BEWEIS VON PROGRAMMIEREIGENSCHAFTEN

1. Gleichung: sum (foo xs) = 2 * sum xs - length x für jedes xs :: [Jnd]

Induktionsbasis: Sei xs = []

zz: sum (foo []) = 2 · sum [] - length []

sum (foo []) $\overset{2}{=}$ sum [] $\overset{6}{=}$ 0 ⇓

2 · sum [] - length [] $\overset{6}{=}$ 2 · 0 - length [] $\overset{10}{=}$ 2 · 0 - 0 = 0 ⇓

⇒ beide Seiten gleich, da 0 = 0

Induktionsschritt: Sei xs :: [Jnd] sodass gilt: sum (foo xs) = 2 · sum xs - length xs (IV)
Induktionsvoraussetzung

Für alle x : Jnd wird gezeigt, dass

sum (foo (x : xs) = 2 · sum (x : xs) - length (x : xs) gilt

sum (foo (x : xs)) $\overset{3}{=}$ sum (x : x : (-1) : foo xs)

$\overset{7}{=}$ x + x + (-1) + sum (foo xs)

$\overset{IV}{=}$ x + x + (-1) + 2 · sum xs - length xs

= 2x + 2 · sum xs + (-1) - length xs

= 2 (x + sum xs) - (1 + length xs)

$\overset{7}{=}$ 2 · sum (x : xs) - (1 + length xs)

$\overset{11}{=}$ 2 · sum (x : xs) - length (x : xs) □

2. a) Gleichung: $[x] ++ rev\ ys ++ rev\ xs = rev\ (xs ++ ys ++ [x])$ (H3)

$\overset{(H1),(H2)}{=} rev\ [x] ++ rev\ (xs ++ ys)$

$\overset{(H2)}{=} rev\ (xs ++ ys ++ [x])$ //

b) Gleichung: $preOrder\ t = rev\ (mPostOrder\ t)$

Induktionsbasis: Sei $x :: Ind$

$preOrder\ (Leaf\ x) = rev\ (mPostOrder\ (Leaf\ x))$

$\overset{4}{=} [x]$

$\overset{(H1)}{=} rev\ [x]$

$\overset{8}{=} rev\ (mPostOrder\ (Leaf\ x))$ //

Induktionsschritt: $l, r :: BinTree$, also gilt: $preOrder\ l = rev\ (mPostOrder\ l)$, $preOrder\ r = rev\ (mPostOrder\ r)$ } (IV)

$x :: Ind$

$\Rightarrow preOrder\ (Node\ x\ l\ r) = rev\ (mPostOrder\ (Node\ x\ l\ r))$

$\overset{(5)}{=} [x] ++ preOrder\ l ++ preOrder\ r$

$\overset{IV}{=} [x] ++ rev\ (mPostOrder\ l) ++ rev\ (mPostOrder\ r)$

$\overset{(H3)}{=} rev\ (mPostOrder\ r ++ mPostOrder\ l ++ [x])$

$\overset{(9)}{=} mPostOrder\ (Node\ x\ l\ r)$ //

□