TECHNISCHE UNIVERSITÄT DRESDEN

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Relativistic Quantum Field Theory

Exercise 6

Problem 1: (*Perturbation theory*)

For problems which cannot be solved exactly, it is possible to use a perturbative expansion in powers of a small parameter such as the fine structure constant. This perturbative expansion can be understood as asymptotic series expansion. This can be clarified using a simple integral. Consider the integral

$$I(g) = \int_{-\infty}^{+\infty} \frac{dx}{\sqrt{2\pi}} \exp\left[-\frac{1}{2}x^2 - gx^4\right]$$

which can be written formally as a sum,

$$I(g) = \sum_{n=0}^{\infty} \frac{(-g)^n}{n!}\alpha_n$$

a) What can you tell about the convergence radius of this series?

b) Calculate the coefficients α_n and show that they can be interpreted as combinatorical factor to pair $4n$ objects. For the evaluation of the integrals, use the definition of the Gamma function and the relation

$$\Gamma\left(\frac{z}{2}\right)\Gamma\left(\frac{z+1}{2}\right) = \frac{\sqrt{\pi}}{2^{z-1}}\Gamma(z)$$

c) Use the Stirling formula for large n to show that

$$\frac{(-g)^n}{n!}\alpha_n \sim f(n) = \left(-\frac{16gn}{e}\right)^n \quad \text{for} \quad n \gg 1$$

From which order n_0 on is the perturbative expansion problematic? It might help to treat $|f(n)|$ as a continuous function of n.

Problem 2: (*Schrödinger field with interaction*)

Consider the Schrödinger field with the Hamiltonian ($\hbar = 1$)

$$\hat{H} = \int d^3r \left[\frac{(\nabla\hat{\Psi}^\dagger(\vec{r}))\cdot(\nabla\hat{\Psi}(\vec{r}))}{2m}\right] + \hat{H}_{\text{WW}}$$

and the interaction Hamiltonian

$$\hat{H}_{\text{WW}} = g\int d^3r d^3r' \hat{\psi}^\dagger(\vec{r})\hat{\psi}^\dagger(\vec{r}\,')W(\vec{r}-\vec{r}\,')\hat{\psi}(\vec{r}\,')\hat{\psi}(\vec{r})$$

with the interaction potential $W(\vec{r}-\vec{r}\,')$. Discuss the graphic representations for the orders g^0, g^1 and g^2 (Feynman diagrams) of the interaction, with the incoming and the outgoing states containing 0, 1, 2 or 3 particles.