetry parameter depends on space-time-coordinates?

c) Replace the partial derivative by a covariant derivative $\partial_\mu \to \partial_\mu + iqA_\mu$. How does the vector potential have to transform such that the Lagrangian remains invariant?

Problem 2: (*Schrödinger field*)

Given is the Schrödinger field with the Hamiltonian ($\hbar = 1$)

$$\hat{H} = \int d^3r \left[\frac{(\nabla \hat{\Psi}^\dagger(\vec{r})) \cdot (\nabla \hat{\Psi}(\vec{r}))}{2m} + V(\vec{r})\hat{\Psi}^\dagger(\vec{r})\hat{\Psi}(\vec{r}) \right]$$

and the commutation relations

$$[\hat{\Psi}(\vec{r}_1), \hat{\Psi}(\vec{r}_2)] = [\hat{\Psi}^\dagger(\vec{r}_1), \hat{\Psi}^\dagger(\vec{r}_2)] = 0 \text{ and } [\hat{\Psi}(\vec{r}_1), \hat{\Psi}^\dagger(\vec{r}_2)] = \delta^{(3)}(\vec{r}_1 - \vec{r}_2)$$

a) Diagonalize the Hamiltonian $\hat{H}$ as shown in the lecture.

b) What is the ground state of the system and is there a non-zero vacuum energy?

c) How does the Fock space of the Schrödinger field look like?

d) How does the the general one-particle state read?

Problem 3: (*Casimir effect*)

In the range $0 \leq x \leq l$ a real massless scalar field in $1+1$ dimensions is defined through the Lagrangian

$$\mathcal{L} = \frac{1}{2}[(\partial_t \phi)^2 - (\partial_x \phi)^2]$$

The boundary conditions of the field are given by $\phi(x=0) = \phi(x=l) = 0$.

a) Quantize the field and determine the modes f_I and the associated frequencies Ω_I. How should the f_I be normalized?

b) Calculate the regularized energy of the ground state which is given by

$$E_\infty^\lambda = \frac{1}{2}\sum_I \Omega_I e^{-\lambda \Omega_I}$$

with the regulator $\lambda > 0$.

c) Approximate E_∞^λ for small λ by expanding E_∞^λ into a Laurent series up to λ^1. What is the interpretation of the quantities?