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Relativistic Quantum Field Theory

Exercise 8

Problem 1: (*Amplitudes*)

The amplitude of a scattering process is the sum of the expressions which originate from the amputated Feynman diagrams. Discuss the following cases for the ϕ^4-theory and give the mathematical expressions for the amplitudes to leading order.

a) One particle in the initial state and three particles in the final state
b) Two particles in the initial state and three particles in the final state
c) Two particles in the initial state and four particles in the final state

Problem 2: (*Propagator correction*)

In ϕ^4-theory in D dimensions with the self interaction

$$\mathcal{L}_{\text{int}}^D = -\frac{\lambda \mu^{4-D}}{4!}\,\phi^4(x)\,,$$

the leading order correction to the propagator is determined by the tadpole diagram which corresponds to the mathematical expression

$$\Sigma_{\text{tad}} = \frac{\lambda \mu^{4-D}}{2}\int \frac{d^D k}{(2\pi)^D}\frac{i}{k_\mu k^\mu - m^2 + i\epsilon}\,.$$

Perform a so-called Wick rotation by substituting the energy component of the four-momentum in the loop integral with $k^0 = ik_E^0$ and express the integral in spherical coordinates. The angular integral in D dimensions is given by

$$\int d\Omega = \frac{2\pi^{\frac{D}{2}}}{\Gamma\left(\frac{D}{2}\right)}$$

where Γ denotes the Gamma function. Evaluate the remaining integral by employing the definition of the Euler-beta function

$$B(x,y) = \int_0^\infty dt \frac{t^{x-1}}{(1+t)^{x+y}} = \frac{\Gamma(x)\Gamma(y)}{\Gamma(x+y)}\,.$$

In order to specify the divergent part of the expression, expand the expression for $4 - D = \epsilon \ll 1$. The expansion for the Gamma function around zero is given by $\Gamma(\epsilon) = 1/\epsilon - \gamma + \mathcal{O}(\epsilon)$ with $\gamma = 0.577215...$ being the Euler-Mascheroni constant.

Problem 3: (*Vertex correction*)

The leading order vertex correction is of order λ^2. Give for two incoming particles and two outgoing particles (with different momenta p_1^μ, p_2^μ, p_3^μ and p_4^μ) the second-order diagrams in λ and the associated mathematical expressions in D dimensions. Carry out the Wick rotation as in the first problem and employ the Feynman identity

$$\frac{1}{AB} = \int_0^1 \frac{dx}{[xA + (1-x)B]^2}\,.$$

Perform the momentum integral and separate the finite part from the divergent part. The correction should have the same mass dimension as $\lambda\mu^{4-D}$.