

Relativistic Quantum Field Theory

Exercise 7

Problem 1: (*Wick-Theorem*)

Using the Wick theorem, calculate the following expressions

- a) ${}_0\langle 0 | \mathcal{T} \{ \hat{\phi}(x_1^\mu) \hat{\phi}(x_2^\mu) \hat{\phi}^2(x_3^\mu) \} | 0 \rangle_0$
- b) ${}_0\langle 0 | \mathcal{T} \{ \hat{\phi}(x_1^\mu) \hat{\phi}(x_2^\mu) \hat{\phi}^4(x_3^\mu) \} | 0 \rangle_0$
- c) ${}_0\langle 0 | \mathcal{T} \{ \hat{\phi}(x_1^\mu) \hat{\phi}(x_2^\mu) \hat{\phi}^6(x_3^\mu) \} | 0 \rangle_0$

where $|0\rangle_0$ is the unperturbed vacuum of the free scalar field.

Problem 2: (ϕ^3 -theory)

Consider a scalar field theory with the Lagrangian density

$$\mathcal{L} = \frac{1}{2}(\partial_\mu\phi)(\partial^\mu\phi) - \frac{m^2}{2}\phi^2 - \frac{\lambda}{3!}\phi^3.$$

Draw all possible Feynman diagrams up to the order λ^2 and give the associated mathematical expressions for

a)

$$\lambda \langle 0 | \mathcal{T} \{ \hat{\phi}(x_1^\mu) \hat{\phi}(x_2^\mu) \} | 0 \rangle_\lambda$$

b)

$$\lambda \langle 0 | \mathcal{T} \{ \hat{\phi}(x_1^\mu) \hat{\phi}(x_2^\mu) \hat{\phi}(x_3^\mu) \} | 0 \rangle_\lambda$$

c)

$$\lambda \langle 0 | \mathcal{T} \{ \hat{\phi}(x_1^\mu) \hat{\phi}(x_2^\mu) \hat{\phi}(x_3^\mu) \hat{\phi}(x_4^\mu) \} | 0 \rangle_\lambda$$

d) Which of the diagrams are divergent?

e) Draw all Feynman diagrams in leading order in λ for two incoming particles with the four-momentum k_1^μ and k_2^μ and two outgoing particles with the four-momentum k_3^μ and k_4^μ and give the associated mathematical expression. All k_i^μ are different from each other.