Sebastian Schmidt

Aufgabe 1

a) $log_3 \frac{1}{27} = \frac{log \frac{1}{27}}{\log 3} = \underline{\underline{-3}}$

b) $log_{10}\sqrt{10} = \frac{\log\sqrt{10}}{\log 10} = \underline{\underline{0{,}5}}$

c) $(\sqrt{20})^4 \times \frac{1}{20} - (99^{20})^0 = 20^2 \times \frac{1}{20} - 1 = 20 - 1 = \underline{\underline{19}}$

d) $y = a^b - c$

i. $a^b = y + c \;\; b = log_a(y + c)$

ii. $a^b = y + c \;\; a = \sqrt[b]{(y + c)}$

Aufgabe 2

a) $\sum_{i=1}^{7} i^2 = 49 + 36 + 25 + 16 + 9 + 4 + 1 = \underline{\underline{140}}$

b) $\prod_{i=1}^{3}(3 - i) = (3 - 3) \times (3 - 2) \times (3 - 1) = 0 \times 1 \times 2 = \underline{\underline{0}}$

Aufgabe 3

$$\sum_{i=1}^{n}(2i - 1) = n^2$$

i. Induktionsanfang: $n = 1$: $(2 \times 1 - 1) = 1 = 1^2$

ii. Voraussetzung: gilt für $n = k$

$$(2 \times 1 - 1) + \cdots + (2k - 1) = k^2$$

iii. Behauptung: gilt für $(n = k + 1)$

$$(2 \times 1 - 1) + \cdots + (2k - 1) + (2(k + 1) - 1) = (k + 1)^2$$

iv. Beweis: $V \Longrightarrow B$

$$1 + 3 + 5 + \cdots + (2k - 1) + (2k + 1) = k^2 + 2k + 1$$

$$\cdots = (k + 1)^2 \; q.e.d.$$

Aufgabe 4

$$\sum_{i=0}^{n} 2^i = 2^{n+1} - 1$$

i. Induktionsanfang: $n = 2: 2^0 + 2^1 + 2^2 = 7 = 2^{2+1} - 1$

ii. Voraussetzung: gilt für $n = k$

$$2^0 + \cdots + 2^k = 2^{k+1} - 1$$

iii. Behauptung: gilt für $n = k + 1$

$$2^0 + \cdots + 2^k + 2^{k+1} = 2^{k+2} - 1$$

iv. Beweis: $V \Longrightarrow B$

$$1 + 2 + 4 + \cdots + 2^k + 2^{k+1} = 2^{k+1} - 1 + 2^{k+1}$$

$$\cdots = 2^{k+2} - 1 \; q.e.d.$$

Aufgabe 5

a) $(3{,}5 + 4{,}5i) + (1{,}5 - 2{,}5i) = 5 + 2i$

b) $(3{,}5 - 2{,}5i) - (7{,}25 - 1{,}5i) = -3{,}75 - i$

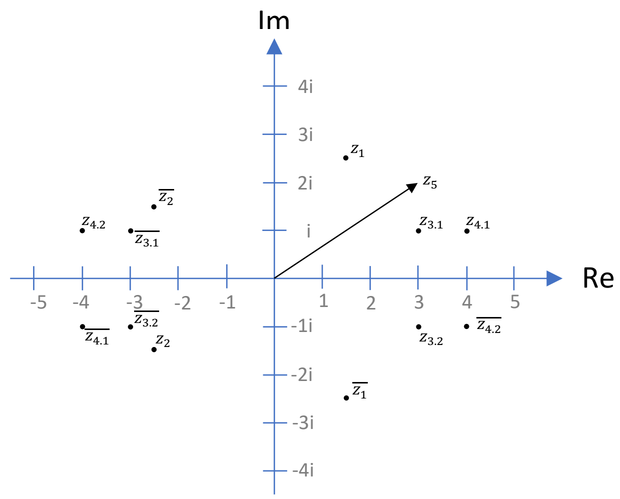
c) $(4 + 2i) \times (4 - 2i) = 20$

d) $(5 - 2i) \times (5 + 2i) = 29$

e) $(2 - i) \times (4 + 3i) = 12 + 2i$

f) $(1 + 2i) \div (3 + i) = \frac{(1+2i)\times(3-i)}{(3+1)\times(3-i)} = \frac{3-i+6i+2}{9-3i+3i+1} = \frac{5+5i}{10} = \frac{1+5i}{2}$

Aufgabe 6



$$z_6 = 2 + 2i = |z|(\cos\varphi + i\sin\varphi) = \sqrt{8}\left(\frac{2}{\sqrt{8}} + i\frac{2}{\sqrt{8}}\right)$$

$$z_7 = 4 + 0i = |z|(\cos\varphi + i\sin\varphi) = \sqrt{16}\left(\frac{4}{\sqrt{16}} + i\frac{0}{\sqrt{16}}\right) = 4(1 + 0) = 4$$