

Relativistic Quantum Field Theory

Exercise 4

Problem 1: (*Massless scalar field in 1 + 1 dimensions*)

Calculate for a real massless scalar field in 1 + 1 dimensions the retarded and the advanced Green function and the Wightman function $W(t_1, x_1; t_2, x_2)$. Evaluate the integrals explicitly.

Problem 2: (*Massless scalar field in 3 + 1 dimensions*)

Calculate for a real massless scalar field in 3 + 1 dimensions the retarded and the advanced Green function and the Wightman function $W(t_1, \vec{r}_1; t_2, \vec{r}_2)$. Evaluate the integrals explicitly.

Problem 3: (*Linearized gravity*)

In general relativity, the Einstein-Hilbert action can be linearized if the gravitational field is weak. Then the metric of the four-dimensional space-time can be linearized around the Minkowski metric, i.e. $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$. In leading-order approximation, the action becomes

$$S = \int d^4x \left(\frac{1}{2} \partial_\sigma h_{\mu\nu} \partial^\sigma h^{\mu\nu} - \partial_\sigma h_{\mu\nu} \partial^\nu h^{\mu\sigma} + \partial_\sigma h^{\mu\sigma} \partial_\mu h - \frac{1}{2} \partial_\mu h \partial^\mu h \right),$$

where $h = h^\mu{}_\mu$. Derive the equations of motion for $h_{\mu\nu}$. Show that the linearized theory is invariant under the gauge symmetry

$$h_{\mu\nu} \rightarrow h_{\mu\nu} + \partial_\mu \Lambda_\nu + \partial_\nu \Lambda_\mu.$$

Which form adopt the equations in the De Donder gauge where Λ is chosen such that $\partial^\mu h_{\mu\nu} - \partial_\nu h/2 = 0$ holds?