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Relativistic Quantum Field Theory

Exercise 11

Problem 1: (*Scalar electrodynamics*)

Obtain to leading order all possible Feynman diagrams in scalar electrodynamics for the following processes and give the corresponding mathematical expression for the total amplitude

a) Particle-particle scattering

b) Particle-antiparticle annihilation to two photons

c) Particle-photon scattering

Problem 2: (*Schrödinger field*)

Consider the Schrödinger field with the Lagrange density

$$\mathcal{L} = i\psi^*(\vec{r},t)\frac{\partial\psi(\vec{r},t)}{\partial t} - \frac{1}{2m}\left(\nabla\psi^*(\vec{r},t)\right)\cdot\left(\nabla\psi(\vec{r},t)\right) .$$

a) Couple the electromagnetic field to the Schrödinger field in analogy to scalar electrodynamics. Assume that the scalar potential is spherically symmetric and derive the Hamiltonian of the system.

b) Utilizing first-order time-dependent perturbation theory, investigate the possible transitions when the Schrödinger field is initially in the

- 1s state $(n = 1, l = 0, m = 0)$
- 2s state $(n = 2, l = 0, m = 0)$
- 2p state $(n = 2, l = 1, m = 0)$

Assume that there is no photon in the initial state. (Hint: Decompose the Schrödinger field according to $\hat{\psi} = \sum_I \hat{a}_I(t) f_I(\vec{r})$ where the functions $f_I(\vec{r})$ are eigenfunctions for $-\triangle/(2m) + \phi(r)$ with $\phi(r)$ being the scalar potential. You can employ the dipole approximation. It is not necessary to evaluate all integrals explicitly.

c) What changes qualitatively when there are initially n photons in the initial state, (The initial state has then the form $|\text{in}\rangle \propto \left(\hat{b}^\dagger_{\vec{p},\lambda}\right)^n |I, 0\rangle$ where I specifies the initial state of the Schrödinger field.)

d) The interaction Hamiltonian involves a component that exhibits a quadratic dependence on the photon field. Explore the qualitative distinctions in permitted transitions resulting from this quadratic interaction in contrast to the interaction which is linear in the photon field.

e) Obtain for the special case $V(|\vec{r}|) = 0$ all Feynman rules of the system. Which diagrams are not possible in comparison to scalar electrodynamics? For simplicity you may set $A_0 = 0$.